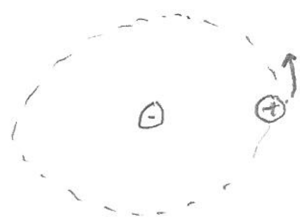
Spin - Orbit (LS) Coupling



electron sees this

Proton makes B field $\left| \frac{\mu_0 I}{2r} \right|$ → Biot Savart law

remind of RHR

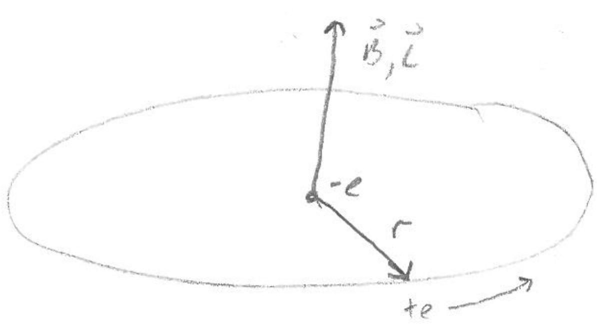
"Spinning" electron experiences a Torque whose Hamiltonian is $-\vec{\mu}\cdot\vec{B} = \hat{H}$

Now Proton current is $\frac{|e|}{T}$ T → Period $T = \frac{2\pi r}{v}$ → 1 revolution Time

but $|\vec{L}| = m_e v r$ so $T = \frac{2\pi r}{m v r} \cdot m r = \frac{2\pi m r^2}{|L|}$

$$|B| = \left| \frac{\mu_0 I}{2r} \right| = \frac{\mu_0 |L|}{2\pi m r^2} \cdot \frac{e}{2r} = \frac{\mu_0 |L| e}{4\pi m r^3}$$ but $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ $c^2 = \frac{1}{\epsilon_0 \mu_0}$ → $\mu_0 = \frac{1}{\epsilon_0 c^2}$

So $|B| = \frac{e|L|}{4\pi\epsilon_0 c^2 m r^3}$ or $\vec{B} = \frac{e\vec{L}}{4\pi\epsilon_0 c^2 m r^3}$



Now $\vec{\mu} = I\vec{A} = \frac{q_e}{T} \cdot \pi r^2 \hat{z}$ $\quad \vec{S}_e = I\vec{\omega}_e = \frac{mr^2 \cdot 2\pi}{T}$

gyromagnetic ratio $\left|\frac{\mu}{S_e}\right| = \frac{q_e}{2m}$ or $\vec{\mu} = \left(\frac{q_e}{2m}\right)\vec{S}_e$

Classically $\vec{\mu}_e = -\left(\frac{e}{2m}\right)\vec{S}_e$ correctly a g-factor needs to be incorporated

or $\vec{\mu}_e = g\left(\frac{q_e}{2m}\right)\vec{S}$ $\quad g_e = 2$ so $\vec{\mu}_e = \frac{q_e}{m}\vec{S} = \frac{-e}{m}\vec{S}$

$\hat{H}' = \left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{1}{m^2c^2r^3}\vec{S}\cdot\vec{L}$ Fraud calculation done in

inertial frame, Frame actually

$$\boxed{\hat{H}'_{so} = \left(\frac{e^2}{8\pi\epsilon_0}\right)\frac{1}{m^2c^2r^3}\vec{S}\cdot\vec{L}}$$ Spin orbit $\hat{H}_{so}' = -\vec{\mu}\cdot\vec{B}$

$\vec{L}, \vec{S}$ Now bad. $[\hat{H}, \hat{L}] \neq 0$ $[\hat{H}, \hat{S}] \neq 0$ does not commute

Need $\vec{J} = \vec{L} + \vec{S}$ $\quad \vec{J}^2 = (\vec{L}+\vec{S})(\vec{L}+\vec{S}) = L^2 + S^2 + 2\vec{L}\cdot\vec{S}$

good eigenstates use L^2, S^2, J^2, J_z

$$\boxed{\text{So } \vec{L}\cdot\vec{S} = \frac{1}{2}\left(J^2 - (L^2+S^2)\right)}$$

$$\boxed{\text{eigenvalues of } \vec{L}\cdot\vec{S} \rightarrow \frac{\hbar^2}{2}\left(j(j+1) - l(l+1) - s(s+1)\right)}$$

Now $j \in [|l-s|, |l+s|]$ $\quad n=1 \quad l=0 \quad s=\frac{1}{2} \quad j=\frac{1}{2}$

$n=3 \quad l=0,1,2 \quad s=\frac{1}{2} \quad j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$

$$\boxed{\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{l(l+\frac{1}{2})(l+1)\,n^3 a^3}}$$

So $\quad \hat{H}'_{so} = \left(\frac{e^2}{8\pi\epsilon_0}\right)\frac{1}{m^2c^2r^3}\,\vec{S}\cdot\vec{L}$

$$E'_{so} = \langle \hat{H}'_{so} \rangle = \frac{e^2}{8\pi\epsilon_0}\cdot\frac{1}{m^2c^2}\left\langle\frac{1}{r^3}\right\rangle\frac{\hbar^2}{2}\left[j(j+1) - l(l+1) - \frac{3}{4}\right]$$

$$E'_{so} = \frac{e^2}{8\pi\epsilon_0}\cdot\frac{1}{m^2c^2}\cdot\frac{(\hbar^2/2)\left[j(j+1) - l(l+1) - 3/4\right]}{l(l+\frac{1}{2})(l+1)\,n^3a^3}$$

$$\boxed{E'_{so} = \frac{(E_n^0)^2}{mc^2}\left[\frac{n\left[j(j+1) - l(l+1) - 3/4\right]}{l(l+\frac{1}{2})(l+1)}\right]}$$

$$\boxed{E'_{fs} = E'_r + E'_{so} = \frac{(E_n)^2}{2mc^2}\left(3 - \frac{4n}{j+\frac{1}{2}}\right)} \;!!$$

$$\boxed{\text{Now } E_{nj} = -\frac{13.6\,\text{eV}}{n^2}\left[\underbrace{1}_{E_n^0} + \underbrace{\frac{\alpha^2}{n^2}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)}_{E'_{fs}}\right]} \;!!!$$

use n, l, s, j, m_j