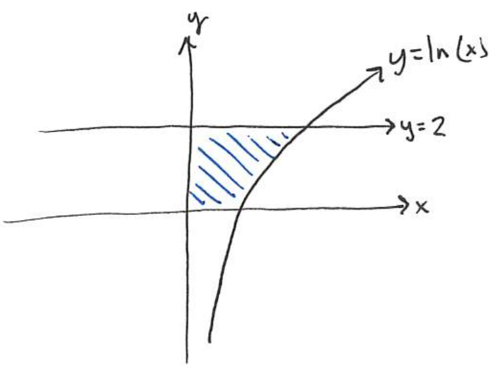
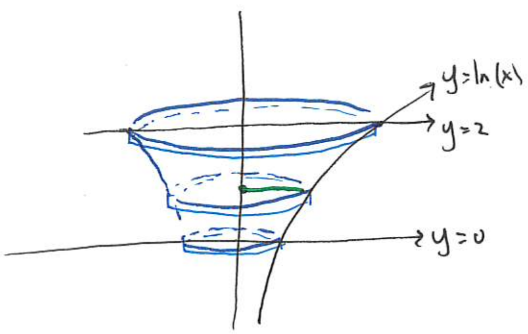
Find the volume of the solid formed by rotating the region bounded by $y = \ln(x)$, $y=2$, the x-axis, and the y-axis, about the y-axis.



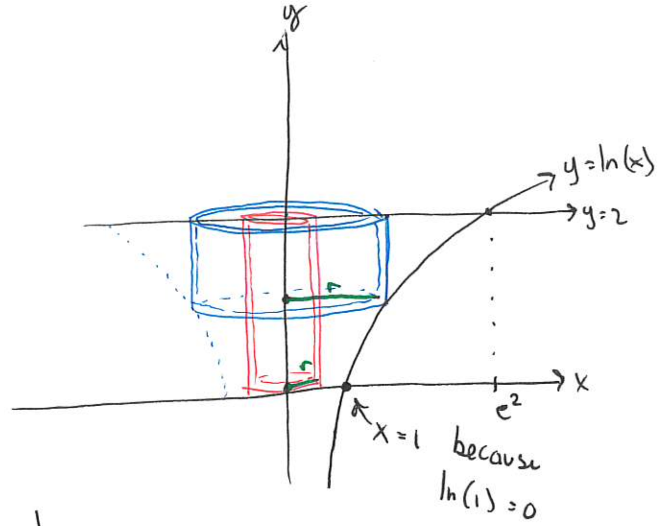


Let's first use the "washer method" to find this volume. Really its the disk method since there is no inner radius, and each cross-section is a disk. We must integrate from $y=0$ to $y=2$ in this method, and see that the ~~radius~~ radii of the circular cross-sections (in terms of y) must be $x = e^y$ (this comes from $y = \ln(x)$).

So the volume is

$$\int_0^2 \pi (e^y)^2 \, dy = \pi \int_0^2 e^{2y} \, dy = \frac{\pi}{2} e^{2y} \Big|_0^2 = \frac{\pi}{2}\left(e^4 - 1\right)$$

Now let's find the same volume using the "shell method." Notice though we'll have to break this up into two integrals since the height of the shells is given by different functions along our interval we'd have to integrate: from $x=0$ to $x=1$ the height is constantly 2, but from $x=1$ to $x=e^2$, the height is given by $2-\ln(x)$, top-curve minus bottom-curve. ~~So the volume will be given by~~ Recalling that our integrand must be the volume of each of these shells, given by the formula $2\pi r h\,dx$ where r is the radius and h is the height and dx is a tiny width, the volume will be



$$\int_0^1 2\pi(x)(2)\,dx + \int_1^{e^2} 2\pi(x)(2-\ln(x))\,dx$$

This'll be tougher to integrate and uses techniques we haven't seen yet. Often when choosing either the disk/washer method vs the shell method, there is an [illegible] better choice.

