Math253 Multivariable Calculus

Third Midterm Exam

Colorado Mesa University · 2025 Fall

NAME: ______________________________

1. Write down an argument for why the following limit doesn't exist.

$$\lim_{(x,y)\to(0,0)} \frac{\left(x+3y\right)^2}{x^2+9y^2}$$

2. For a function $f\colon \mathbf{R}^2 \to \mathbf{R}$ defined by a formula $f(x,y)$, what is the precise, mathematical definition of $f_x(x,y)$, the partial derivative of f with respect to x?

3. Consider the point $(3,1)$ in $\mathbf{R}^2$ and consider the function $f: \mathbf{R}^2 \to \mathbf{R}$ defined by the following formula.

$$f(x,y) = 2xy - \tfrac{2}{3}x^3 - y^2$$

(a) At the given point, in what direction is the value of f *decreasing* most rapidly?

(b) What is the directional derivative of f at the given point in the direction $\langle -8, -15 \rangle$?

(c) At the given point, does there exist a direction in which the rate-of-change along the surface of the graph of f — the slope of the line tangent to the graph of f at the point in that direction — is exactly eleven? Why or why not?

(d) Write down (and clearly label) explicit formulas for all the first- and second-order partial derivatives of f and for the discriminant of the function f.

(e) List the critical points of f and classify each of them as corresponding to either a local minimum, a local maximum, or a saddle point.

(f) Do any of the local extrema of f correspond to global extrema? How do you know?

(g) What is an equation for the plane tangent to the graph $z = f(x,y)$ at the point $(3,1)$? Express the equation in the form $z = A + Bx + Cy$ for rational numbers A, B, and C.

(h) Identify any points at which the vector normal to the graph $z = f(x,y)$ is parallel to $\langle 2,2,1 \rangle$.

(i) Consider the constrained optimization problem "*Maximize f subject to the constraint $x^2 + y = 2$.*" Per the method of Lagrange multipliers, any points (x,y) that provide a solution to this problem, along with a factor λ called the Lagrange multiplier of that solution, must satisfy a specific system of equations. Write down this system of equations. CHALLENGE: Once you are satisfied with your responses on the rest of this exam, on the back of this page, try to solve the system of equations and identify the solution to the constrained optimization problem.