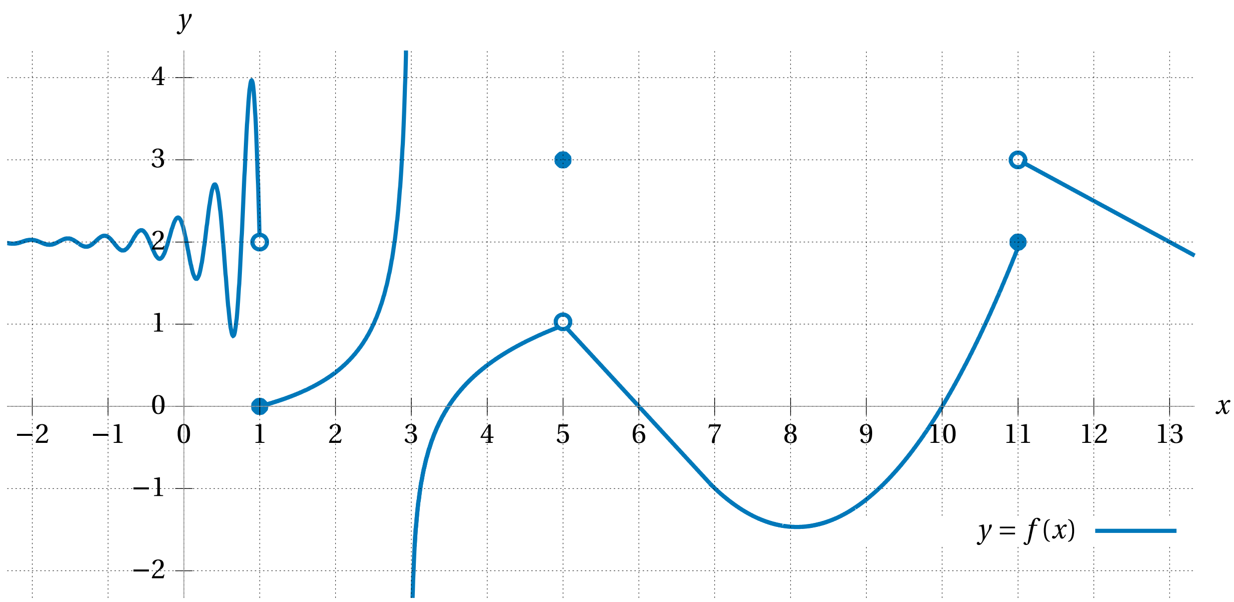
Math135 Engineering Calculus I

Just a Pop Quiz

Colorado Mesa University

NAME: ____

1. Based on this graph of $y = f(x)$ estimate the values of the following expressions. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.



$\lim_{x\to-\infty} f(x) = 2$	$\lim_{x\to 1^-} f(x) = 2$	$\lim_{x\to 1^+} f(x) = 0$	$f(1) = 0$
$\lim_{x\to 3^-} f(x) = \infty$	$\lim_{x\to 3^+} f(x) = -\infty$	$\lim_{x\to 5^-} f(x) = 1$	$\lim_{x\to 5^+} f(x) = 1$
$\lim_{x\to 5} f(x) = 1$	$f(5) = 3$	$\lim_{x\to 7} f(x) = -1$	$f(7) = -1$
$\lim_{x\to 11^-} f(x) = 2$	$\lim_{x\to 11^+} f(x) = 3$	$\lim_{x\to 11} f(x) \times$	$f(11) = 2$

2. What are the exact values of these limits? If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.

(a) $\lim\limits_{x\to 2} \dfrac{x^2-5x+6}{x-1}$

The function is continuous at $x=2$, so the value of the limit is the same as the value of the function evaluated at 2.

$$\lim_{x\to 2} \frac{x^2-5x+6}{x-1} \;=\; \frac{(2)^2-5(2)+6}{(2)-1} \;=\; 0$$

(b) $\lim\limits_{t\to 7} \dfrac{t^2-5t-14}{t^2-12t+35}$

Factoring the numerator and denominator, $\frac{t^2-5t-14}{t^2-12t+35} = \frac{(t-7)(t+2)}{(t-7)(t-5)} = \frac{t+2}{t-5}$, so

$$\lim_{t\to 7} \frac{t^2-5t-14}{t^2-12t+35} \;=\; \lim_{t\to 7} \frac{t+2}{t-5} \;=\; \frac{(7)+2}{(7)-5} \;=\; \tfrac{9}{2}.$$

(c) $\lim\limits_{x\to 1} \dfrac{x^2+1}{3-\sqrt{x+8}}$

Scaling the numerator and denominator by the denominator's conjugate,

$$\begin{aligned}\left(\frac{x^2-1}{3-\sqrt{x+8}}\right)\left(\frac{3+\sqrt{x+8}}{3+\sqrt{x+8}}\right) \;=\; \frac{\left(x^2-1\right)\left(3+\sqrt{x+8}\right)}{9-(x+8)} \;&=\; \frac{(x-1)(x+1)\left(3+\sqrt{x+8}\right)}{-(x-1)} \\ &=\; -(x+1)\left(3+\sqrt{x+8}\right)\end{aligned}$$

so we have that

$$\lim_{x\to 1} \frac{x^2-1}{3-\sqrt{x+8}} = -\lim_{x\to 1}(x+1)\left(3+\sqrt{x+8}\right) = -\big((1)+1\big)\left(3+\sqrt{(1)+8}\right) = -12.$$

3. A drag race, commonly called a *pass*, is run on a 1320 ft (¼ mile) long track. Suppose $f(t) = t^3 + 32t$ serves as a model for how far a certain dragster travelled from the starting line t seconds after it began its pass until $t = 10$ seconds when it crossed the finish line.

