

Math114 Trigonometry

Yet Another Pop Quiz

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NAME: _____

1. Consider the points $O = (0,0)$ and $P = (12,4)$ and $Q = (7,24)$, and let $\mathbf{u} = \overrightarrow{OP}$ and $\mathbf{v} = \overrightarrow{OQ}$.

(a) Calculate $\mathbf{v} - 3\mathbf{u}$.

$$\mathbf{u} = \langle 12, 4 \rangle \text{ and } \mathbf{v} = \langle 7, 24 \rangle$$

$$\mathbf{v} - 3\mathbf{u} = \langle 7, 24 \rangle - 3\langle 12, 4 \rangle = \langle 7 - 3(12), 24 - 3(4) \rangle = \langle -29, 12 \rangle$$

(b) Calculate $\mathbf{u} \cdot \mathbf{v}$.

$$\mathbf{u} \cdot \mathbf{v} = (7 \times 12) + (4 \times 24) = 180$$

(c) Calculate the distance between P and Q .

$$\sqrt{(5)^2 + (20)^2} = \sqrt{425} = 5\sqrt{17} \approx 20.615528$$

(d) Calculate the coordinates of the midpoint between P and Q .

$$\bar{x} = \frac{12+7}{2} = \frac{19}{2} \quad \bar{y} = \frac{4+24}{2} = 14$$

so the midpoint is $(\frac{19}{2}, 14)$.

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- (e) Calculate $\hat{v}$, the unit vector in the direction of v .

Since $|v| = \sqrt{7^2 + 24^2} = 25$, we have $\hat{v} = \left\langle \frac{7}{25}, \frac{24}{25} \right\rangle$.

- (f) Calculate $\text{proj}_v(u)$, the projection of u onto v .

Recalling that in general $\text{proj}_v(u) = \left(\frac{u \cdot v}{|v|^2} \right) v$, for our specific u and v we get that

$$\text{proj}_v(u) = \left(\frac{180}{625} \right) \langle 7, 24 \rangle = \left\langle \frac{252}{125}, \frac{864}{125} \right\rangle.$$

- (g) Calculate the measure of the angle between u and v .

Since $u \cdot v = |u||v|\cos(\theta)$ where θ is the angle between u and v , we have

$$\theta = \arccos\left(\frac{u \cdot v}{|u||v|}\right) = \arccos\left(\frac{180}{(4\sqrt{10})(25)}\right) = \arccos\left(\frac{9}{5\sqrt{10}}\right) \approx 55.30484647^\circ.$$

- (h) Calculate the area of the triangle with vertices O and P and Q .

The area of this triangle will be

$$\frac{1}{2}|u||v|\sin(\theta) \approx \frac{1}{2}(4\sqrt{10})(25)\sin(55.30484647^\circ) \approx 130.$$

2. Consider the vectors $\mathbf{u} = \langle 3, 1, 4 \rangle$ and $\mathbf{v} = \langle x, 7, 1 \rangle$. What must x be so that $\mathbf{u} \cdot \mathbf{v} = 23$?

Since $23 = \mathbf{u} \cdot \mathbf{v} = 3x + 11$ we must have $x = 4$.

3. Suppose an airplane has a velocity vector of $\langle 520, -231 \rangle$ mph, but there is a strong 50 mph wind blowing from the southwest. What is the course of the plane as an angle? What is the actual speed of the plane?

