

Homework #7

Use Excel and VBA to solve the following problem and document your solution in a report.

Include the following elements in your report using Microsoft Word and copy the Excel output into the Word document. When copying parts of Excel spreadsheets and graphs, it is best to initiate the copy in Excel and then use Home → Clipboard → Paste → Paste Special → Picture(Enhanced Metafile). You can rescale the figure to suit.

- statement of the problem (brief description of the problem in your own words)
- strategy for solution (describe your approach to solving the problem – what steps you will take – this should include a flowchart)
- basic data (present any basic parameter values that do not change in your solution)
- problem solution (this is the main part – your Excel/VBA solution – include comments)
- limitations (discuss the limitations of your solution – under what circumstances will it fail or give inaccurate results)
- references (cite any sources of information you used to solve the problem)
- independent check (confirm your VBA results with your Excel spreadsheet prototype result)

Submit your report in printed form and your Excel spreadsheet electronically to your TA as an email attachment.

Implementing the Golden Section search method using Excel and VBA

The ratio of adjacent numbers in the Fibonacci series [0, 1, 1, 2, 3, 5, 8, 13, ...] approaches a value called the Golden Ratio,

$$GR = \frac{\sqrt{5} - 1}{2}$$

It has the unique property that

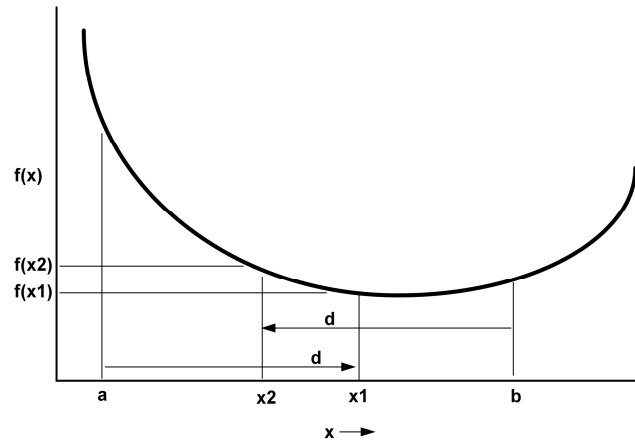
$$GR = \frac{1}{1 + GR} \quad (\text{test this!})$$

This Fibonacci numbers and the Golden Ratio appear in many different phenomena, both in nature and civilization¹. In numerical methods, it occurs in an optimization technique called the Golden Section search method. In solving single nonlinear equations with a method like bisection, the goal was to find the value of the variable **x** which yields a *zero* of the function **f(x)**. *Single-variable optimization* has the goal of finding the value of the variable **x** which yields an *extremum*, either maximum or minimum, of the function **f(x)**.

The Golden Section method is perhaps the best, general-purpose, single-variable search technique. Just as with the bisection method, we start with two values of **x**, **a** and **b**, which

¹ A pinecone shows two spirals, five and eight of each. A daisy has 21 and 34. These are adjacent numbers in the Fibonacci sequence. The Golden Ratio is embodied in Greek architecture, e.g., in the design of the Parthenon based on a sequence of diminishing rectangles where the ratio of the lengths of the sides correspond to GR.

bracket the extremum. See the figure below which shows the case for finding the minimum. The method presumes that there is one local extremum of $f(x)$ in the interval from a to b .



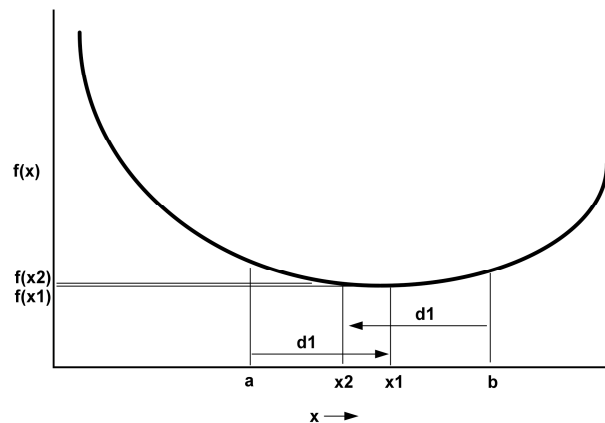
To start the search method, two x values interior to the interval, x_1 and x_2 , are chosen according to the Golden Ratio. That is, x_1 is a distance d to the right of a , where

$$x_1 = d + a$$

and

$$d = \frac{\sqrt{5} - 1}{2} \cdot (b - a)$$

and x_2 is the same distance d to the left of b . The function $f(x)$ is evaluated at x_1 and x_2 . If $f(x_1) < f(x_2)$, as in the figure, then the domain of x to the left of x_2 , from a to x_2 , can be eliminated as not containing the minimum. If it had occurred that $f(x_2) < f(x_1)$, then the domain of x to the right of x_1 , from x_1 to b , can be eliminated. In the first case, x_2 would become the new a for the next round of the method. In the second case, x_1 would become the new b for the next round.



The next round of the method is shown in the figure above. The old x_2 has become the new a and the old b becomes the new b . The *neat trick* that occurs now is that because the original x_1 and x_2 were chosen using the Golden Ratio, the old x_1 becomes the new x_2 . That also means

that we already have the value of $f(x_2)$ already computed, since it is the same as $f(\text{old } x_1)$. To carry the method forward, we now need to place the new x_1 . This is done with the same proportionality as before. As the method is repeated, the interval containing the extremum is reduced rapidly. In fact, each round it is reduced by a factor of the Golden Ratio (about 61.8%). That means that after 10 rounds, the interval is about 0.618^{10} or 0.008 or 0.8% -- after 20 rounds it is about 0.000066 or 0.0066%, quite small. Practically speaking, after 10 to 20 rounds, we have found the value of x which provides the extremum of $f(x)$.

1. Develop a table-based prototype of the Golden Section search technique on an Excel spreadsheet that solves the problem described below.

Suggestion: Lay out the spreadsheet table in a similar manner to the diagrams above, that is,

Iteration	a	x_2	$f(x_2)$	x_1	$f(x_1)$	b	d
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After an initialization row, you will need IF functions in the next row to determine a and b following the logic described above. The second row should be able to be copied down to complete, for example, 20 iterations of the method.

Problem description: Getting that ladder around the corner

A hallway of a certain width, for example, 2 meters, intersects another hallway at right angles. The second hallway has another width, for example, 3 meters. A contractor is concerned that their ladders may not be able to make it through the path from one hallway into another. Set up an Excel spreadsheet to determine the ladder of maximum length that can make it around the corner. Consider that the ladder will be held horizontally only. Your spreadsheet should be able to handle hallways of different widths.

Suggested approach: The longest ladder that will make it around the corner is also the shortest ladder that (barely) gets stuck. Set up a diagram of the hallways and a ladder (straight line) that touches two walls and the intervening corner. Label the ladder length L and the part of its length in one hallway x . Equate two ratios by similar triangles: $x / \text{width of its hallway} = L / \text{similar side of triangle}$. Solve this equality for L . Find the minimum L . That is the longest ladder than doesn't (quite) get stuck.

2. Develop a VBA function called GoldenSection that has input arguments **a** and **b** and returns the minimum of $f(x)$. You can use a similar approach to that used in the Class14 notes for bisection.