Physics 342, Homework 8 This is long.

1: Easy

A) Calculate the magnitude of the centrifugal acceleration due to the Earth's rotation at the equator, at a latitude of 45 degrees and at the north pole. Notice, this represents a local correction to gravitational acceleration that varies with latitude. Comment on the direction of this fictitious force. I want your answer in m/s^2. Notice, this is a correction to the local direction of $\vec{g}$.

B) There is also a centrifugal acceleration due to our orbit around the sun, calculate it's magnitude and compare it to the centrifugal force due to the Earth's rotation.

C) Calculate the speed you would need to be going in a car when turning in a circle of radius 100 meters that would produce an acceleration on the driver equivalent to the acceleration due to gravity near the surface of the Earth. Report this speed in terms of miles per hour.

D) What velocity would you need to be traveling at for the magnitude of the coriolis force to equal the magnitude of the centripetal force at a latitude of 45 degrees. Decompose your velocity as using the local plane geometry of figure 10-9. Pick which direction you should move to maximize the coriolis effect and put all of your velocity in that direction.Write your answer in miles per hour.

Hint - to calculate the MAGNITUDE at 45 degrees you do not need to form a vector cross product. Remember what the cross product means and look at figure 10-3

2: (short) You find yourself in a spinotron - the amusement park ride where you stand against a wall and the whole thing begins to rotate. Once it is rotating fast enough the floor drops out from under you and you find yourself suspended in midair. There are 4 forces acting on you - friction, the normal force of the wall, gravity, and the centripetal force. Free body the equilibrium situation and determine the time it takes for one complete rotation of the ride such that you are held in place against the wall neither slipping rising or falling off. I want a purely algebraic answer. What happens to you if the ride slows down or speeds up it's down rate from this equilibrium spin rate?

Start with the equation at the end of example 1.8 and set $\sum \vec{F}_{external} = m\vec{A}$ You'll get an equation in $\hat{z}$ and $\hat{r}$ for force and acceleration, solve them and then tell me what happens if $\omega = \dot{\theta}$ increases or decreases.

3: (long) Imagine a ball is set on a merry go round which is rotating counterclockwise at a fixed angular frequency ω. The ball starts at x_0 and y_0. Determine the equation of motion if only coriolis acceleration comes into play. Remember, the rotation is in the z direction and the motion is only in the x and y direction. See example 2.10 for help. Allow for arbitrary x_0 and y_0. Set $\vec{V}(0) = v_{x0}\hat{i} + v_{y0}\hat{j}$. Pick values for your angular frequency and initial velocities and initial positions to plot $\vec{r}(t)$. You'll need to pick numbers for initial conditions and solve for all the constants in $\vec{r}(t)$. This and the next problem are essentially two dimensional x and y problems. Neglect gravity, the normal force, and friction. Do this in the rotating frame, see equation 10.30.

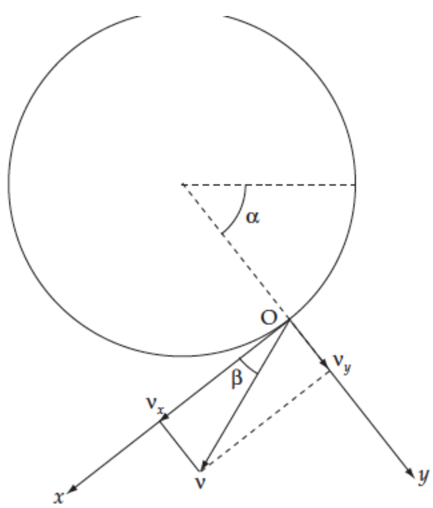
4: (Long) Repeat Problem 3:, this time do it for both the centripetal force and the coriolis force. Assume. You'll find your equations are coupled in x and y. Try to combine the two equations into a single equation using the complex variable $\eta = x + iy$. When you have combined the two equations into one equation in terms of $\eta, \dot{\eta}, \ddot{\eta}$, etc. This will be another heavily weighted problem. Graph your solution for various choices of the spin rate ω and various initial conditions. Do this in the rotating frame, see equation 10.30. Give me the full solutions for $x(t)$ and $y(t)$.

