Math253 Multivariable Calculus

Second Midterm Exam

Colorado Mesa University · 2025 Fall

NAME: _____

1. Each of the following statements is either true or false. Indicate which by writing TRUE or FALSE in the blank next to the statement. Note that you may choose to write nothing in any blank; an incorrect response counts as much against you a correct response counts for you.

_____ For the tangent, normal, and binormal vectors **T** and **N** and **B** at any point point on a smooth curve, the magnitude of $\mathbf{T} \times \mathbf{N} \times \mathbf{B}$ must be equal to one.

_____ The osculating circle at a point on a smooth curve will have a radius equal to the curve's curvature at that point.

_____ The velocity vector and the acceleration vector of a particle travelling smoothly along a smooth curve will always be orthogonal.

_____ For a smooth curve that lies entirely within the plane $x + y + z = 1$, the unit normal vector **N** to the curve at any point on the curve will be $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$.

_____ The "unit helix" $\langle \cos(t), \sin(t), t \rangle$ has constant curvature *and* constant torsion.

2. Suppose a particle is travelling through space, measured in feet, along a curve parameterized by the following vector-valued function for $t \geq 0$ seconds.

$$\boldsymbol{r}(t) = \left\langle \cos(t) + t\sin(t),\ \sin(t) - t\cos(t),\ 3t^2 \right\rangle$$

(a) What are the coordinates of the particle's position after $t = \pi$ seconds?

(b) For which value(s) of the parameter t will the particle intersect the surface given by $x^2 + y^2 + 7 = z$?

Continued from the previous page. $\boldsymbol{r}(t) = \left\langle \cos(t) + t\sin(t),\ \sin(t) - t\cos(t),\ 3t^2 \right\rangle$

(c) At what angle of inclination (ascent) does the particle's path cross the plane $z = 3$?

(d) What are the explicit formulas for the components of the vectors that form the Frenet-Serret (TNB) frame of the particle's trajectory through space along the curve.

Continued from the previous page. $\boldsymbol{r}(t) = \left\langle \cos(t) + t\sin(t),\ \sin(t) - t\cos(t),\ 3t^2 \right\rangle$

(e) How many seconds elapse before the particle has travelled 368 feet along the curve?

(f) At $t = 4\pi$ seconds the particle leaves the curve, flying off along a linear tangential trajectory. What are the coordinates of the particle's position one second after it flies off the curve?