

Math113 College Algebra
First Midterm Exam
 Colorado Mesa University Fall 2023

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1. If $f(x) = 3x^2 - 1$, what is the value of $f(2)$?

$$\begin{aligned} f(2) &= 3(2)^2 - 1 \\ &= 12 - 1 = 11 \end{aligned}$$

2. This table reports input/output pairs for a function g . What is the value of $g(3)$?

x	-1	0	2	3	4	5
$g(x)$	3	22	0	9	32	-1

$$g(3) = 9$$

3. What's an equation for the line that intersects the y -axis at 3 and has a slope of $-\frac{5}{2}$?

$$y = -\frac{5}{2}x + 3$$

4. What's the value of the x -intercept of the line $y = 3x - 4$?

The x -intercept occurs where $y = 0$.

$$\begin{aligned} 0 &= 3x - 4 \\ \Rightarrow 4 &= 3x & \Rightarrow x &= \frac{4}{3} \end{aligned}$$

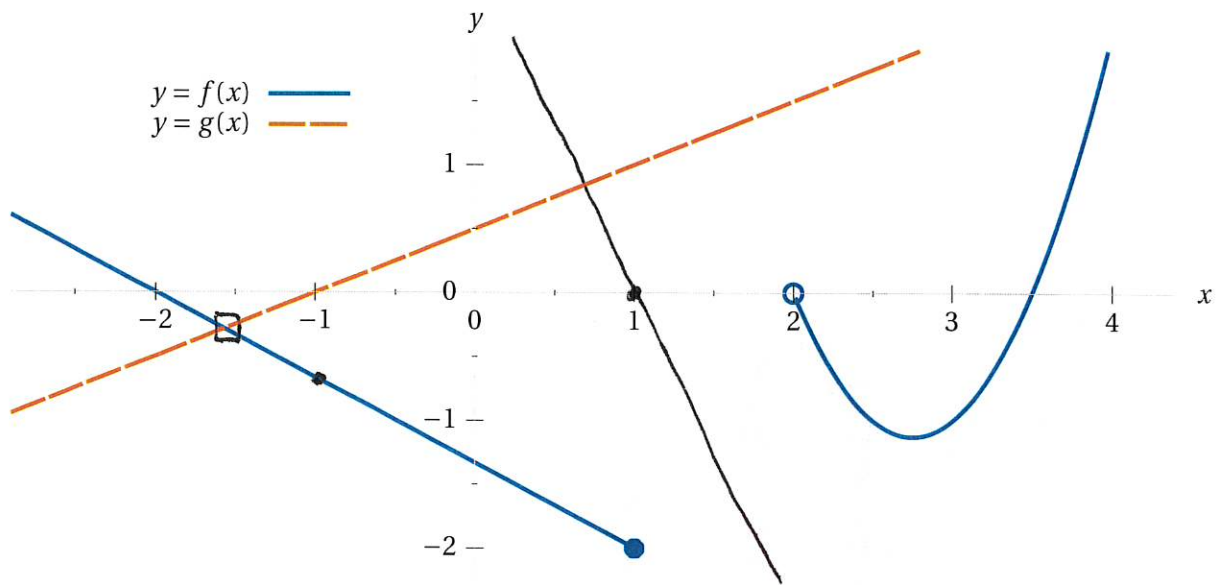
5. Does the point $(3, 4)$ lie on the graph of the function $h(x) = \frac{1}{2}x^2 - \frac{1}{5}x$? In one sentence, state how you figured out the answer this question.

Does $h(3) = 4$? If it does, then it lies on the graph. If not, then it doesn't.

$$\begin{aligned} h(3) &= \frac{1}{2}(3)^2 - \frac{1}{5}(3) \\ &= \frac{9}{2} - \frac{3}{5} \\ &= \frac{45 - 6}{10} = \frac{39}{10} \neq 4 \end{aligned}$$

So it doesn't.

