

Math136 Engineering Calculus II  
**Third Midterm Exam**  
Colorado Mesa University Fall 2023

NAME: \_\_\_\_\_

1. Explain, as if explaining to a peer in the class, what a *sequence* is and what a *series* is.
2. Explain, as if explaining to a peer in the class, the difference between a series  $\sum a_n$  being *absolutely convergent* versus *conditionally convergent*.
3. Decide if the following claim is true or false. If it is true, simply declare it to be TRUE. If is false, declare that it is FALSE and present an example to demonstrate that it's false.

If  $\sum_{n=1}^{\infty} a_n$  converges, then  $\sum_{n=1}^{\infty} |a_n|$  converges too.

4. Below are three series. Choose two of them, and demonstrate how to determine whether they converge absolutely, converge conditionally, or diverge. Cross-out the third series. Explicitly name any technique or test you use in your demonstration. It may be a good idea to also annotate your demonstration with other comments to guide a reader.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{9n-4}}$$

$$\sum_{n=1}^{\infty} \left( \frac{5n-2n^3}{6n^3+3} \right)^n$$

$$\sum_{n=2}^{\infty} (-1)^n \left( \frac{3}{\ln(\sqrt{n})} \right)$$

Fall 2023

5. Write this convergent series in sigma ( $\Sigma$ ) notation and then demonstrate how to find its exact value.

$$\frac{7 \times 3}{4^2} + \frac{7 \times 3^2}{4^3} + \frac{7 \times 3^3}{4^4} + \frac{7 \times 3^4}{4^5} + \frac{7 \times 3^5}{4^6} + \dots$$

6. Consider the polynomial function  $f(x) = 3x^2 - x + 2$ .

(a) What is the Maclaurin series of  $f$ ?

(b) What is the Taylor series of  $f$  centered at  $x = 1$ ?

7. First write down the *definition* of the Maclaurin series (Taylor series centered at zero) for a general function  $f$  that has derivatives of all orders. Then using this definition, and using the definition of  $\cosh$  below, write down the Maclaurin series for  $\cosh(x)$  in sigma ( $\Sigma$ ) notation.

$$\cosh(x) = \frac{e^x + e^{-x}}{2}.$$

Fall 2023

8. Consider the function  $f(x) = \frac{\arctan(2x)}{x}$ .

(a) Write down the Maclaurin series for  $f$ . (HINT: don't build it from scratch.)

(b) Find a power series representation for the function

$$F(t) = \int_0^t f(x) dx.$$

\* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, write about anything you've learned in this class that you haven't already gotten a chance to demonstrate on this exam.

Fall 2023