Math136 Engineering Calculus II
Fourth Midterm Exam
Colorado Mesa University Fall 2023

NAME: _____

1. Explain, as if explaining to a peer in the class, what a *differential equation* is, and the difference between a *general solution* and a *particular solution* to a differential equation.

2. Consider the point with polar coordinates $(r, \theta) = (12.34, 5.678)$. What are the rectangular coordinates (x, y) for this same point? Express each coordinate as a decimal rounded to three decimal places.

3. Verify that $y = e^{2x} + x$ is a solution to this differential equation.

$$\dot{y}x - y = (1 - 2x)(x - y)$$

4. Find an *explicit* general solution to this differential equation.

$$\frac{1}{2}\dot{y} = \sqrt{y}\ln(x - 1)$$

5. Consider the curve segment in rectangular space defined parametrically by the coordinate functions $x(t) = \ln(t)$ and $y(t) = \frac{\ln(t)}{t}$ for t ranging from 1 to e^3.

   (a) Sketch this curve, and label the initial and the terminal points with their explicit coordinates.

   (b) What are the coordinates of the point on this curve at which the tangent line is horizontal?

   (c) Write down, but do not evaluate, an integral that computes the length of this curve segment.

   (d) Can you *eliminate the parameter* t and write down a function f such that the graph $y = f(x)$ will coincide with this curve segment?

6. Let $f(t) = t\sin(3t)$, and consider the graph $r = f(\theta)$ in polar coordinates for θ ranging from 0 to π.

   (a) Sketch this graph, and label the point $(\pi/4, f(\pi/4))$ with its explicit coordinates.

   (b) What is the area of the region enclosed by this graph?