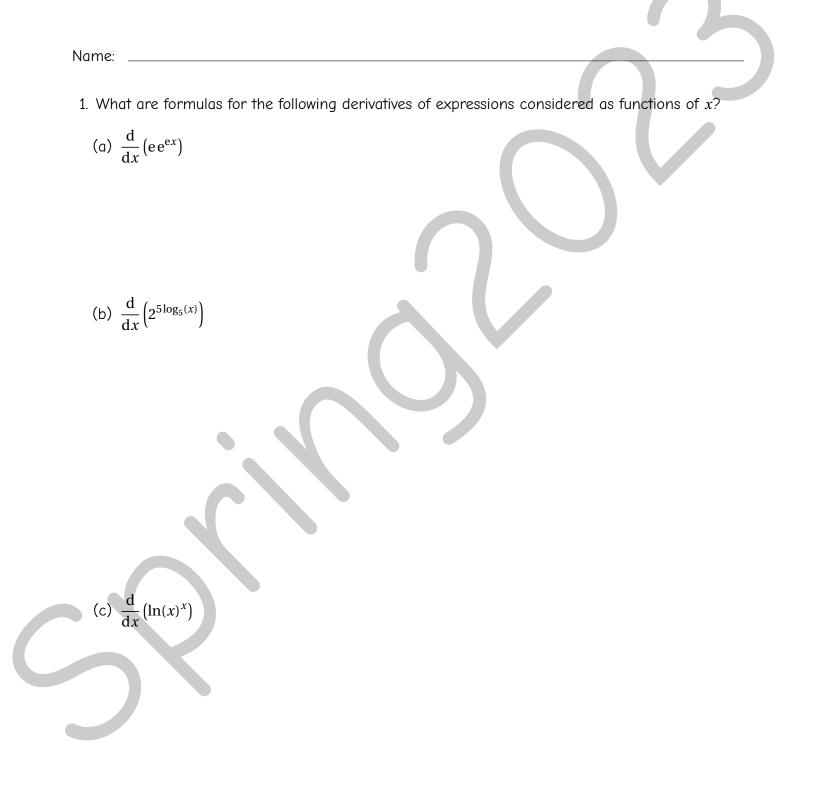
## Midterm Exam One

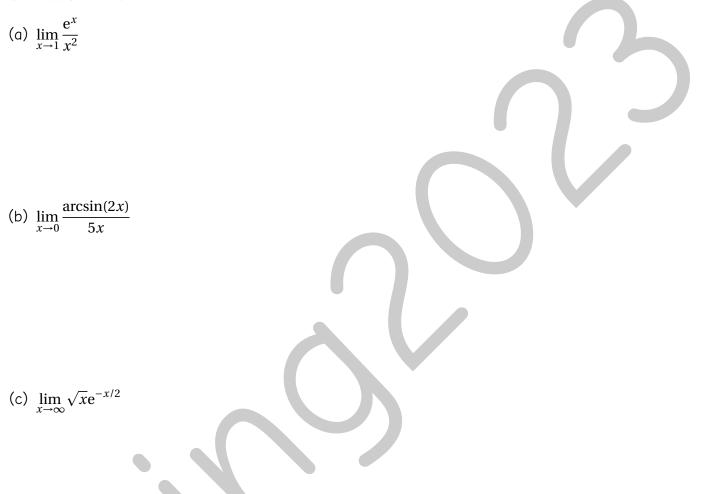
Math 136-001 Engineering Calculus II Colorado Mesa University Spring 2023



2. Demonstrate how to calculate the following integrals.

(a) 
$$\int \frac{3x^3 + 5x + 7}{x^2} dx$$
  
(b)  $\int \frac{\tan(\ln(x))}{x} dx$   
(c)  $\int_{0}^{10} \frac{e^x}{1 + e^{2x}} dx$ 

3. Demonstrate how to calculate the values of the following limits. If the limit is  $\pm \infty$  or if the limit does note exist, be sure to clearly indicated this. In your demonstration, place a small "LH" anywhere you apply L'Hospital's Rule.



4. Demonstrate how to calculate the derivative of the arccosine function knowing only that the derivative of  $\cos(x)$  is  $-\sin(x)$ , and express the derivative as an algebraic function.

- 5. Consider the function  $\ell(x) = \frac{1}{1 + e^{-x}}$ .
  - (a) Prove that this function attains no global minimum or maximum value.
  - (b) Find an algebraic formula for the derivative of  $\ell^{-1}(x)$ .

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Name:

This is due at the beginning of the next class period. While working on this and while writing up your report, you may consult only static, non-sentient resources that require no responsive interaction from an intelligent multi-cellular being besides yourself. I.e pretend you're the last human alive.

1. When a cable of uniform linear density is hung between two poles (in a uniform gravitational field) it'll sag in a "u"-like shape under its own weight. This curve is called a *catenary*. Considered as a curve y = f(x) in the (x, y)-plane, symmetric about the y-axis, and with its lowest point on the y-axis, it can be shown that the catenary satisfies the differential equation

$$\ddot{y} = \frac{\rho g}{T} \sqrt{1 + \dot{y}^2}$$

where  $\rho$  is the cable's linear density, g is acceleration due to gravity, T is the tension on the cable at its lowest point,  $\dot{y}$  is the first derivative of f(x), and  $\ddot{y}$  is the second derivative of f(x).

(a) Verify that this function f is a solution to that differential equation. Note that C is just some vertical offset.

$$y = f(x) = \frac{T}{\rho g} \cosh\left(\frac{\rho g x}{T}\right) + C$$

- (b) Investigate the units of every parameter involved in the formula f(x). In particular verify that the argument of cosh and the output of cosh are both scalar, unit-less quantities.
- (c) Suppose you have two utility poles embedded in the ground 100 meters apart with a power line of linear density 2kg/m running between them fastened 10 meters up each pole. Use  $g = 9.80 \text{m/s}^2$  for gravity in Grand Junction due to elevation.
  - i. If the power line is rated for a maximum tension of 20,000N. How much will the line sag at its lowest point if we hang it such that it experiences this tension at its lowest point?
  - ii. If we hang the power line at this maximum tension, at what angle  $\theta$  will the line be descending where it initially hangs from the pole?
  - iii. We don't want to push the line near its limits, maxing out its tension, though. We only need to allow for a few meters of clearance under the wire at the lowest point. Hanging the line such that it allows for 5 meters of clearance, compute the force of tension the cable is experiencing at its lowest point as precisely as possible.