Math135 Engineering Calculus I

Third Midterm Exam

Colorado Mesa University Fall 2023

NAME: _____

1. What is the result of evaluating these indefinite integrals? Don't forget your $+C$s!

   (a) $\displaystyle\int t^2 + t + \sqrt{t} + 1 + \frac{1}{t^2}\,\mathrm{d}t$

   (b) $\displaystyle\int \sin(3x)\,\mathrm{d}x$

   (c) $\displaystyle\int \tan(\theta)\sec(\theta)\,\mathrm{d}\theta$

2. Explain, as if explaining to a peer in the class, what an *indefinite integral* of a function f is and what a *definite integral* of a function f on some domain is.

3. What is the value of this integral?

$$\int_{-1}^{2} 3|x| - 4 \, \mathrm{d}x$$

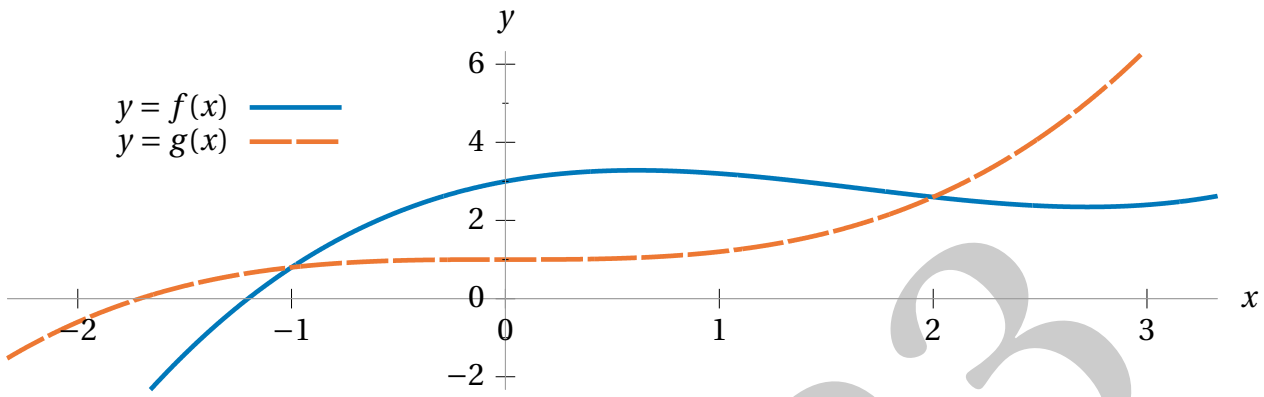
4. Demonstrate how to calculate the *exact* value of this integral.

$$\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}} \, \mathrm{d}x$$

5. Write down a calculation to verify this equality.

$$\int x \sin(x) \, \mathrm{d}x \;=\; \sin(x) - x\cos(x) + 3C$$

6. Here are the graphs of functions $f(x) = \frac{1}{5}x^3 - x^2 + x + 3$ and $g(x) = \frac{1}{5}x^3 + 1$.



Let R denote the region bounded by the graphs of f and g.

(a) Write down an integral that represents the area of R.

(b) Write down an integral that represents the volume of the solid generated by revolving R about the x-axis.

(c) Visually estimating, for what value of b will $\int_0^b g(x)\,\mathrm{d}x = 3$?

7. You begin observing a particle, the movement of which is constrained to a straight directed line, subject to a variable force causing the particle to accelerate along the line according to the function

$$a(t) = -\frac{1}{4}t^2 + \cos\left(\frac{8t}{\pi}\right) + 10\,\mathrm{m/s^2}\,.$$

t seconds after you begin observing it.

(a) Suppose the velocity of the particle is $-24\,\mathrm{m/s}$ when you begin observing it. Write down a formula for a function $v(t)$ that returns the particle's velocity t seconds after you begin observing it.

(b) After 6 seconds, how far is the particle from where you first observed it? Express your answer as a decimal number accurate to five decimal places.

8. You are on the rooftop of a 500 m tall building. A 40 kg chimpanzee is strapped into a harness dangling at the end of a 200 m long rope anchored to the edge of the rooftop. You are determined to pull this chimp up to safety. The rope itself has a mass of 1.16 kg/ m. Using 9.81 m/ s^2 for earth's gravitational acceleration, write down an integral that represents the amount of work you've done after pulling ℓ meters of the rope up towards the roof.

* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, demonstrate anything you've learned in this class that you haven't already gotten a chance to present on this exam.