

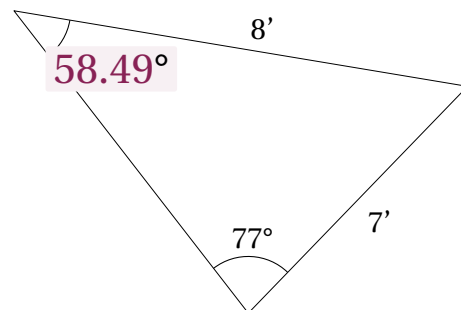
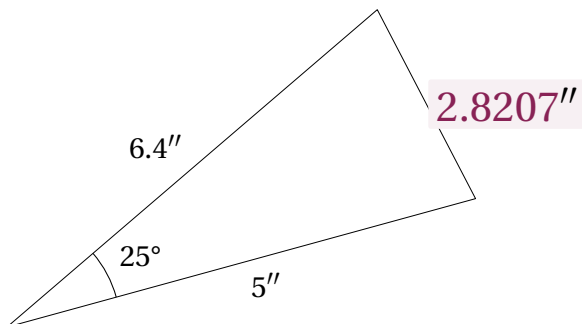
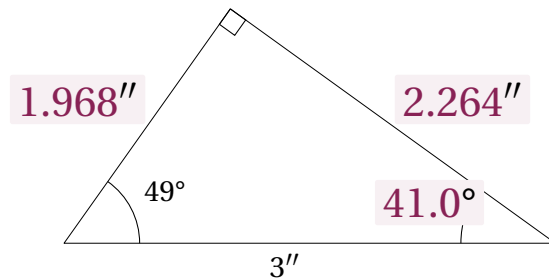
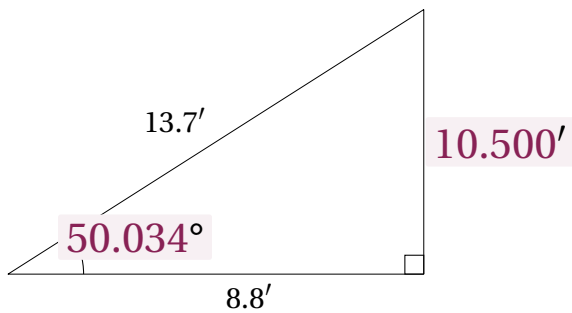
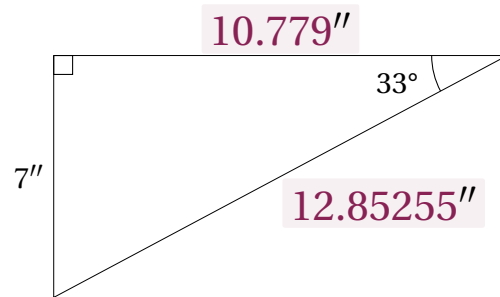
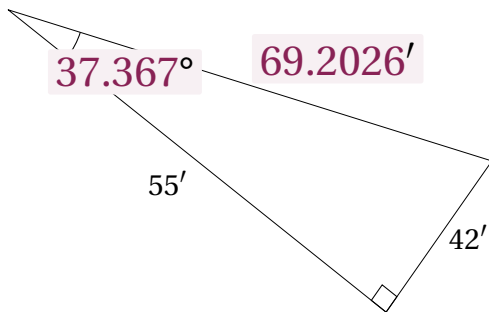
Math114 Trigonometry

Just Another Pop Quiz

Colorado Mesa University

NAME: _____

1. For each of the following triangles, fill in each shaded box with the measure of that side or angle.



2. What are three different angles θ for which $\cos(\theta) = \frac{1}{3}$?

Since $\arccos\left(\frac{1}{3}\right) \approx 70.52878^\circ$ that is one possibility for the angle θ . Another possible angle is -70.52878° . Another possible angle is $360^\circ - \arccos\left(\frac{1}{3}\right) \approx 289.47122^\circ$. In general, any number of full rotations away from $\pm \arccos\left(\frac{1}{3}\right)$ will result in an angle θ such that $\cos(\theta) = \frac{1}{3}$

3. The ADA (Americans with Disabilities Act) requires that wheelchair ramps to access public structures must have a rise:run ratio of no more than 1:12.

(a) What slope (grade) does this requirement correspond to? Express the slope as a percent.

$$\frac{1}{12} = 0.08\bar{3} \approx 8.3\%$$

(b) At what angle should such a ramp be inclined?

$$\arctan\left(\frac{1}{12}\right) \approx 4.7636417^\circ$$

(c) If you have to install such a ramp up to a stoop landing that is 1'8" above the level ground, how far from the landing does the base of the ramp need to start?

In terms of ratios, $\frac{\text{rise}}{\text{run}} = \frac{1}{12} = \frac{1'8''}{???}$ so the unknown distance ??? is $12 \times 1'8'' = 20'$.

Alternatively since the angle of inclination is 4.7636417° the distance will be $\frac{1'8''}{\tan(4.7636417^\circ)}$.

(d) If you have to install such a ramp up to a stoop landing that is 1'8" above the level ground, and then cover the length of the ramp's surface with with that slip-resistant traction material (the sandpaper-like stuff), what length of that material do you need?

Using the Pythagorean theorem we'd need $\sqrt{(1'8'')^2 + (20')^2} \approx 20.07' \approx 20'1''$.

Note that $\frac{1'8''}{\sin(4.7636417^\circ)}$ and $\frac{20'}{\cos(4.7636417^\circ)}$ give the same result.

