Midterm Exam Two

Math 136-001 Engineering Calculus II

Colorado Mesa University Spring 2023

Name: ______________________________

1. Demonstrate how to evaluate each of the following integrals.

   (a) $\displaystyle\int_{1}^{\pi/2} x\sin(x)\,\mathrm{d}x$

   (b) $\displaystyle\int \frac{7x^2-3x+4}{(x+1)(x^2+1)}\,\mathrm{d}x$

2. Below are three integrals. Demonstrate how to evaluate two of them, and cross-out the third.

$$\int \left(\cos(\theta) + 3\tan(\theta)\right)^2 \, d\theta \qquad \int \frac{\ln(x)\ln\left(\ln(x)\right)}{x} \, dx \qquad \int \frac{\arcsin(x)}{\sqrt{1-x^2}} \, dx$$

3. Below are three integrals. Prove the convergence/divergence of two of them, and cross-out the third.
   Hint: remember you don't need to *evaluate* an integral to determine its convergence/divergence.

$$\int_{-2}^{2} \frac{\mathrm{d}x}{x^4 + x^2} \qquad \int_{\mathrm{e}}^{\infty} \frac{x^3+1}{x^4-1}\,\mathrm{d}x \qquad \int_{\ln(3)}^{\infty} \mathrm{e}^{-t}\sin^2(t)\,\mathrm{d}t$$

4. Let C be the curve $y = 2\sqrt{x}$. Consider the region R in the (x, y)-plane bound by C, the line $y = 4$, and the y-axis. Let S be the solid generated by revolving R about the line $y = 5$. Write down expressions—likely involving integrals—that compute the following values. You do not need to evaluate the integrals.

(a) The arclength of C.

(b) The x-coordinate of the centroid of R.

(c) The y-coordinate of the centroid of R.

(d) The surface area of S.

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This is due at the beginning of class on Thursday. While working on this and while writing up your report, you may consult only static, non-sentient resources that require no responsive interaction from an intelligent multi-cellular being besides yourself. I.e pretend you're the last human on earth.

1. Your calculator is broken, but you need to compute a decimal approximation of $\ln(4)$.

   (a) First recall that $\ln$ is *defined* in terms of an integral. Write down the integral that equals $\ln(4)$.

   (b) Use the *trapezoid rule* to estimate the value of $\ln(4)$ with six equal subintervals.

   (c) Suppose you need to know the value of $\ln(4)$ with a margin-of-error no greater than $\pm 10^{-5}$. Using the trapezoid rule, how many equal subintervals n would you have to divide the domain of integration into to achieve this precision?

   (d) (Extra Credit) Write a computer program to test your previous answer. Email me your code. Maybe the n you found in the previous part was excessive, and a much smaller n would yield a margin-of-error no greater than $\pm 10^{-5}$. Using your program, can you the *smallest* n that calculates $\ln(4)$ at this precision?

2. Sketch the curve $y = \frac{1}{x}$ for $x \geq 1$, and sketch the surface that results from rotating this curve about the x-axis. This surface is popularly referred to as *Gabriel's horn*.

   (a) Write down an integral that computes the surface area of this horn, and demonstrate that this integral is divergent.

   (b) Write down an integral that computes the volume of the interior of this horn, and demonstrate that this integral is convergent.

   (c) (Extra Credit) Suppose you pour paint into the horn. Since the volume of the interior of the horn is finite, you'll need only finitely much paint to "fill" it. But this finite amount of paint would seem to coat the entire inner surface of the horn with paint, thus painting the horn's infinite amount of surface with finitely much paint. How can this be?

3. A vertical dam has a semicircular gate as illustrated on the back of this page.

   (a) Calculate the hydrostatic force against the gate.

   (b) (Extra Credit) How would your calculation have to change if the surface of the dam against the water wasn't perfectly vertical, but instead inclined at $15°$?