Math113 College Algebra

Just a Pop Quiz

Colorado Mesa University

NAME: ____

1. How do you write the factored quadratic polynomial $(5x-1)(2x+3)$ in "standard form" ax^2+bx+c?

$$(5x-1)(2x+3) \quad = \quad 10x^2-2x+15x-3 \quad = \quad 10x^2+13x-3$$

2. What are the *exact* values, expressed in terms of arithmetic and square-root operations on whole numbers, of the roots of x^2+x-1?

For $a=1$ and $b=1$ and $c=-1$ the roots must be equal to

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad = \quad \frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)} \quad = \quad \frac{-1\pm\sqrt{5}}{2}.$$

So the roots are $\frac{-1+\sqrt{5}}{2}$ and $\frac{-1-\sqrt{5}}{2}$.

3. What is the minimum output value of the function $g(x)=\frac{2}{3}(x-2)(x+7)$?

Since g is a quadratic polynomial function, its graph $y=g(x)$ will be a parabola. Since its leading coefficient will be $\frac{2}{3}$, a positive number, this parabola will be opening upwards and its vertex (h,k) will correspond to a minimum value. Explicitly, the y-coordinate of the vertex of the graph of g will be the minimum output value.

The roots of g are 2 and -7, and the x-coordinate of vertex h must be the midpoint of those roots: $h=\frac{(2)+(-7)}{2}=-\frac{5}{2}$. The minimum we are looking then is the y-coordinate corresponding to this x-coordinate h.

$$\tfrac{2}{3}\left(-\tfrac{5}{2}-2\right)\left(-\tfrac{5}{2}+7\right) \quad = \quad \tfrac{2}{3}\left(-\tfrac{9}{2}\right)\left(\tfrac{9}{2}\right) \quad = \quad -\tfrac{27}{2}$$

4. Demonstrate how to algebraically find the x-coordinates of the two points where the graphs of the functions $f(x) = 2x + 5$ and $g(x) = x^2 + x - 1$ intersect.

The graphs will intersect where the functions' outputs are equal, at any x where $f(x) = g(x)$.

$$2x + 5 = x^2 + x - 1 \quad \Longrightarrow \quad 0 = x^2 - x - 6 \quad \Longrightarrow \quad 0 = \left(x - 3\right)\left(x + 2\right)$$

So the x-coordinates of the intersection points are $x = 3$ and $x = -2$.

5. Demonstrate how to algebraically find the positive value of t that satisfies this equation, then express this value as a decimal number accurate to within ± 0.001.

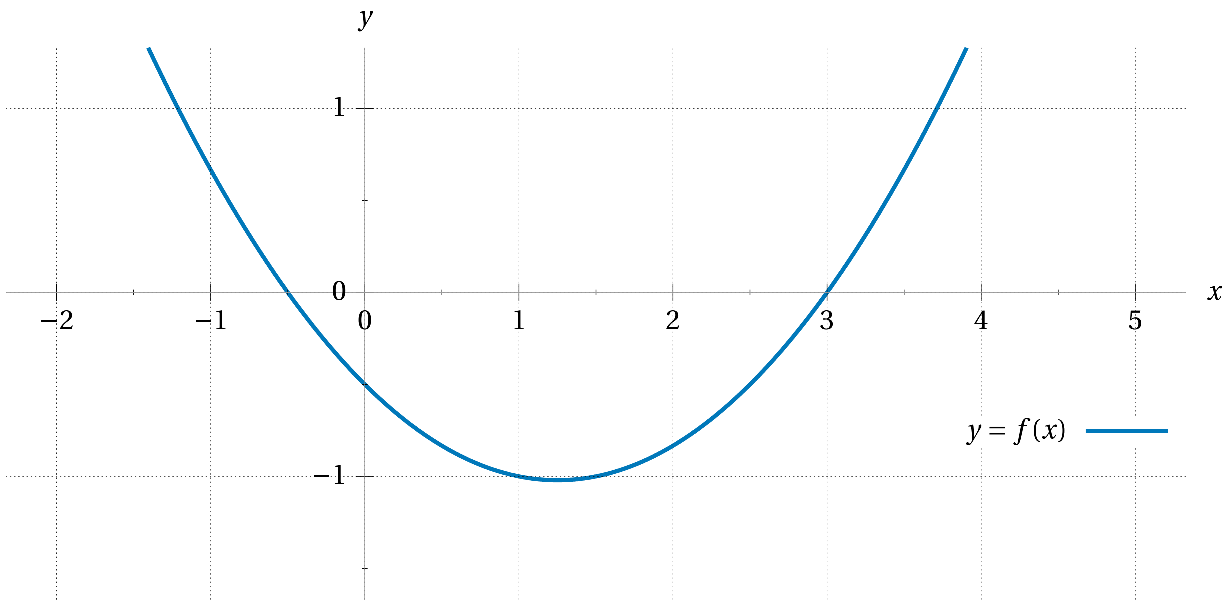
$$\tfrac{1}{5}\left(\tfrac{2}{5}t - \tfrac{3}{5}\right)^{\frac{4}{5}} = \tfrac{5}{5}$$