4. Adeline and Casper are in the center of a lake near a buoy, each standing on their own paddle-board. They take off paddling in separate directions straight away from the buoy. After a moment they each stop. Adeline pulls a total station out of nowhere and, seeing Casper and the buoy, measures that she is 123' from the buoy and that the angle between Casper and the buoy from her position is 46° . She wants to know the same measurements from Casper's spot, but upon contacting him over their walkie-talkies learns Casper only brought his theodolite onto the lake, and can only report that from his vantage the angle between Adeline and the buoy is 50° . How far apart are Adeline and Casper?

The measure of the angle between Adeline and Casper from the perspective of the buoy will be $180^\circ - (50^\circ + 46^\circ) = 84^\circ$. If ℓ is the distance between Adeline and Casper, by the law of sines we have

$$\frac{\sin(84^\circ)}{\ell} = \frac{\sin(50^\circ)}{123'} \implies \ell = 123' \times \frac{\sin(84^\circ)}{\sin(50^\circ)} \approx 159.6855'.$$

5. After having sailed northeast from Puerto Rico towards the heart of the north Atlantic Ocean for three days and three nights, Captain Olimar spots two shipping vessels at sea on the distant horizon. His ship's radar indicates that one vessel is 2.4 miles away and 14° off the port side (left/counterclockwise) of their ship's heading while the other vessel is 3.1 miles away and 35° off the starboard side (right/clockwise). How far apart are the two shipping vessels?

From Captain Olimar's perspective the total angle between the vessels is 49° . Letting ℓ be the distance between the two vessels, per the law of cosines we have

$$\begin{aligned} \ell^2 &= (2.4)^2 + (3.1)^2 - 2(2.4)(3.1)\cos(49^\circ) \\ \implies \ell &= \sqrt{(2.4)^2 + (3.1)^2 - 2(2.4)(3.1)\cos(49^\circ)} \approx 2.368088 \text{ miles.} \end{aligned}$$

6. What is the area of a triangle with side-lengths 15'' and 28'' and 41''?

Let A and B and C be the sides of lengths 15'' and 28'' and 41'' respectively, and let θ be the measure of the angle between the sides A and B . The area of the triangle is $\frac{1}{2}|A||B|\sin(\theta)$. We can use the law of cosines to calculate θ :

$$41^2 = 15^2 + 28^2 - 2(15)(28)\cos(\theta) \implies \theta = \arccos\left(\frac{15^2 + 28^2 - 41^2}{2(15)(28)}\right) \approx 143.1301^\circ$$

So the area must be $\frac{1}{2}(15)(28)\sin(143.1301) = 126\text{in}^2$.

7. Here are two common definitions of the sine function:

- For a right triangle with angle of measure θ , the output $\sin(\theta)$ is the ratio between the length of the side of the triangle opposite θ with the length of the hypotenuse.
- The output $\sin(\theta)$ is the y -coordinate of the point on the unit circle $x^2 + y^2 = 1$ corresponding to an angle of θ inclined from the positive x -axis.

To someone new to studying trigonometry these definitions may seem unrelated, or even in conflict. However, they are quite related, and they do not contradict one another. Explain, as if explaining to someone new to studying trigonometry, how these definitions really do agree with each other and are just different interpretations of the same sine function. Your explanation may certainly be accompanied by illustrations.

Explanations will vary, but should contain this core idea: if you think of the hypotenuse of a right triangle as the radius of a circle of length h and align the side of the triangle adjacent to the angle in question with the x -axis, the height of the opposite side will be $h\sin(\theta)$. Scaling all of this down by h will align our circle with the unit circle, and the length of the opposite side, now scaled down to $\sin(\theta)$, will be the y -coordinate of the point on the unit circle corresponding to the angle θ . In a sense the second definition is a special case of the first for when $h = 1$.

In more detail, from the opposite perspective: referring to the second definition imagine a ray emanating from the origin, inclined at an angle θ from the positive x -axis, and intersecting the unit circle at the point $(\cos(\theta), \sin(\theta))$, and imagine the triangle with that point along with the point $(\cos(\theta), 0)$ directly below it on the x -axis and the origin $(0, 0)$ as vertices. Being a radius of the unit circle, the length of the hypotenuse of this triangle is 1, and the height of the leg of this triangle opposite θ is $\sin(\theta)$, so $\sin(\theta) = \frac{\sin(\theta)}{1}$, which matches the first "definition" in a rather degenerate way. But imagine that you scale the unit circle by a factor of h , increasing all the lengths by a factor of h , but maintaining the value of θ . The length of the hypotenuse is now h and the height of the side opposite θ is now $h\sin(\theta)$, and so the same ratio $\frac{\sin(\theta)}{1} = \frac{h\sin(\theta)}{h}$, is equal to $\sin(\theta)$ as the first definition indicates.

8. Three friends Anita, Becca, and Charleston are challenged to a game by the Game Maestra. The Game Maestra paints two colored dots on each of the friends' foreheads and tells the friends that each dot is either blue or yellow, but neither color is used more than four times. She then places the three friends in a circle so that each of them can see the dots on their friends' foreheads, but not the dots on their own. The game proceeds like this: The Maestra will ask the friends in turn, first Anita, then Becca, then Charleston, then Anita again, then Becca again, and so on, if they know the colors of the dots on their foreheads. When someone responds "no," the Maestra asks the next person. If someone responds "yes" and is right, the friends win! However if someone responds "yes" and is wrong, all three friends will be banished to the shadow realm for eternity.

The friends were given no time to strategize, but they begin playing. Their responses in turn are "no," "no," "no," "no," "no," "yes," and the three friends win! Where are the colors of the dots on Becca's forehead?