$$\int_0^1 2\pi(x)(2)\,dx + \int_1^{e^2} 2\pi(x)(2-\ln(x))\,dx$$

$$= 2\pi\int_0^1 2x\,dx + 2\pi\cdot\int_1^{e^2} 2x\,dx - 2\pi\int_1^{e^2} x\ln(x)\,dx$$

$$= 2\pi\int_0^{e^2} 2x\,dx - 2\pi\int_1^{e^2} x\ln(x)\,dx$$

↓ integration by parts

$$= 2\pi x^2\Big|_0^{e^2} - 2\pi\left(\frac{1}{2}x^2\ln(x)\Big|_1^{e^2} - \int_1^{e^2}\frac{1}{2}x\,dx\right)$$

$$= 2\pi e^4 - 2\pi\left(\frac{1}{2}(e^2)^2\ln(e^2) - \frac{1}{4}x^2\Big|_1^{e^2}\right)$$

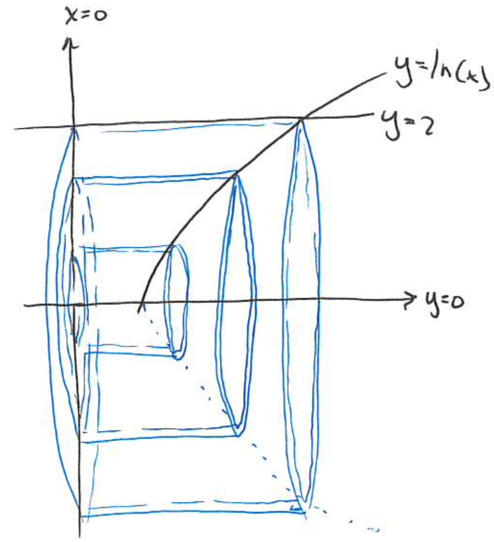
$$= 2\pi e^4 - 2\pi\left(e^4 - \left(\frac{1}{4}e^4 - \frac{1}{4}\right)\right)$$

$$= 2\pi\left(e^4 - e^4 + \frac{1}{4}e^4 - \frac{1}{4}\right)$$

$$= \frac{\pi}{2}\left(e^4 - 1\right) \quad //$$

Which is what we got using the other method too! :)

Now let's write down the integrals that correspond to the volume of the solid formed by rotating that same region about the x-axis instead. Using the Shell method this time, we'd integrate in the

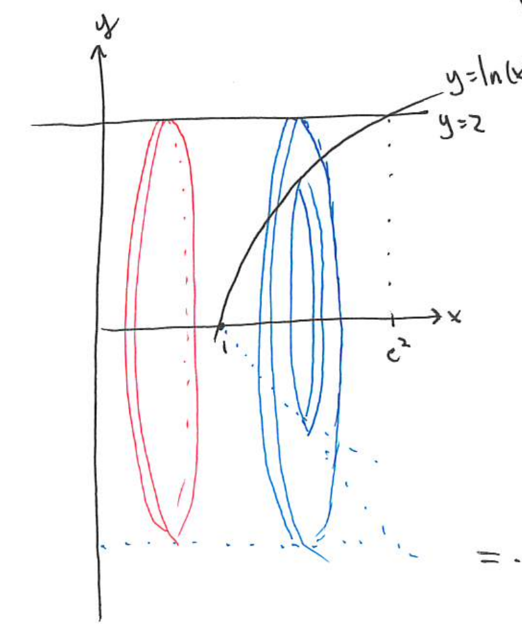


y direction from y=0 to y=2, our radii are just y, and the heights of each shell are given by ~~y = ln(x) − 0~~ $x = e^y$.

$$\int_0^2 2\pi(y)(e^y)\,dy = 2\pi\int_0^2 ye^y\,dy = \cdots = 2\pi(e^2+1)$$

integration by parts

Now using the washer method we'll have to break up the volume into two integrals because the radii involved with the washer/disk changes:



$$\int_0^1 \underline{\pi R^2}\,dx + \int_1^{e^2} \underline{\pi R^2 - \pi r^2}\,dx$$

$$= \pi\int_0^1 (2)^2\,dx + \pi\int_1^{e^2} (2)^2 - \underline{(\ln(x))^2}\,dx$$

tough: substitute, then integrate by parts twice.

$$= \cdots = 2\pi(e^2+1)$$