Midterm Exam One

Math 136-001 Engineering Calculus II

Colorado Mesa University Spring 2023

Name: ____

1. What are formulas for the following derivatives of expressions considered as functions of x?

   (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}\,\mathrm{e}^{\mathrm{e}x}\right)$

   (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2^{5\log_5(x)}\right)$

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x)^x\right)$

2. Demonstrate how to calculate the following integrals.

(a) $\displaystyle\int \frac{3x^3+5x+7}{x^2}\,\mathrm{d}x$

(b) $\displaystyle\int \frac{\tan(\ln(x))}{x}\,\mathrm{d}x$

(c) $\displaystyle\int_0^{\ln(\sqrt{3})} \frac{\mathrm{e}^x}{1+\mathrm{e}^{2x}}\,\mathrm{d}x$

3. Demonstrate how to calculate the values of the following limits. If the limit is $\pm\infty$ or if the limit does note exist, be sure to clearly indicated this. In your demonstration, place a small "LH" anywhere you apply L'Hospital's Rule.

(a) $\lim\limits_{x\to 1} \dfrac{\mathrm{e}^x}{x^2}$

(b) $\lim\limits_{x\to 0} \dfrac{\arcsin(2x)}{5x}$

(c) $\lim\limits_{x\to \infty} \sqrt{x}\mathrm{e}^{-x/2}$

4. Demonstrate how to calculate the derivative of the arccosine function knowing only that the derivative of $\cos(x)$ is $-\sin(x)$, and express the derivative as an algebraic function.

5. Consider the function $\ell(x) = \dfrac{1}{1+e^{-x}}$.

(a) Prove that this function attains no global minimum or maximum value.

(b) Find an algebraic formula for the derivative of $\ell^{-1}(x)$.

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This is due at the beginning of the next class period. While working on this and while writing up your report, you may consult only static, non-sentient resources that require no responsive interaction from an intelligent multi-cellular being besides yourself. I.e pretend you're the last human alive.

1. When a cable of uniform linear density is hung between two poles (in a uniform gravitational field) it'll sag in a "u"-like shape under its own weight. This curve is called a *catenary*. Considered as a curve $y = f(x)$ in the (x, y)-plane, symmetric about the y-axis, and with its lowest point on the y-axis, it can be shown that the catenary satisfies the differential equation

$$\ddot{y} = \frac{\rho g}{T}\sqrt{1 + \dot{y}^2}$$

where ρ is the cable's linear density, g is acceleration due to gravity, T is the tension on the cable at its lowest point, $\dot{y}$ is the first derivative of $f(x)$, and $\ddot{y}$ is the second derivative of $f(x)$.

(a) Verify that this function f is a solution to that differential equation. Note that C is just some vertical offset.

$$y = f(x) = \frac{T}{\rho g}\cosh\left(\frac{\rho g x}{T}\right) + C$$

(b) Investigate the units of every parameter involved in the formula $f(x)$. In particular verify that the argument of $\cosh$ and the output of $\cosh$ are both scalar, unit-less quantities.

(c) Suppose you have two utility poles embedded in the ground 100 meters apart with a power line of linear density 2kg/m running between them fastened 10 meters up each pole. Use $g = 9.80$m/s^2 for gravity in Grand Junction due to elevation.

i. If the power line is rated for a maximum tension of 20,000N. How much will the line sag at its lowest point if we hang it such that it experiences this tension at its lowest point?

ii. If we hang the power line at this maximum tension, at what angle θ will the line be descending where it initially hangs from the pole?

iii. We don't want to push the line near its limits, maxing out its tension, though. We only need to allow for a few meters of clearance under the wire at the lowest point. Hanging the line such that it allows for 5 meters of clearance, compute the force of tension the cable is experiencing at its lowest point as precisely as possible.