Math135 Engineering Calculus I

First Midterm Exam

Colorado Mesa University Fall 2023

NAME: _____

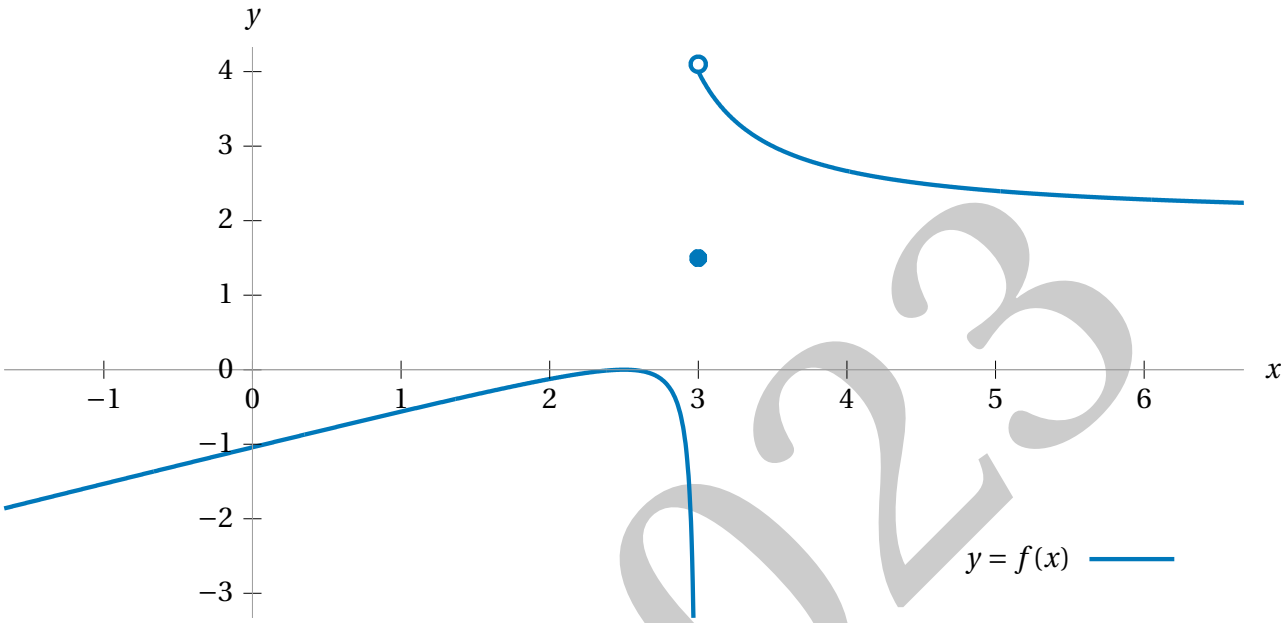
1. Explain, as if explaining to a peer in the class, what the notation $\lim_{x\to 3} f(x)$ means.

2. What is the precise, mathematical definition of a function f being *continuous at a point c*?

3. What's an example of a single function f that satisfies all three of these requirements?

$$\lim_{x\to-\infty} f(x) = \infty \qquad \lim_{x\to 8} f(x) = 8 \qquad \lim_{x\to\infty} f(x) = \infty$$

4. Based on this graph of $y = f(x)$ estimate the values of the following expressions. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.



(a) $\lim_{x\to-\infty} f(x)$

(b) $\lim_{x\to\infty} f(x)$

(c) $\lim_{x\to3^-} f(x)$

(d) $\lim_{x\to3^+} f(x)$

(e) $\lim_{x\to3} f(x)$

(f) $f(3)$

5. Using technology, approximate the value of the following limit accurate to five decimal places.

$$\lim_{x\to0}\left(\frac{1}{\sin^2(x)} - \frac{1}{x^2}\right)$$

6. What are the values of the following expressions? If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out.

(a) $\lim_{x\to 2} 5$

(b) $\lim_{x\to 2} 5x$

(c) $\lim_{x\to 0} \frac{1}{x}$

(d) $\lim_{x\to 2} \frac{(x-2)(x+2)}{x}$

(e) $\lim_{x\to 7^-} \frac{1}{(x-7)^2}$

(f) $\lim_{x\to \infty} \frac{\sin(x)}{x}$

7. *Demonstrate* how to manually (algebraically) calculate the values of the following limits. If the value of any limit is not defined (does not exist), indicate this by simply crossing the expression out. You may freely use the fact that $\lim_{x\to 0} \sin(x)/x = 1$. You may use L'Hospital's rule for evaluating limits only if you can *prove* that L'Hospital's rule works on the last page of this exam.

(a) $\lim_{x\to 0} \frac{\sin(7x)}{8x}$

(b) $\lim_{x\to 3} \frac{x-3}{\sqrt{2x-5}-1}$

8. You are standing on the surface of the planet Mercury. It is hot. Despite your extreme discomfort you toss a stone straight up into the air at a speed of 70 ft/s. The height of the stone above you t seconds after you throw it is roughly modelled by the function $h(t) = 70t - 6.06t^2$. What's the highest the stone gets? How long after you throw it does it get this high? What is the average velocity of the stone in the three seconds just before it hit this highest point? Express your answers to each of these questions as a decimal number accurate to five decimal places.

9. Let g be the function defined piecewise as

$$g(x) = \begin{cases} 1-x & \text{for } x < -7 \\ f(x) & \text{for } -7 \le x < 2 \\ 2x-1 & \text{for } x \ge 2 \end{cases}$$

Write down *any* formula $f(x)$ that defines a function f such that g will be continuous. Demonstrate that your definition of f satisfies the requirement.

* (TRIVIA) What 2004 comedy starring Tina Fey and Lindsay Lohan featured the following limit?

$$\lim_{x\to 0} \frac{\ln(1-x)-\sin(x)}{\sin^2(x)}$$

* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, write about anything you've learned in this class that you haven't already gotten a chance to demonstrate on this exam.