(a) What percentage of the total pass's distance does the dragster cover in the first seven seconds?

The pass is 1320 ft long and the dragster travels $f(7) = 576$ ft in the first 7 seconds:

$$\frac{f(7)\text{ ft}}{1320\text{ ft}} = \frac{567}{1320} \approx 42.95\%$$

(b) What was the dragster's average speed between the start of the pass and the seven second mark?

The average speed can be calculated as the ratio of the change in position (displacement) to the change in time between the $t = 0$ mark and the $t = 7$ mark:

$$\frac{\text{change-in-position}}{\text{change-in-time}} = \frac{f(7)\text{ ft} - f(0)\text{ ft}}{7\text{ s} - 0\text{ s}} = \frac{(567-0)\text{ ft}}{(7-0)\text{ s}} = 81\text{ ft/s}$$

(c) How fast was the dragster going at exactly the seven second mark?

We can calculate this "instantaneous" speed as the limit of average speeds various different ways. Let's look at the average speed of the dragster during progressively shorter intervals of time just before the seven second mark:

h	1	0.1	0.01	0.001	0.0001	0.00001
$\frac{f(7)-f(7-h)}{h}$	159	~176.91	~178.79	~178.979	~178.9979	~178.99979

This sequence of values appear to be approaching 179, so it appears that

$$\lim_{h\to 0} \frac{f(7)-f(7-h)}{h} = 179\,.$$

If this is true, the dragster's speed at exactly the seven second mark was 179 ft/s.

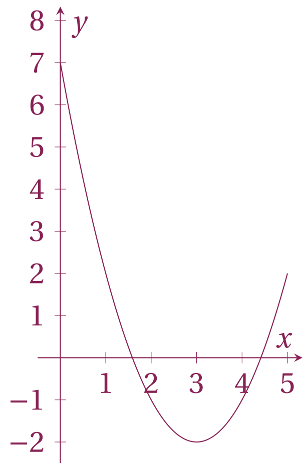
(d) Recall that 5280 ft = 1 mi. Convert the previous two answers to miles-per-hour (mph).

$$81\text{ ft/s} = \left(\frac{81\text{ ft}}{1\text{ s}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right) = \frac{81\times 60\times 60\text{ mi}}{5280\text{ hr}} \approx 55.227\text{ mph}$$

Similarly 179 ft/s ≈ 122.045 mph.

4. Consider the function f defined by the formula $f(x) = x^2 - 6x + 7$ on the domain $[0,5]$, and consider the parabolic graph of f.

(a) How far apart are the x-intercepts of the graph of f?



The x-intercepts of the graph of f are the points $(r_1, 0)$ and $(r_2, 0)$ where r_1 and r_2 are the roots of f, which are:

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)} = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$$

These two roots are $\left|(3+\sqrt{2}) - (3-\sqrt{2})\right| = 2\sqrt{2}$ apart.

(b) What is the slope of the line secant to the graph of f between its endpoints?

The left endpoint is $\big(0, f(0)\big)$ and the right endpoint is $\big(5, f(5)\big)$. The slope of the secant line between these points is

$$\frac{f(5) - f(0)}{5 - 0} = \frac{-5}{5} = -1\,.$$

(c) What is an equation for the line secant to the graph of f passing through its vertex and right endpoint? Express this equation in the form $y = mx + b$.

The vertex's coordinates are $\big(h, f(h)\big)$ for $h = \frac{-(-6)}{2(1)} = 3$ and the right endpoint's coordinates are $\big(5, f(5)\big)$. The secant line's slope will be

$$\frac{f(5) - f(3)}{5 - 3} = \frac{4}{2} = 2\,,$$

and so the equation of this secant line will be $\big(y - 2\big) = 2\big(x - 5\big)$, or $y = 2x - 8$.

(d) Does there exist a point on the graph of f at which the tangent line at that point will be horizontal? If so, what is an equation of this tangent line?

Yes, the line tangent to the parabola at its vertex will be horizontal. An equation of this line is $y = -2$.

5. Determine the value of this limit *accurate to within* ± 0.001. PROTIP: use technology to help.

$$\lim_{x \to 0^+} \sqrt[x]{2x+1}$$

We can approximate this limit numerically by evaluating $\sqrt[x]{2x+1}$ at positive values of x that are progressively closer to zero until confident the thousandths digit will no longer change:

x	1	0.01	0.0001	0.000001	0.00000001	0.000000001
$\sqrt[x]{2x+1}$	3	~7.2446	~7.3876	~7.3890	~7.3891	~7.3891

It appears that $\lim_{x \to 0^+} \sqrt[x]{2x+1} \approx 7.389$.

For reference, note that the exact value of this limit is 7.38905609893065022723042746057500781318031557055

6. Consider a very peculiar car that when travelling uphill must go exactly 56mph, when travelling on level ground must go exactly 63mph, and when travelling downhill must go exactly 72mph. You drive this car from your home to the next town over and the trip takes four hours. The return trip however, from town back home, takes four hours and forty minutes. How far is the town from your home?