

Math114 Trigonometry
Just a Pop Quiz
Colorado Mesa University

NAME: _____

1. Consider the circle given by the equation $x^2 + y^2 = 21025$.

(a) What is the radius of this circle?

The radius is $\sqrt{21025} = 145$.

(b) Does the point $(123, -45)$ lie inside or outside this circle?

Inside, because $\sqrt{123^2 + (-45)^2} \approx 130.97 < 145$.

(c) What are the x -coordinates of the points on this circle that have y -coordinate 100?

Since y must be 100, x must satisfy the equation $x^2 + (100)^2 = 21025$. Solving that equation for x we get that $x = \pm\sqrt{11025} = \pm 105$.

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Verify real quick that the points $(17, 144)$ and $(143, -24)$ lie on the perimeter of this circle.

(d) What are the coordinates of the midpoint of those two points?

The midpoint has coordinates $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{17 + 143}{2} = 20.5 \quad \bar{y} = \frac{144 + (-24)}{2} = 143.5.$$

(e) What is the straight-line distance between these two points?

$$\text{The distance is } \sqrt{(143 - 17)^2 + (-24 - 144)^2} = \sqrt{44100} = 210.$$

(f) What is the distance between these two points along the arc of the circle?

Measuring from the origin, the point $(17, 144)$ is inclined at an angle of $\arctan(144/17)$ from the x -axis, and the point $(143, -24)$ is declined at an angle of $\arctan(-24/143)$ from the x -axis. The total angle between them then is $\arctan(144/17) - \arctan(-24/143) \approx 92.794362^\circ$. A full rotation corresponds to an angle measure of 360° so the length of arc of the circle between those points will be the fraction $92.794362^\circ/360^\circ$ of the full circumference of the circle:

$$\frac{92.794362}{360} (2\pi \times 145) \approx 234.837236$$

(g) What is the area of the sector of the circle subtended by the arc between these two points?

Similar idea to the last question, since that arc constitutes the fraction $92.794362^\circ/360^\circ$ of the full circumference of the circle, the subtended sector will constitute that same fraction of the full area of the circle:

$$\frac{92.794362}{360} (\pi \times (145)^2) \approx 17025.699613$$

2. What are the rectangular coordinates of the point whose polar coordinates are $(7, 33^\circ)$?

$$(7 \cos(33^\circ), 7 \sin(33^\circ)) \approx (5.870694, 3.81247)$$

3. What are the *principal* polar coordinates of the point whose rectangular coordinates are $(-123, 456)$?

$$\left(\sqrt{(-123)^2 + (456)^2}, 180^\circ + \arctan(-456/123) \right) \approx (473.065, 105.09552^\circ)$$

4. What is the degree measure of an angle that has a radian measure of six? Express the degree measure as a decimal of a degree *and* in degrees-minutes-seconds (DMS).

$$6 = 6 \times \left(\frac{360^\circ}{2\pi} \right) \approx 343.774677^\circ$$

$$\begin{cases} 0.774677^\circ \times \left(\frac{60'}{1^\circ} \right) \approx 46.4806247' \\ 0.4806247' \times \left(\frac{60''}{1'} \right) \approx 28.83748'' \end{cases} \implies 343.774677^\circ \approx 343^\circ 46' 29''$$

5. What is the acute angle at which the line $y = \frac{2}{7}x - 6$ and the x -axis intersect?

The arctangent function takes a slope and returns the rightward angle of inclination/declination from the horizontal axis:

$$\arctan(2/7) \approx 15.945396^\circ$$

6. What is the smallest *positive* angle θ for which $\sin(\theta) = -\frac{1}{2}$?

$$\theta = 210^\circ$$

7. What are the *exact values*, expressed in terms of radicals if necessary, of each of the following outputs of a trigonometric function? No need to rationalize the denominator.

(a) $\cos(405^\circ)$

(b) $\sin(60^\circ)$

(c) $\tan(60^\circ)$

(d) $\sec(45^\circ)$

(e) $\csc(-120^\circ)$

(f) $\cot(40020^\circ)$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

$$-\frac{2}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}}$$

8. Explain, as if explaining to a twelve-year-old, the significance of *radian* measure of an angle. Feel free to compare/contrast it to degree measure, to use examples, or to draw any diagrams to help your explanation.

Explanations will vary, but should contain this core idea: thinking of an angle as corresponding to a sector of some circle, the radius r of the circle and the length ℓ of the arc of the circle that subtends the sector will be directly proportional, the proportion depending on the angle. The radian measure of the angle is *defined* to be this proportion, the ratio ℓ / r . In other words, the radian measure of an angle is the number of circle-radii that will lay along an circular arc subtended by the angle.

Another, less direct, way to get at this core idea: while a full rotation corresponds to an angle measuring 360° (an arbitrary choice) a full rotation corresponds to an angle with radian measure of 2π since there are 2π radii-lengths along a circle's (full rotation's) circumference regardless of the circle's size.

9. Imagine a planet covered entirely in water with the exception of a single tiny island. On this island is a large fleet of airplanes and a bottomless supply of fuel. These airplanes have two peculiar features: on a single full tank of fuel each plane can each fly exactly half-way around their planet, and each plane is capable of exchanging fuel with another plane instantaneously midair. What is the fewest number of planes that would be needed to allow a single plane to fly completely around the planet along a great circle, such that every plane makes it safely back to the island?