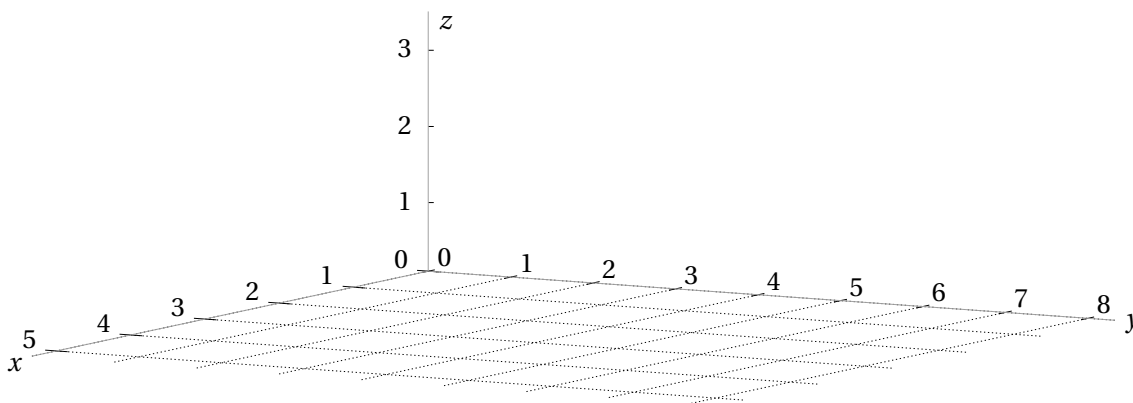
Blowing from the southwest the direction of the wind is $\langle 1, 1 \rangle$. The unit vector in this direction is $\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$ so a velocity vector with a magnitude of 50 mph in this direction will be $\langle 25\sqrt{2}, 25\sqrt{2} \rangle$. The course of the airplane will be the sum of this wind velocity vector with its own velocity vector:

$$\langle 25\sqrt{2}, 25\sqrt{2} \rangle + \langle 520, -231 \rangle \approx \langle 555.35534, -195.64466 \rangle .$$

So since $\arctan\left(-\frac{195.64466}{555.35534}\right) \approx -19.4067^\circ$ the course of the plane is S 70.5933° E. The speed of the plane will be

$$|\langle 555.35534, -195.64466 \rangle| \approx 588.809 \text{ mph}.$$

4. Consider the points $P = (2, 1, 3)$ and $Q = (1, 6, 2)$. Plot these points in xyz -space below, sketch the line segment between them, and calculate the length of that segment.



The length of the segment is $\sqrt{(1-2)^2 + (6-1)^2 + (2-3)^2} = \sqrt{27} \approx 5.19615$.

5. Here are two common definitions of the sine function:

- For a right triangle with angle of measure θ , the output $\sin(\theta)$ is the ratio between the length of the side of the triangle opposite θ with the length of the hypotenuse.
- For a vector pitched at an angle of θ from a given vector, the output $\sin(\theta)$ is the factor by which the length of the first vector and the length of its projection onto the plane to which the second vector is normal differ.

To someone new to studying trigonometry these definitions may seem unrelated, or even in conflict. However, they are quite related, and they do not contradict one another. Explain, as if explaining to someone new to studying trigonometry, how these definitions really do agree with each other and are just different interpretations of the same sine function. Your explanation may certainly be accompanied by illustrations.

Explanations will vary, but should contain this core idea: Referring to the second situation, let $\mathbf{v}$ be the first vector and $\mathbf{n}$ be the second given vector and let θ be the angle between them. Consider the right triangle having hypotenuse length $|\mathbf{v}|$ and side of length $|\text{proj}_{\mathbf{n}}(\mathbf{v})|$ adjacent θ . The length ℓ of the remaining side of this triangle will be the same as the length of the projection of $\mathbf{v}$ onto the plane to which $\mathbf{n}$ is normal, which means $|\mathbf{v}| \sin(\theta) = \ell$. Rewriting that last equation as $\sin(\theta) = \frac{\ell}{|\mathbf{v}|}$ presents us with exactly the ratio the first situation mentions.

Conversely one could argue from the first situation to the second by embedding the triangle in question into space and aligning vectors $\mathbf{v}$ and $\mathbf{n}$ to its hypotenuse and leg adjacent θ respectively.

6. There are five houses of different colors next to each other on the same road. In each house lives a woman of a different nationality. Each woman has her favorite drink, her favorite brand of cigarettes, and keeps a particular kind of pet.

The Englishwoman lives in the red house.

The Swede keeps dogs.

The Dane drinks tea.

The green house is just to the left of the white one.

The owner of the green house drinks coffee.

The Pall Mall smoker keeps birds.

The owner of the yellow house smokes Dunhills.

The woman in the center house drinks milk.

The Norwegian lives in the first house.

The Blend smoker has a neighbor who keeps cats.

The woman who smokes Blue Masters drinks bier.

The woman who keeps horses lives next to the Dunhill smoker.

The German smokes Prince.

The Norwegian lives next to the blue house.

The Blend smoker has a neighbor who drinks water.

Who keeps fish?