Math136 Engineering Calculus II

Second Midterm Exam

Colorado Mesa University Fall 2023

NAME: _____

1. Briefly demonstrate how to evaluate each of the following integrals.

   (a) $\int x e^x \, dx$

   (b) $\int \cos^3(x) \, dx$

   (c) $\int \sin(2x) \, dx$

   (d) $\int_2^\infty \frac{1}{x^5} \, dx$

2. Below are four integrals. On this page and the next page, demonstrate how to evaluate *three* of them, and cross-out the fourth. It may be a good idea to annotate your demonstration with comments to guide a reader.

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}}\,\mathrm{d}x \qquad \int 2x\,\mathrm{arccsc}\left(x\right)\,\mathrm{d}x \qquad \int_0^{1} \frac{3x^2+3x+2}{(x+1)(x^2+1)}\,\mathrm{d}x \qquad \int \left(\sec(x)-3\tan(x)\right)^2\,\mathrm{d}x$$

3. What is the *definition* of the natural logarithm function ln that we've established for this class?

4. Determine if the following integral converges or diverges, and explain how you determined it.

$$\int_{\mathrm{e}}^{\infty} \mathrm{e}^{-t} \cos^2(t)\, \mathrm{d}t$$

5. Write down, but do not evaluate, an integral that computes the length of the curve $y = 5x^2 - 7$ between the points $(1, -2)$ and $(3, 38)$.

6. Consider the region R in the (x, y)-plane bounded by the parabolas given by the equations $y = x^2$ and $y = \frac{1}{2}x^2 + 1$. Suppose this region has uniform density 1.

   (a) Explicitly what are the coordinates of the centroid of R?

   (b) (CHALLENGE) Suppose instead of $y = \frac{1}{2}x^2 + 1$ the second parabola is given by the equation $y = ax^2 + 1$ for some variable $a > 1$. Show that the position of the centroid doesn't depend on a.

* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, write about anything you've learned in this class that you haven't already gotten a chance to demonstrate on this exam.