# Midterm Exam Four

Math 136-001 Engineering Calculus II

Colorado Mesa University Spring 2023

Name: ______________________________

1. Explain, as if explaining to a peer in the class, what a *sequence* is and what a *series* is.

2. Explain, as if explaining to a peer in the class, the difference between a series $\sum a_n$ being *absolutely convergent* versus *conditionally convergent*.

3. Write this series in summation ($\Sigma$) notation and then find its value.

$$\frac{2^2}{5\times 3}-\frac{2^3}{5\times 3^2}+\frac{2^4}{5\times 3^3}-\frac{2^5}{5\times 3^4}+\cdots$$

4. Below are three series. Choose two of them, and demonstrate how to determine whether they converge absolutely, converge conditionally, or diverge. Explicitly name any technique or test use in your demonstration. Cross-out the third series.

$$\sum_{n=1}^{\infty} \frac{n}{3n-1} \qquad \sum_{n=1}^{\infty} \frac{\cos(n\pi)\sqrt[10]{n}}{n^{0.2}} \qquad \sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!}$$

5. Let $f(x) = \left(\frac{x}{2}\right)^{1/3}$. Write down $T_2$, the Taylor polynomial of degree two, centered at $x = 2$.



6. Determine the radius and interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

7. Let $f(x) = x^3 e^{2x}$.

(a) Write down the Taylor series presentation of $f(x)$ centered at $x = 0$. Express this series "nicely" in summation ($\Sigma$) notation with initial index $n = 0$. (Hint: don't build it from scratch.)

(b) The notation $f^{(n)}$ denotes the $n$th derivative of $f$. What is the exact value of $f^{(13)}(0)$? Please don't write it as a decimal number.

(c) What is the exact value of this sum? Again please don't write it as a decimal number. (So you may check your answer, the value is *about* $437$.)

$$\sum_{n=0}^{\infty} \frac{2^{2n+3}}{n!}$$

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This is due at the beginning of the next class period. While working on this and while writing up your report, you may consult only static, non-sentient resources that require no responsive interaction from an intelligent multi-cellular being besides yourself. I.e. pretend you're the last human alive.

1. Recall that *every* decimal number that either terminates or has a "tail" that repeats indefinitely is actually a rational number (a fraction). Demonstrate how to figure out an expression for the repeating decimal number $0.1642857\overline{142857}$ as a rational number using a calculator capable only of integer multiplication and division.

2. It's a mild tragedy that the Taylor series for $\ln(1+x)$ only converges for $|x| < 1$. But wait! It's not so bleak! Using the algebraic *properties of logarithms* you can write any logarithm in terms of $\ln(x+1)$ for $|x| < 1$.

   (a) Using properties of logarithms and the Taylor series for $\ln(1+x)$ write down a series that converges to $\ln(4)$.

   (b) (Extra Credit) An approximation for $\ln(4)$ is $1.386294$. Using a computer or programmable calculator, figure out how many terms of this series you'd need to add up before you get within $\pm 10^{-2}$ of the exact value. How many terms before you get within $\pm 10^{-3}$? How many terms before you get within $\pm 10^{-4}$? Notice the pattern and tell me why this happens. Hint: you'll think it's "obvious" once you realize why.

   (c) (Extra Credit) Suppose you need to write an algorithm to calculate $\ln(x)$ for any positive input $x$ accurate to within some fixed precision as quickly as possible. Using what you've learned about the convergence of the Taylor series for $\ln$, the algebraic rules of logarithms, and how $\ln$ can be computed from its integral (e.g. using the *trapezoid rule*), how would you implement such an algorithm? Consider doing some research on how professional programmers have implemented a natural logarithm function.