

Midterm Exam Three

Math 113-007/8 College Algebra
Colorado Mesa University Spring 2023

Name: _____

1. Write the following expression in a simpler form that only has a single x to a positive power and no parentheses.

$$\frac{x^{100}}{(2x^{-2})^3}$$

2. The expression

$$2\log_7(y) + \frac{1}{2}\log_7(x^5)$$

can be written as a single logarithm $\log_7(\square)$. What must \square be?

3. Demonstrate how to find the value(s) of x that satisfy this equation.

$$\log_{13}(2x - 1) = \frac{1}{3}$$

Write the value(s) as a decimal number rounded to five decimal places.

4. According to the CDC¹, here is historical data on the prevalence of *diagnosed* cases of diabetes in the US among adults (adjusted for age) as a percentage of the total population.

years since 2000	1	4	8	12	16	21
percent	6.4	7.0	7.9	8.4	8.5	8.5

- (a) Do you think an exponential model or a logarithmic model would fit the data best? Justify your response to a skeptical critic.
- (b) Based on your choice in the previous part, perform regression to find a function of t years after 2000 that models the data. Write your formula for your model below with parameters rounded to two decimal places.
- (c) What does your model predict the percent of US adults diagnosed with diabetes to be this year to be?
- (d) According to your model, by what year will 10% of the US adult population be diagnosed with diabetes?

¹gis.cdc.gov/grasp/diabetes/diabetesatlas-surveillance.html

5. Coloramo Credit Union offers *Certificates of Deposit* (CDs) to its members². A CD offers a higher interest rate than a typical savings account, in exchange for agreeing that you won't withdraw your funds for a certain fixed amount of time.

(a) Coloramo offers a four-year CD with a 2.869% interest rate compounded monthly. If you invest \$1000 into this CD, how much will your CD be worth after those four years?

(b) How much more would the CD be worth if instead of compounding the interest monthly Coloramo compounded the interest daily?

(c) Suppose you plan on graduating college in four years. You're living on campus now, so you don't have a car, but you know you'll need a car to commute to work after you graduate. You estimate that \$21,000 should be enough to buy an adequate used car in four years when you graduate, and figure why not deposit some money into Coloramo's four-year CD (in which they compound the interest monthly) and collect that \$21,000 in four years. How much money should you deposit in the CD today such that after four years it will be worth \$21,000?

²coloramo.org/rates/#certificate

6. Suppose that you bought a home in Grand Junction for \$314,000. After making a down payment, you took out a 15-year fixed-rate mortgage on the remaining balance of \$251,200 with your bank at an annual interest rate of 5.99%. To the right is the formula that describes a mortgage with *monthly* payments, where S is the value of the mortgage loan, P is the monthly payment, r is the annual interest rate, and t is the duration (the *term*) of the mortgage.

$$S = P \left(\frac{1 - \left(1 + \frac{r}{12}\right)^{-12t}}{\frac{r}{12}} \right)$$

(a) According to this formula, what are your monthly payments going to be?³

(b) Uh-oh! You can't afford those monthly payments! You're only comfortably able to pay \$1550 towards your mortgage each month. Your bank offers mortgages for longer terms though. You go to re-negotiate a longer-term mortgage with your bank; what's the shortest mortgage duration for which you can afford the monthly payments?⁴ Round your answer *up* to the nearest five-year term.

³Usually mortgage payments also include tax and insurance charges; ignore this fact.

⁴In reality your mortgage lender will be very displeased with this. At the very least they'll impose a higher interest rate for a longer mortgage, and probably also charge you some fees re-negotiating the original contract.