Homework #11

Your solution t should contain three m-files, two functions and an m-script. No written report is required.

Based on your solution to Homework #7, create a function in Matlab that implements the Golden Section search method. Your function must have three arguments, a function handle (@) for the problem to be minimized and the two initial bracketing estimates.

Test your function by solving the following problem.

> A pipeline is to transfer crude oil from a tanker docking area to a large oil refinery. The power required to pump the oil, P in hp, is determined from
>
> $$P = 4 \times 10^{-13} \frac{w^3}{\rho^2 d^5}$$
>
> where w: oil flow rate in lb/hr
> ρ: density of the oil in lb/ft^3
> d: pipe diameter in ft
>
> The cost incurred, in dollars, is given by
>
> $$C = 10000d^2 + 170P$$
>
> where the first term on the right-hand side represents the capital cost of the pipeline and the second term represents the cost of the pump and its operation.
>
> Suppose the oil flow is set at 10^7 lb/hr and the oil density is 50 lb/ft^3. Find the pipe diameter that will minimize the cost. Using information on standard pipe sizes, find the size of pipe with inside diameter closest to your minimum. Compare the costs for your minimum and the actual (commercial) pipe.

Here is the information on the Golden Section search method from Homework #7:

The ratio of adjacent numbers in the Fibonacci series [0,1,1,2,3,5,8,13,...] approaches a value called the Golden Ratio,

$$GR = \frac{\sqrt{5} - 1}{2}$$

This Fibonacci numbers and the Golden Ratio appear in many different phenomena, both in nature and civilization[1]. In numerical methods, it occurs in an optimization technique called the Golden Section search method. In solving single nonlinear equations with a method like

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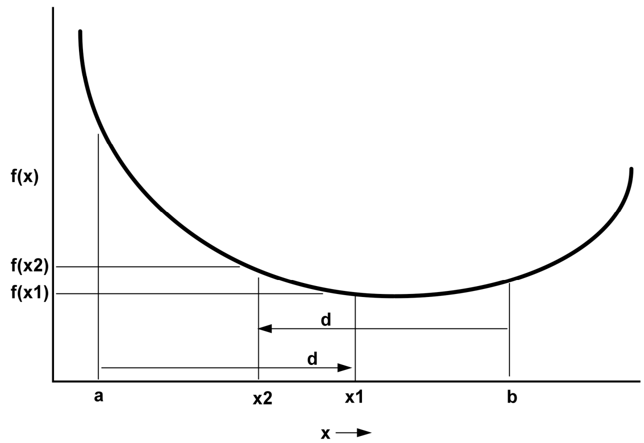
[1] A pinecone shows two spirals, five and eight of each. A daisy has 21 and 34. These are adjacent numbers in the Fibonacci sequence. The Golden Ratio is embodied in Greek architecture, e.g., in the design of the Parthenon based on a sequence of diminishing rectangles where the ratio of the lengths of the sides correspond to GR.

bisection, the goal was to find the value of the variable **x** which yields a *zero* of the function **f(x)**. *Single-variable optimization* has the goal of finding the value of the variable **x** which yields an *extremum*, either maximum or minimum, of the function **f(x)**.

The Golden Section method is perhaps the best, general-purpose, single-variable search technique. Just as with the bisection method, we start with two values of **x**, **a** and **b**, which bracket the extremum. See the figure below which shows the case for finding the minimum. The method presumes that there is one local extremum of **f(x)** in the interval from **a** to **b**.



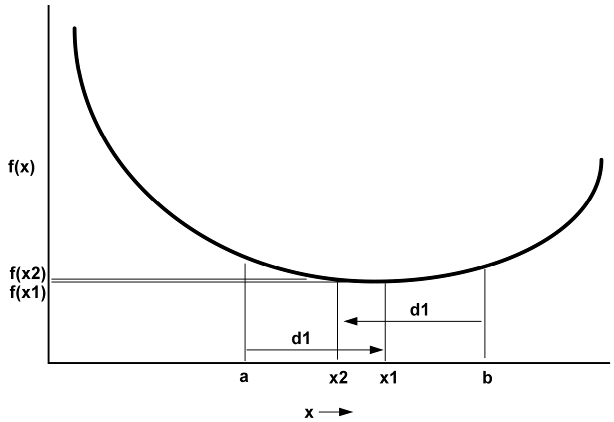
To start the search method, two **x** values interior to the interval, x_1 and x_2, are chosen according to the Golden Ratio. That is, **x1** is a distance **d** to the right of **a**, where

$$x_1 = d + a$$

and

$$d = \frac{\sqrt{5}-1}{2} \cdot (b-a)$$

and x_2 is the same distance **d** to the left of **b**. The function **f(x)** is evaluated at x_1 and x_2. If $f(x_1) < f(x_2)$, as in the figure, then the domain of **x** to the left of x_2, from **a** to x_2, can be eliminated as not containing the minimum. If it had occurred that $f(x_2) < f(x_1)$, then the domain of **x** to the right of x_1, from x_1 to **b**, can be eliminated. In the first case, x_2 would become the new **a** for the next round of the method. In the second case, x_1 would become the new **b** for the next round.



The next round of the method is shown in the figure above. The old $\mathbf{x}_2$ has become the new **a** and the old **b** becomes the new **b**. The *neat trick* that occurs now is that because the original $\mathbf{x}_1$ and $\mathbf{x}_2$ were chosen using the Golden Ratio, the old $\mathbf{x}_1$ becomes the new $\mathbf{x}_2$. That also means that we already have the value of $\mathbf{f(x_2)}$ already computed, since it is the same as **f(old** $\mathbf{x}_1$**)**. To carry the method forward, we now need to place the new $\mathbf{x}_1$. This is done with the same proportionality as before. As the method is repeated, the interval containing the extremum is reduced rapidly. In fact, each round it is reduced by a factor of the Golden Ratio (about 61.8%). That means that after 10 rounds, the interval is about 0.618^{10} or 0.008 or 0.8% -- after 20 rounds it is about 0.000066 or 0.0066%, quite small. Practically speaking, after 10 to 20 rounds, we have found the value of **x** which provides the extremum of **f(x)**.