

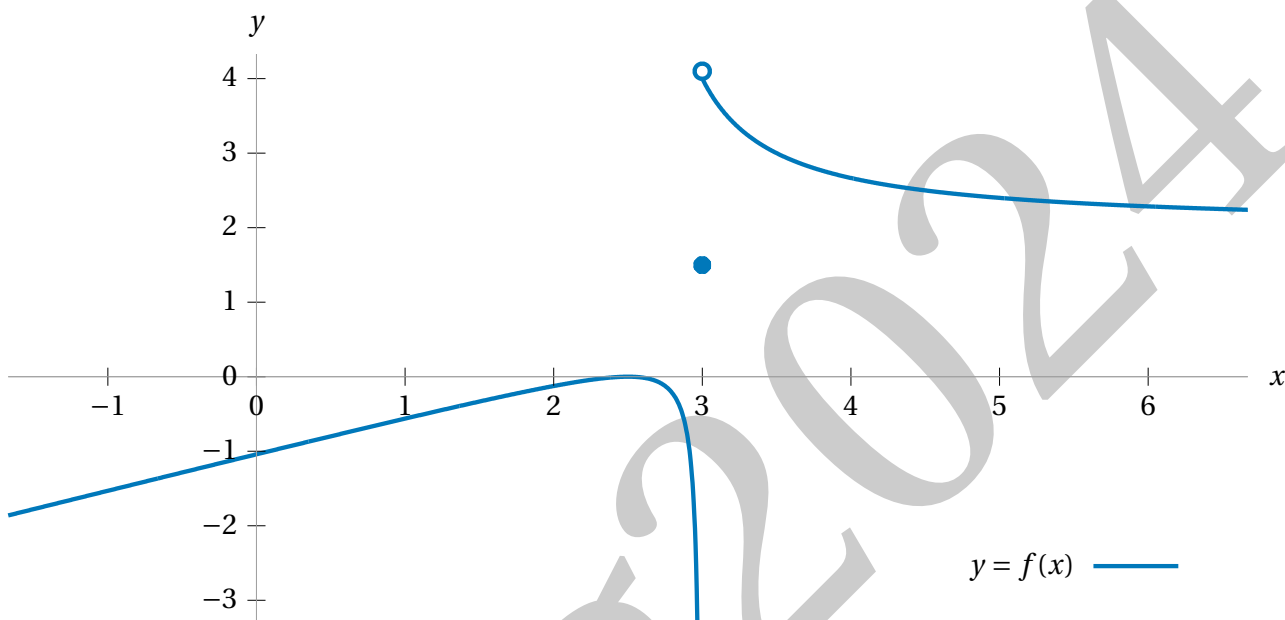
Math135 Engineering Calculus I
First Midterm Exam
Colorado Mesa University Spring 2024

NAME: _____

1. Explain, as if explaining to a peer in the class, what it means to say " $\lim_{x \rightarrow 3} f(x)$ does not exist."

2. What is the precise, mathematical definition of a function f being *continuous on its domain*?

3. Based on this graph of $y = f(x)$ estimate the values of the following expressions. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.



(a) $\lim_{x \rightarrow -\infty} f(x)$

(b) $\lim_{x \rightarrow \infty} f(x)$

(c) $\lim_{x \rightarrow 3^-} f(x)$

(d) $\lim_{x \rightarrow 3^+} f(x)$

(e) $\lim_{x \rightarrow 3} f(x)$

(f) $f(3)$

4. Using technology, approximate the value of the following limit accurate to six decimal places.

$$\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$$

5. Sketch the graph of an example of a function f that satisfies the following conditions.

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \lim_{x \rightarrow 0^-} f(x) = \infty \quad \lim_{x \rightarrow 8} f(x) = 0 \quad f(8) = 8 \quad \lim_{x \rightarrow \infty} f(x) = -\infty$$

6. What are the values of the following expressions? If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.

(a) $\lim_{x \rightarrow 3} 42$

(b) $\lim_{x \rightarrow 3} 42x$

(c) $\lim_{x \rightarrow 0} \frac{42}{x^2}$

(d) $\lim_{x \rightarrow 42} \frac{(x-42)(x+42)}{x}$

(e) $\lim_{x \rightarrow 42^+} \frac{1}{x-42}$

(f) $\lim_{x \rightarrow \infty} \frac{\sin(42x)}{x}$

7. *Demonstrate* how to manually (algebraically) calculate the values of the following limits. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out. You may use L'Hospital's rule for evaluating limits only if you can *prove* that L'Hospital's rule works.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$

(b) $\lim_{x \rightarrow 8} \frac{8 - x}{5 - \sqrt{3x + 1}}$

* (TRIVIA) What 2004 comedy starring Tina Fey and Lindsay Lohan featured the following limit?

$$\lim_{x \rightarrow 0} \frac{\ln(1 - x) - \sin(x)}{\sin^2(x)}$$

8. You are standing on the surface of the planet Mars. It is rather comfortable. You toss a stone straight up into the air at a speed of 80 ft/s. The height of the stone above you, in feet, measured t seconds after you throw it is roughly modelled by the function $h(t) = 5.5 + 80t - 6.09t^2$.

- (a) How long after you throw it does the stone reach its maximum height?
- (b) What is this maximum height?
- (c) What is the average velocity of the stone between the moment you throw it and the moment it reaches its maximum height?

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