Math113 College Algebra

Second Midterm Exam

Colorado Mesa University Spring 2024

NAME: _____

1. What are the exact solution(s) to this equation?

   $0 = (x+1)^2 - 9$

2. What are the exact solution(s) to the equation?

   $|2x-3| = -5$

3. What are the *coordinates* of the **x**-intercept(s) of the parabola given by the graph of $f(x) = x^2 + 6x - 55$?

4. What are the *coordinates* of the **y**-intercept(s) of the parabola given by the graph of $f(x) = x^2 + 6x - 55$?

5. On what interval of its domain is the function $f(x) = (x+3)^2 - 64$ decreasing?

6. Given the equation below, what must the value of r be? Express r as a decimal rounded to three decimal places.

$$1492 = 420(1 + r)^{2.718}$$

7. Consider the functions g and h defined as follows:

$$g(x) = 3(x - 1) \qquad h(x) = \begin{cases} |x| & \text{if } x < 2 \\ 6 - x^2 & \text{if } x \geq 2 \end{cases}$$

What are the values of the following expressions?

(a) $(hg)(-2)$

(b) $(g - 2h)(3)$

(c) $(h \circ g)(-3)$

8. The frequency at which Hollywood is producing Batman movies is increasing dramatically. Here is data correlating the rank of each Batman movie (the 1st movie, 2nd movie, 3rd movie, etc) since the original 1943 film *Batman* starring Lewis Wilson, with the year it was released *since 1900*[1].

Movie Rank	1	2	3	4	5	6	7	8	9	10	11	12	13
Year Since 1900	43	49	66	82	92	95	97	105	108	112	116	117	122

The function $B(x) = 44x^{\frac{2}{5}}$ provides a fairly good model for this data.

(a) According to this model, what year should we expect the 30th Batman movie to be released?

(b) The function B is invertible. Write down a formula for its inverse B^{-1}, and describe what the inputs and outputs of this inverse are in the context of the model.

(c) According to this model, how many Batman movies should we expect to exist by the year 2100?

(d) (CHALLENGE) According to this model, by what year should we start to expect that multiple Batman movies will be released *per year*, each and every year into the future?

[1] From en.wikipedia.org/wiki/Batman_in_film#List_of_films, excluding DC universe films not *about* Batman.

9. Here's historical data for the population of Cuba (in millions) according to The World Bank[2].

Year	1960	1981	1995	2011	2022
Population (in millions)	7.27	9.85	10.93	11.29	11.21

(a) Use technology to perform *quadratic regression* to find a quadratic model for this data as a function of *x years after 1950*. Write the formula for your model below. Round the parameters of your model to have two non-zero digits after the decimal place.[3]

(b) According to the model, what will the population of Cuba be in the year 2050?

(c) According to the model, eventually the population of Cuba will begin declining, and then diminish to zero. What year will the population of Cuba vanish? Briefly explain, as if explaining to a peer in the class, how you answered this question.

[2] data.worldbank.org/indicator/SP.POP.TOTL?locations=CU

[3] For example, the number 0.00018753 should be rounded to 0.00019. Also, recall that the capital "E" notation is scientific notation. For example the notation $3.7\text{E}{-2}$ means 3.7×10^{-2}.

(d) According to your model, during what year did/will the population hit a maximum, and what will this maximum population be? Briefly explain, as if explaining to a peer in the class, how to answers these questions *algebraically* using the formula for your model.

(e) Discuss the pros and cons of using a *quadratic* function to model the population of Cuba, versus using a *power* function. In your opinion, which of these two functions would serve as a better model for population in the long term? Why?

(f) How would the formula for your model have to change if instead of letting x be the number of years since 1950, you let x be the literal year? HINT: Think about *transformations* of functions.

* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, demonstrate anything you've learned in this class that you haven't already gotten a chance to present on this exam.