Pop Quiz 4

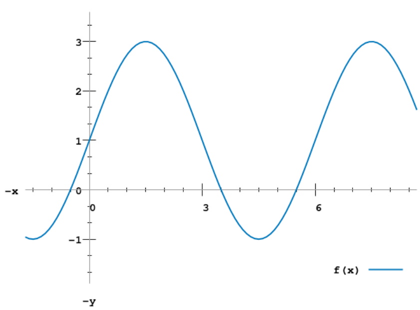
Precalculus: Functions, Geometry, Trigonometry, & Modelling

UCR Math-005-E01, Summer 2020

1. A population of lactobacillus is growing exponentially is a jar of freshly-made kimchi. Suppose we know that the size of the population triples every 48 hours, and that the population reaches 3 billion members after one week. How many lactobacillus were initially in the jar when the kimchi was made?

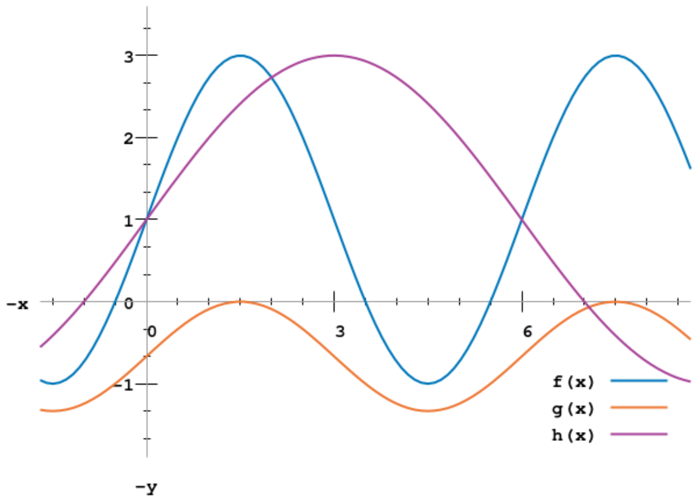
   Let t denote the number of days since the kimchi was made. Then we can model the size of the population of lactobacillus as
   $$P(t) = P_0 3^{\frac{1}{2}t}$$
   where P_0 is the initial size of the bacteria population. We know that $P(7) = 3 \times 10^9 = P_0 3^{\frac{1}{2}(7)}$, and so
   $$P_0 = 3 \times 10^9 \times 3^{-\frac{1}{2}(7)} = 3^{-\frac{5}{2}} \times 10^9 \approx 1.56 \times 10^8.$$

2. Below is the graph of a function f.



Sketch the graph of the function $g(x) = \frac{1}{3}f(x) - 1$. What is the minimum and maximum value of g on the interval $[0,7]$? Sketch the graph of the function $h(x) = f\left(\frac{1}{2}x\right)$. What are the roots of h on the interval $[-1,8]$?

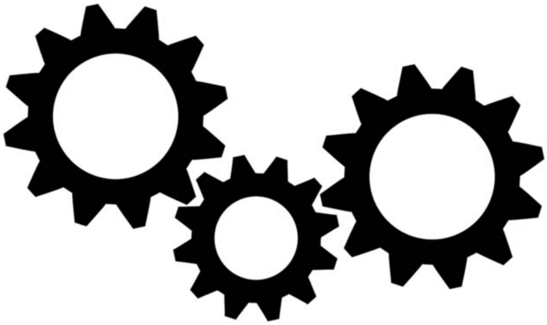
Here are the graphs of g and h.



Since the minimum and maximum values of f were -1 and 3 respectively, and g is simply a transformation of f resulting from a vertical scaling of $\frac{1}{3}$ (squishing the graph of f towards the x-axis by a factor of 3) and then a vertical shift of -1 moving the graph down by 1, the minimum and maximum values of g are $\frac{1}{3}(-1) - 1 = -\frac{4}{3}$ and $\frac{1}{3}(3) - 1 = 0$ respectively.

Similarly the roots of f appear to be at the x values $-\frac{1}{2}$, $\frac{7}{2}$, and $\frac{11}{2}$. Since h is a transformation of f, the result of multiplying its parameter by $\frac{1}{2}$ and thus visually stretching the graph horizontally outward from the y-axis by a factor of 2, we see that -1 and 7 and 11 will all be roots of h. But only the roots -1 and 7 fall inside the interval we're interested in.

3. Suppose you have three gears interlocked like so:



The radii of the gears in order from left to right is 5 miles, 2 miles, and 4 miles. Suppose that the gear on the far right is rotating at a rate of 366 revolutions per day. How fast is the gear on the left rotating in miles per hour?

Since the three gears are interlocked, their linear speeds, the speed at which their outermost edge is moving, are all the same So the speed of the left-most gear is the same as the speed of the right-most gear, which will be

$$\left(\frac{366\,\text{revolutions}}{\text{day}}\right)\left(\frac{8\pi\,\text{miles}}{1\,\text{revolution}}\right)\left(\frac{\text{day}}{24\,\text{hours}}\right) = 122\pi\frac{\text{miles}}{\text{hour}}$$