Math113 College Algebra

Second Midterm Exam

Colorado Mesa University Fall 2023

NAME: _____

1. What are the exact solution(s) to this equation?

   $0 = x^2 + 5x + 6$

2. What are the exact solution(s) to the equation?

   $|x - 5| = -4$

3. Given $g(x) = 2x + 1$ and $h(x) = x^3$, what's a formula for $h(x) - 2g(x)$?

4. How do you write the factored quadratic polynomial $(2x - 1)(x + 7)$ in "standard form" $ax^2 + bx + c$?

5. What are the coordinates of the vertex of the parabola given by the graph of $f(x) = (x + 3)^2 + 2$?

6. On what interval of its domain is the function $f(x) = (x + 3)^2 + 2$ increasing?

7. What must r be if $1992 = 777(1+r)^{3.141}$? Express r as a decimal rounded to three decimal places.

8. The function with formula $f(t) = \frac{5}{9}(t-32)$ takes a temperature t measured in °F and returns that temperature measured in °C. The function with formula $g(t) = t + 273.15$ takes a temperature t measured in °C and returns that temperature measured in Kelvins. Write down a formula for the function that take a temperature measured in °F and returns that temperature in Kelvins.

9. Here's historical data for the population of South Korea (in millions) according to The World Bank[1].

year	1960	1984	2002	2021
population (in millions)	25.0	40.4	47.6	51.7

(a) Use technology to perform *quadratic regression* to find a quadratic model for this data as a function of *x years after 1960*. Write the formula for your model below with parameters rounded to four decimal places.[2]

(b) According to your model, what will the population of South Korea be in the year 2060?

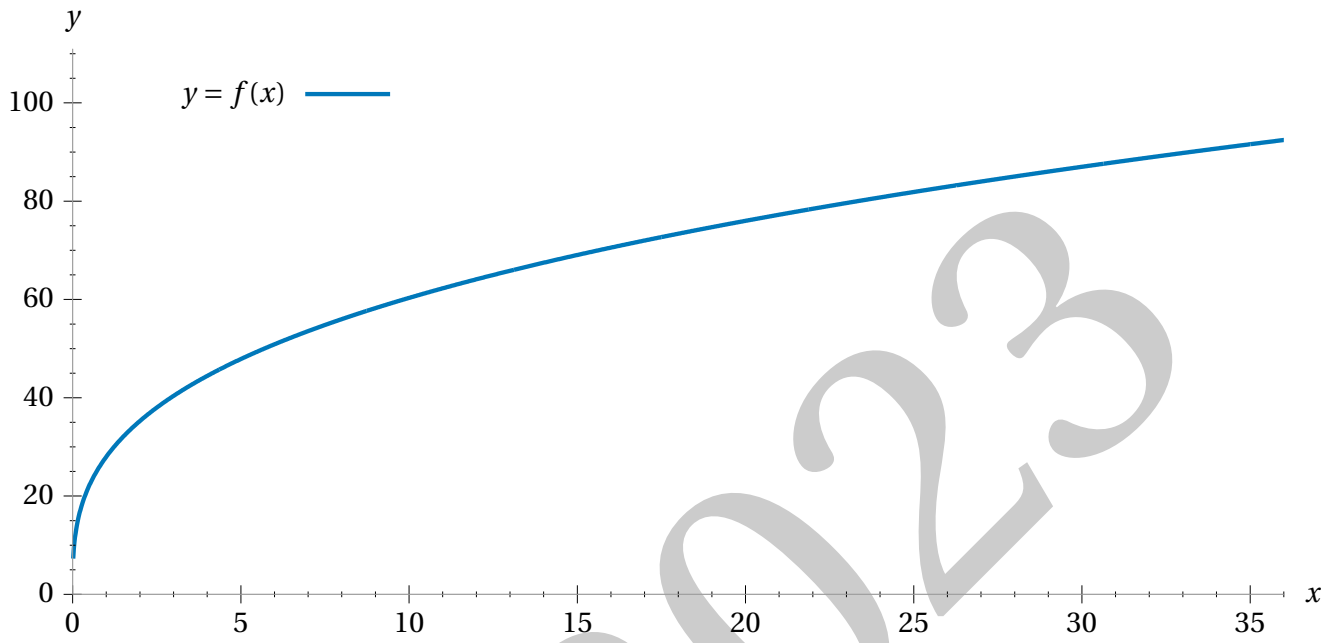
(c) According to your model, during what year will the population hit a maximum, and what will this maximum population be? Briefly explain, as if explaining to a peer in the class, how to answers these questions algebraically using the formula for your model.

(d) Do you think a quadratic function is a good choice of model for population in the long term? If you do, discuss the benefit of this model over others. If not, discuss why another model would be a better choice.

[1] data.worldbank.org/indicator/SP.POP.TOTL?locations=KR

[2] Recall that the capital "E" notation is scientific notation. E.g. the notation $3.7\,\text{E}-2$ means 3.7×10^{-2}.

10. Use this pair of axes and the graph of f to answer the following questions.



(a) The function f is a *power function*, so its formula looks something like ax^b for real numbers a and b. Based on the graph of f, what can you infer about the values of a and b? I.e. are they positive? Negative? Greater than one? Less than one? Etc.

(b) On the same set of axes, *accurately* sketch the graph the function g given by the formula:

$$g(x) = \begin{cases} 40 - 4x & \text{if } 0 \le x \le 10 \\ 120 - 4x & \text{if } 10 < x \end{cases}$$

(c) Label the point $(10, g(10))$, with its (explicit) coordinates.

(d) For which value(s) of x does $g(x) = 0$?

(e) Using technology or estimating using your sketch, what are the coordinates of the point(s) for which $f(x) = g(x)$? Try to be as accurate as possible.

11. What are the exact solution(s) to this equation?

    $|2x-4| = 11-3x$?

12. What are the values of the roots of the quadratic polynomial $16(x+7)^2-3$? Bonus: what are the *exact* values of the roots, expressed in terms of radicals (e.g. square roots).

13. (CHALLENGE) For which value(s) of c does the equation $x^2+16x+c=0$ have only a single solution?

* (OPTIONAL) The prompts on this exam were designed to elicit evidence of your understanding of the mathematics we've discussed in this course. But perhaps you've learned things that weren't prompted for. Perhaps you've gained some mathematical understanding that you haven't had an opportunity yet to exhibit on this exam. Now is your opportunity. On this page, write about anything you've learned in this class that you haven't already gotten a chance to demonstrate on this exam.