Math136 Engineering Calculus II

First Midterm Exam

Colorado Mesa University Fall 2023

NAME: _____

1. What is the *definition* of the natural logarithm function ln that we've established for this class?

2. The essential fact we get from the *inverse function theorem* can be illustrated by a single, succinct formula. Write down this formula.

3. Write down a formula for an *anti*derivative of $f(x) = e^{\pi x}$.

4. Write down concise formulas for the derivatives of these expressions considered as functions of x. It may be a good idea to annotate your calculations with comments to guide a reader.

   (a) $\cosh(x+3) + 2^{2x}$

   (b) $x^2 \arctan\left(e^{5x}\right)$

   (c) $\ln(x)^{\ln(x)}$

5. Demonstrate how to calculate formulas for the following definite integrals. It may be a good idea to annotate your demonstration with comments to guide a reader.

(a) $\displaystyle\int \frac{t^4 - t^3 + t^2 - t + 1}{t^2}\,\mathrm{d}t$

(b) $\displaystyle\int \frac{1}{t\sqrt{(3t)^2 - 1}}\,\mathrm{d}t$

(c) $\displaystyle\int \frac{\tan\big(\ln(t)\big)}{t}\,\mathrm{d}t$

6. Demonstrate how to calculate the values of the following limits. If the limit is $\pm\infty$ or if the limit does not exist, be sure to clearly indicated this. It may be a good idea to annotate your demonstration with comments to guide a reader. E.g. place a small "LH" anywhere you apply L'Hospital's Rule.

   (a) $\lim_{x\to 0} \dfrac{\arcsin(3x)}{7x}$

   (b) $\lim_{x\to 8} \dfrac{e^x}{x^2}$

   (c) $\lim_{x\to 2^+} \left(\ln(x-2) - \ln\left(x^2-4\right)\right)$

7. Demonstrate how to use the inverse function theorem to calculate a formula for the derivative of the arcsecant function knowing only that the derivative of $\sec(x)$ is $\sec(x)\tan(x)$. Express the formula as an *algebraic* function, i.e. not in terms of trigonometric functions.

* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, write about anything you've learned in this class that you haven't already gotten a chance to demonstrate on this exam.