

Math135 Engineering Calculus I
Second Midterm Exam
Colorado Mesa University Fall 2023

NAME: _____

1. Without the aid of technology, demonstrate how to find a formula for the derivative of each of the following functions. Express the derivative formula concisely.

(a) $a(x) = 42$

(b) $b(x) = x^{42} + \frac{1}{x^{42}}$

(c) $c(x) = 42(\cos(x) - 42)$

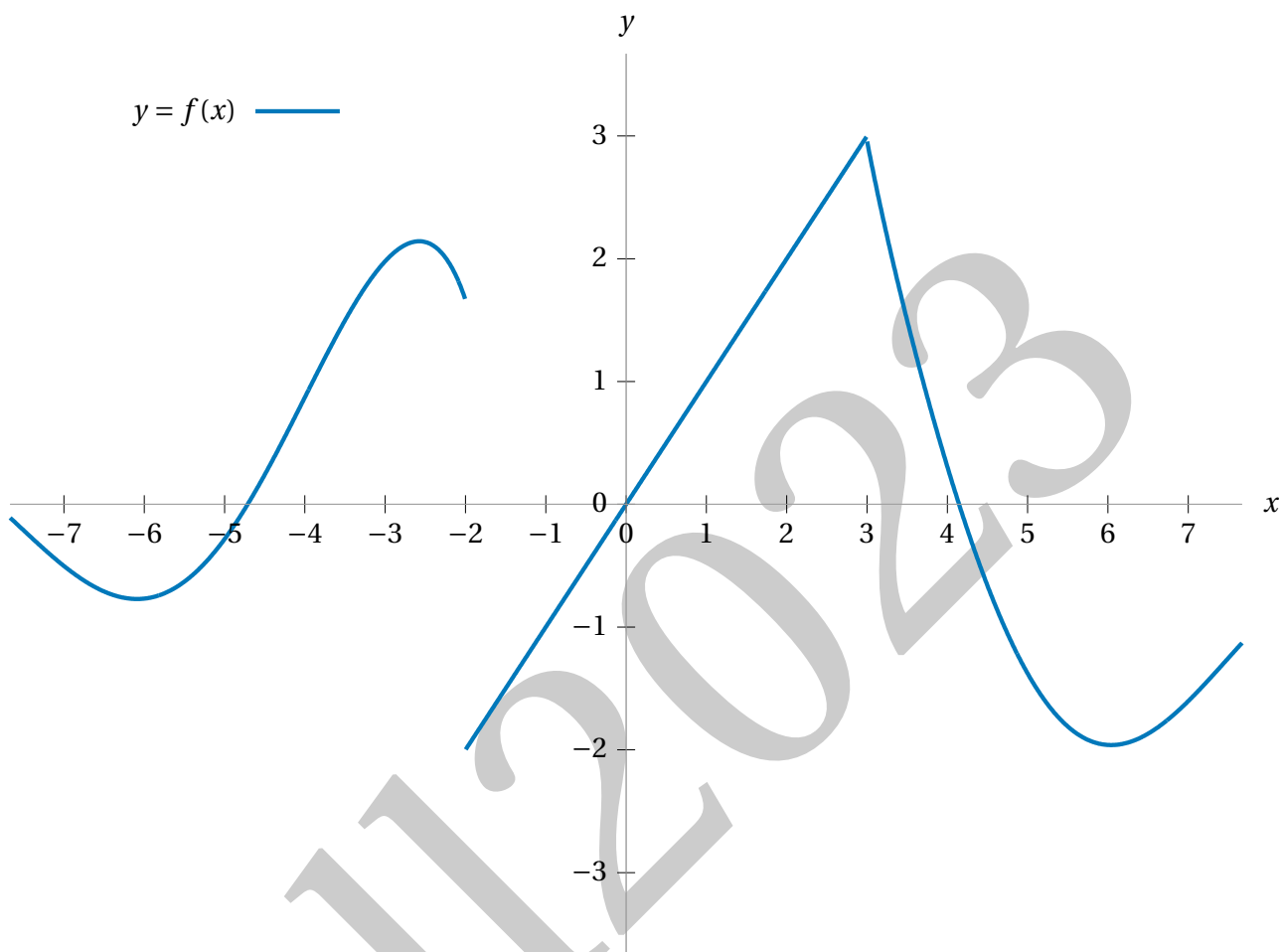
(d) $d(x) = \cos(42x) + \cos(x^{42})$

2. What is the *exact* value of the slope of the line tangent to $y = x \sec(x)$ at the point where $x = \frac{\pi}{3}$?

3. What function is the result of evaluating this limit?

$$\lim_{h \rightarrow 0} \frac{\tan(x+h) - \tan(x)}{h}$$

4. Here's the graph of a function $y = f(x)$.



(a) Estimating, on what intervals is $f''(x) > 0$?

(b) Estimating, what are the values of the following?

$$f(-3)$$

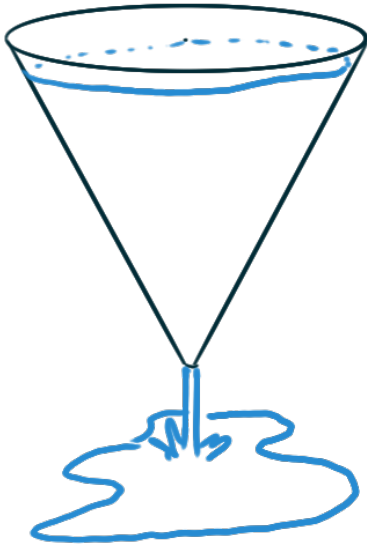
$$f'(-3)$$

$$f(4)$$

$$f'(4)$$

(c) On the same set of axis, sketch the graph of $y = f'(x)$. Be sure to label any point where the output of f' is not defined with an open bubble.

5. An inverted-conical tank of height 10 meters and base radius 4 meters is initially full of water. Then water begins draining from a spigot at the bottom point of the tank at the rate of $5 \text{ m}^3/\text{min}$. How fast is the water level dropping at the moment the depth of the water is 6 meters? Express your answer accurate to five decimal places. NOTE: the volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the base of the cone and h is its height.



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6. Let x and y be two positive numbers under the constraint that $x + 2y = 4$. Determine the values of x and y that maximize the value of $(x + 1)(y + 2)$.

Additionally, demonstrate how a second-derivative can be used to verify those values of x and y do indeed correspond to a maximum.

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* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, write about anything you've learned in this class that you haven't already gotten a chance to demonstrate on this exam.

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