Midterm Exam Three

Math 136-001 Engineering Calculus II

Colorado Mesa University Spring 2023

Name: _____

1. Find an *explicit* solution to this initial value problem.

$$\csc^2(x)\,\dot{y} = y\sec^2(x) \quad \text{where} \quad y\left(\frac{\pi}{4}\right) = \mathrm{e}^{-\pi/4}$$

2. Consider the parametric curve defined by $x(t) = e^{t/2}$ and $y(t) = e^t$ for t ranging from $-\infty$ to ∞.

(a) Find an equation for the line tangent to the curve at the point where $t = 3$.

(b) Write down an integral that computes the arclength of the curve between $t = 1$ and $t = 7$.

(c) Can you *eliminate the parameter* t and write down a function $y = f(x)$ that represents the same curve?

3. Let $f(\theta) = 2 - 4\cos(\theta)$, and consider the graph $r = f(\theta)$ for θ ranging from 0 to 2π. Notice that the graph encloses two regions: an inner region and an outer "crescent roll"-shaped region.

   (a) What is the area of the "crescent roll"-shaped region?

(b) Explain how you'd find *all* values of θ ranging from 0 to 2π for which the line tangent to the graph will be perfectly horizontal. (You're just being asked to explain because this computationally difficult.)

4. Find an explicit general solution to the differential equation $\dot{y} = \frac{1}{3}y(100 - y)$.