Math253 Multivariable Calculus

Fourth Midterm Exam

Colorado Mesa University · 2025 Fall

NAME: _____

1. Consider the expanse bound below by the paraboloid $z = x^2 + y^2$ and bound above by the top sheet (where $z > 0$) of the two-sheeted hyperboloid $z^2 = 6 + x^2 + y^2$.

   (a) Compute the volume of this expanse. Hint: use polar/cylindrical coordinates.

   (b) Write (but don't evaluate) a double integral in polar coordinates that expresses the surface area of the lower/bottom portion of the expanse's boundary defined by the paraboloid.

Continued from the previous page. $z = x^2 + y^2$ $z^2 = 6 + x^2 + y^2$

(c) Suppose that T is a differentiable transformation of $\mathbf{R}^3$ that maps an expanse of volume π onto our expanse in question for which the Jacobian determinant, J_T, is constant. What must the value of J_T be?

(d) Determine the coordinates of the expanse's center of mass (centroid).

2. Demonstrate how to evaluate the following iterated integrals.

$$\int_0^{\frac{1}{\sqrt{2}}} \int_y^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy$$

$$\int_0^{25} \int_{\sqrt{y}}^{5} \sqrt{x^3+1}\,dx\,dy$$

3. The following integral calculates the volume of a tetrahedron in $\mathbf{R}^3$: $\int_0^{15} \int_0^{10-\frac{2}{3}x} \int_0^{6-\frac{2}{5}x-\frac{3}{5}y} \mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

(a) Sketch the tetrahedron, and label its vertices with their coordinates.

(b) Write (but do not evaluate) an iterated integral in rectangular coordinates that expresses the volume of this tetrahedron with respect to the order of differentials $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

(c) Write (but do not evaluate) an iterated integral in spherical coordinates that expresses the volume of this tetrahedron with respect to the order of differentials $\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\varphi$.