Math113 College Algebra

Second Midterm Exam

Colorado Mesa University 2024 Fall

NAME: _____

1. Consider the function f defined by the formula $f(x) = (x-1)^2 - 7$

   (a) What is the smallest (minimum) possible output value of f?

   (b) What is the y-coordinate of the y-intercept of the graph of f?

   (c) What are the *exact* values, expressed in terms of radicals, $\sqrt{\ }$, of the roots of f?

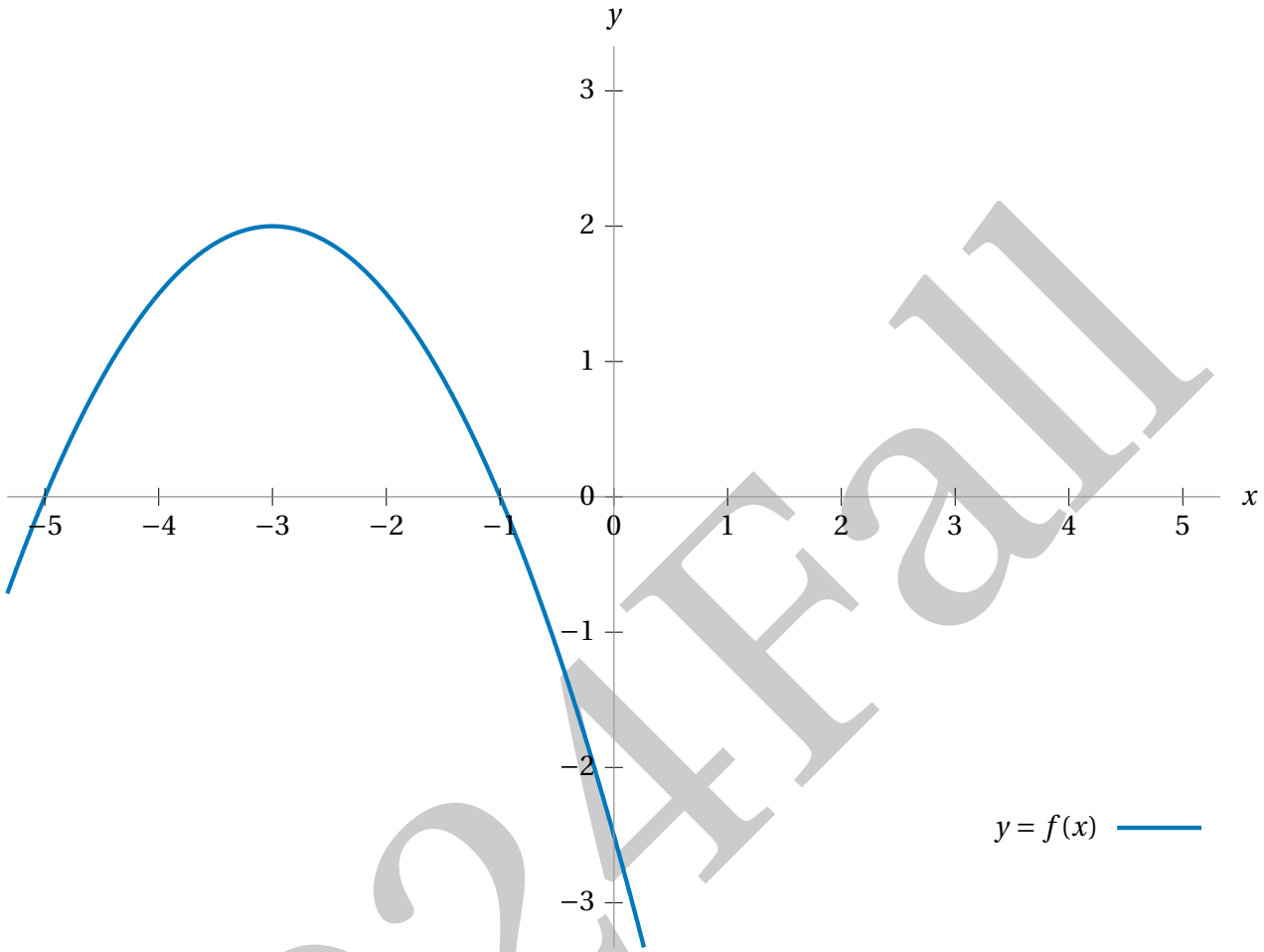
   (d) On what interval of its domain is f increasing?

   (e) What is the value of $\big(f \circ f\big)(2)$?

2. For the equation below, demonstrate how to *algebraically* solve for the value of r. Express r as a decimal accurate to four decimal places.

$$1776 = 123(r-5)^{3.1415}$$

3. Below is the graph of a function f.



(a) Assuming f is a quadratic polynomial, what's a plausible formula for $f(x)$?

(b) On these same axes sketch the graph of the piecewise-defined function

$$g(x) = \begin{cases} -2 - x & \text{if } x < 1 \\ 1.414x^{2.718} & \text{if } x \geq 1 \end{cases}.$$

4. Suppose you are on a space station orbiting the planet Jupiter. Bored, you are playing with a ball. Despite its extreme weight, you manage to toss the ball straight up into the air. The altitude (height) of the ball above you, in feet, measured x seconds after you throw it can be modelled by the function

$$A(x) = 5.8 + 34.4x - 40.2x^2.$$

(a) Rewrite the formula $A(x)$ in "vertex form" $a(x-h)^2 + k$ *and* in "factored form" $a(x-r_1)(x-r_2)$.

(b) What is the highest altitude the ball reaches? Express the answer accurate to four decimal places.

(c) Suppose you don't try to catch the ball on its way down; exactly how long after you initially threw it does it hit the ground? Express the answer accurate to four decimal places.

5. Here is data recorded on Wikipedia[1] for the population of Grand Junction for select decades.

Year	2020	2010	2000	1990	1980	1970
Population (in thousands)	65.6	58.6	42.0	29.0	28.0	20.2

(a) Which do you think would be a better choice of function to model the population of Grand Junction in the long term: a *quadratic* function or a *power* function? Why?

(b) Based on your choice of model in the previous prompt, use technology to perform *regression* to find a model f for the population of Grand Junction x years *since 1950*. (e.g. for the year 2008, $x = 58$.) Write the formula for your model here.Round parameters to four decimal places.

(c) (EXTRAPOLATE) Assuming this model remains accurate beyond the domain of the data, what does it predict the current population of Grand Junction to be?

(d) The population of Grand Junction is increasing; both a quadratic and power model will indicate this. But consider the *rate* at which the population is increasing. Is this rate increasing or decreasing? I.e. is the graph of your model concave up or concave down? What parameter of your model, and what aspect of that parameter, indicates this?

[1] From en.wikipedia.org/wiki/Grand_Junction,_Colorado#Demographics.