Math253 Multivariable Calculus

Fifth Midterm Exam

Colorado Mesa University · 2025 Fall

NAME: ______________________________

1. Consider the curve C with parameterization $\boldsymbol{r}(t) = \left\langle \mathrm{e}^t, t^2 + t \right\rangle$ for $0 \leq t \leq 1$.

   (a) What is the value of the line integral $\int_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}$ for the vector field $\boldsymbol{F} = 3\mathbf{i} + 2x\mathbf{j}$?

   (b) What is the value of the line integral $\int_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}$ for the vector field $\boldsymbol{F} = \frac{y}{x}\mathbf{i} + \ln(x)\mathbf{j}$?

2. Let C be the triangle in the plane, oriented counterclockwise, with vertices $(0,0)$ and $(1,1)$ and $(1,3)$.

(a) What is the value of the line integral $\oint_C \boldsymbol{F}\cdot \mathrm{d}\boldsymbol{r}$ for the vector field $\boldsymbol{F} = (2xy-3y+1)\mathbf{i}+(x^2-3x)\mathbf{j}$?

(b) What is the value of the line integral $\oint_C \boldsymbol{F}\cdot \mathrm{d}\boldsymbol{r}$ for the vector field $\boldsymbol{F} = (3x^2y-\frac{1}{2}y^2)\mathbf{i}+(xe^y+3x^2)\mathbf{j}$?

3. Consider a sphere of radius one centered at the origin and consider the vector field $\boldsymbol{F}$ defined by the component formulas $\boldsymbol{F} = (xz^2)\mathbf{i} + (x^2y + 2y^3z)\mathbf{j} + (y^2z - 3y^2z^2)\mathbf{k}$. Imagine the surface of the sphere as a thin permeable membrane and the vector field as modelling a fluid flowing through the membrane.

(a) What is the total flux across the surface of this sphere?

(b) What does Stokes' Theorem have to say about the following surface integral that computes the total circulation of the vector field $\boldsymbol{F}$ restricted to the surface of the sphere?

$$\oiint_S \operatorname{curl} \boldsymbol{F} \cdot \mathbf{N} \, \mathrm{d}A$$

4. Consider the vector field $\boldsymbol{F} = \big(x\cos(y)\big)\mathbf{i} + \big(z^2\cos(y) + y^3\big)\mathbf{j} + \big(2z\sin(y)\big)\mathbf{k}$.

(a) What is the divergence of $\boldsymbol{F}$? What is the curl of $\boldsymbol{F}$?

(b) Suppose a particle in $\mathbf{R}^3$ starts at the point $(1,0,0)$ and then travels along straight line segments first to the point $(1,\pi,0)$, then to the point $(0,\pi,1)$, then to the point $(0,0,1)$, then back to $(1,0,0)$. Demonstrate Stokes' Theorem by calculating the total amount of work done by the vector field $\boldsymbol{F}$ moving the particle along this path.