$$\tfrac{1}{5}\left(\tfrac{2}{5}t - \tfrac{3}{5}\right)^{\frac{4}{5}} = \tfrac{5}{5} \quad \Longrightarrow \quad \left(\tfrac{2}{5}t - \tfrac{3}{5}\right)^{\frac{4}{5}} = 5 \quad \Longrightarrow \quad \tfrac{2}{5}t - \tfrac{3}{5} = (5)^{\frac{5}{4}} \quad \Longrightarrow \quad \tfrac{2}{5}t = (5)^{\frac{5}{4}} + \tfrac{3}{5}$$

$$\Longrightarrow \quad t = \tfrac{5}{2}\left((5)^{\frac{5}{4}} + \tfrac{3}{5}\right)$$

$$\Longrightarrow \quad t \approx 20.191859765265256773898737420$$

6. Below is the graph of some function f.



(a) Estimating, based on its graph, what appears to be the value of $f(0)$?

It appears $f(0) \approx -\frac{1}{2}$.

(b) Estimating, based on its graph, for what value(s) of x does $f(x) = 1$?

It appears $f\left(-\frac{4}{3}\right) \approx 1$ and $f\left(\frac{11}{3}\right) \approx 1$.

(c) Estimating, based on its graph, for what appears to be the range of f?

The range appears to be all real numbers greater than or equal to -1. Said another way, the range appears to consist of all x such that $x \geq -1$. Said another way, the range appears to be the interval $[-1, \infty)$.

(d) Assuming f is a quadratic polynomial function, what's a plausible formula for $f(x)$?

Based on the roots that appear to be at $x = -\frac{1}{2}$ and $x = 3$, a formula for f plausibly matches the template $f(x) = a\left(x + \frac{1}{2}\right)\left(x - 3\right)$ for some value of the parameter a. Since it appears $f(0) = -\frac{1}{2}$ we discover $a = \frac{1}{3}$ by solving the equation $-\frac{1}{2} = a\left((0) + \frac{1}{2}\right)\left((0) - 3\right)$. So altogether we have $f(x) = \frac{1}{3}\left(x + \frac{1}{2}\right)\left(x - 3\right)$.

7. You are standing near the edge of a 113-foot high cliff on the surface of the planet Mercury. It is hot. Despite your extreme discomfort you toss a rock upwards, over the edge of the cliff, with an initial upwards velocity of 50 feet-per-second, and soon hear it clack against the hard pebbled ground at the cliff's base. The altitude of the rock above the ground t seconds after you toss it is modelled by the function $A(t) = 113 + 50t - 6.06t^2$.

(a) Demonstrate algebraically how to calculate the amount of time that passes between the rock being tossed and the rock reaching its maximum altitude (apex).

Since $t = 0$ corresponds to the moment the rock is tossed, the value of t that corresponds to the vertex of the parabolic graph of A will be the time the rock hits its apex:

$$h = \frac{-(50)}{2(-6.06)} = 4.\overline{1254}\,\text{seconds}$$

(b) Demonstrate algebraically how to calculate the amount of time that passes between the rock being tossed and the rock hitting the ground below.

Since $t = 0$ corresponds to the moment the rock is tossed, the value of t that corresponds to the positive root of the parabolic graph of A will be the time the rock hits the ground:

$$r = \frac{-(50) - \sqrt{(50)^2 - 4(-6.06)(113)}}{2(-6.06)} = \frac{50 + \sqrt{5239.12}}{12.12} \approx 10.0975\,\text{seconds}$$

8. (CHALLENGE) Suppose f is a quadratic polynomial function that satisfies the following property: for some numbers p and q and r we have

$$f(3) = pf(0) + qf(1) + rf(2).$$

What must the value of $p - q + r$ be?