6. Consider these graphs of functions f and g .



- (a) Estimating, what is $f(-1)$? $-\frac{3}{4}$
- (b) Estimating, for what values(s) of x does $f(x) = 0$? -2 and 3.5
- (c) What appears to be the domain of f ? $(-\infty, 1] \cup (2, \infty)$
- (d) What appears to be the range of f ? $[-2, \infty)$
- (e) Estimating, for what input(s) x do f and g share a common output? -1.5
- (f) Notice from its graph that g appears to be a linear function. What is a plausible formula for $g(x)$?

$$g(x) = \frac{1}{2}x + \frac{1}{2}$$

- (g) On this same set of axes above, plot a line perpendicular to the graph of g that passes through the point $(1, 0)$. What must the slope of this line be?

$$-2$$

7. Recall the formula for the future value A of an initial investment of P dollars at a simple annual interest rate r invested for t years is given by the formula $A = P(1 + rt)$.

(a) Solve this equation for t in terms of the other variables.

$$A = P(1 + rt)$$

$$\Rightarrow \frac{A}{P} = 1 + rt$$

$$\Rightarrow \frac{A}{P} - 1 = rt$$

$$\Rightarrow \frac{\frac{A}{P} - 1}{r} = t$$

(b) If you make an initial investment of \$900 at a simple annual interest rate of 0.7%, how many years until your investment appreciates to be worth \$1000?

$$t = \frac{\frac{A}{P} - 1}{r} \quad P = \$900 \quad A = \$1000 \quad r = 0.007$$

$$\Rightarrow t = \frac{\frac{1000}{900} - 1}{0.007} \approx 15.87301587$$

So about 16 years

8. In Spring 2023 a new Elkay ezH₂O water bottle refill station was installed outside the CMU Math Department main office. It has a digital display that reports the number of "Bottles Saved". Here's a record of that number on varies days since the beginning of that semester.

Days since the beginning of the semester	15	29	43	64	99	106
"Bottles Saved"	393	762	1193	1591	2890	3117

- (a) Using technology perform *linear regression* to find a linear model f for the number of bottles saved x days after the beginning of the semester. Write the formula for your model below with parameters rounded to two decimal places.

$$f(x) = 29.96x - 120.09$$

- (b) According to your model, at what *rate* are bottles saved per day?

↑
about 30 bottles/day

- (c) (INTERPOLATE) There's a notable gap in the data between $x = 64$ and $x = 99$. According to your model how many bottles had been saved by the 77th day of the Spring 2023 semester?

$$f(77) = 2186.83, \text{ so about } 2187 \text{ bottles}$$

- (d) (EXTRAPOLATE) According to the model, assuming this model remains accurate well beyond the domain of the data, on what day since the beginning of the Spring 2023 semester will the number of "Bottles Saved" surpass 6,890? In one sentence, explain how you figured this out.

"For what x will $f(x) = 6890$?"

$$6890 = 29.96x - 120.09$$

$$\Rightarrow x \approx 233.9816422$$

≈ 234 days since the beginning of Spring 2023

Just set the function's formula equal to 6890 and solve for x .

- (e) (BONUS) The data point for $x = 64$ is further from the linear model than the other data points. There is a good reason for this. Can you think of what this reason is?

Spring Break

9. What is an equation of the line that passes through the points $(-1, 2)$ and $(4, -4)$? In one sentence, state how you can tell from the equation if this line is *increasing* or *decreasing* as x increases.

$$(x_1, y_1) = (-1, 2)$$

$$(x_2, y_2) = (4, -4)$$

The slope of the line is

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (2)}{4 - (-1)} = \frac{-6}{5}$$

So an equation for the line is

$$(y + 4) = -\frac{6}{5}(x - 4) //$$

10. Demonstrate algebraically how to find the coordinates of the point where the lines corresponding to these two equations intersect.

$$3x + 4y = 17$$

$$x - 8y = 1$$

$$\begin{cases} 3x + 4y = 17 \\ x - 8y = 1 \end{cases}$$

$$x = 5$$

$$\Rightarrow \begin{cases} 6x + 8y = 34 \\ x - 8y = 1 \end{cases}$$

$$\Rightarrow (5) - 8y = 1$$

$$\Rightarrow -8y = -4$$

$$\Rightarrow 7x = 35$$

$$\Rightarrow y = \frac{1}{2} //$$

$$\Rightarrow x = 5 //$$

$$(5, \frac{1}{2})$$