Hint - add the the a_x to $i \times a_y$ and determine a formula of the form $a\ddot{\eta} + b\dot{\eta} + c\eta = 0$. Attempt to find solutions proportional to $Ae^{-\Omega t}$ and determine Ω by direct substitution. You should find a repeated root in which case $\eta(t) = (A + Bt)e^{-i\Omega t}$ By inspection $A = x_0 + iy_0$ and $B = v_{x0} + iv_{y0}$. Equate the components and describe the subsequent evolution, graph it in x and y if you can.

5: A projectile is launched directly upwards with an initial $v_y = v_{y0}$. Determine an expression for its east/west deflection due to the coriolis force. Your answer should be in terms of λ, ω, g, and v_{y0}. This has essentially been done in the book and in class. Plug in real numbers for as high as you could throw a ball and a missile launched vertically to a height of 10,000 meters launched at Grand Junction's latitude.

6: A missile is fired due north at a latitude of 40 degrees south. and longitude 120 degrees east. The launch angle is 35 degrees with respect to the horizon. The initial velocity of the shell is 1000 m/s. What is the total deflection of the missile due to the coriolis force? Hint - calculate the coriolis deflection velocity and the time before the projectile lands. The total deflection is then just $v_{coriolis} \times t_{land}$. Please note that the solution to a very similar problem in the book is wrong. If you use the solution I'll know and this will be given a grade of zero. ω may be decomposed as $-\omega(cos(\alpha)\hat{i} + sin(\alpha)\hat{j})$. Your flight time should be 117 seconds, get this to get the deflection.

hint - use this geometry



7: (Longish but graded as such) Model a toilet bowl as a circular truncated cone with $r_1 = 1ft$ and $r_2 = r_1/4$ When you flush the toilet water begins to fall at approximately the free fall rate (gravitational rate $v_z = -gt$).

A) Now, the toilet will drain when all the water goes through the exit which can only happen at a rate of $\dot{M} = \rho_{water} v_{water} Area_{drain}$ (take this as given from fluid dynamics).The total time for a toilet to drain is then given by $\int \dot{M} dt = m_{waterinitiallyintoilet}$. Make some reasonable assumptions about the mass of water in a toilet and estimate the time it takes for a toilet to drain. Go flush a toilet and see if your number is remotely reasonable. The velocity here can be approximated as $|v| = |gt|$. Why is your answer too small? What happens that we neglect to account for? Go flush a question and answer this. I've given you water draining from a tank, not what happens in a toilet. Make an analogy to what you observe and what you learned in the chapter on why planets orbit.

B) Treating a single particle of H_2O as a point mass falling through a toilet height h in a time t given by your estimate above calculate the eastward/westward deflection and velocity in said direction due to the coriolis effect. Also calculate the magnitude of the maximum coriolis force per unit mass on a parcel of water falling at the velocity it reaches when it is going down the drain. Specifically, give me the z magnitude of $\vec{F}_{coriolis}/m$.

Hint If you're clever there is no new calculus here, look at example 10.3. Does it make a big difference if you use twice the time you calculated in part 1? 5 times the time?

C) Compare this force to the normal force acting on a molecule of H_2O sitting on a flat surface. Compare the two forces. Now, think about this. When a ball rolls down an inclined plane the normal force of the plane is taking the force due to gravity and re-directing the motion of the ball so that it rolls down at whatever the plane angle is. If you look at a toilet it is the geometry of the bowl and the resulting normal force which controls the sense of rotation.There IS a size of a tank of water for which the coriolis force is appreciable assuming no curvature. What would change if I replaced the toilet with one of the big pools at Sea World draining through the same size drain? What Changes?

8: A huge bucket of water is set spinning about its center. Determine the equilibrium profile of the water, specifically $z(h)$. There is a pressure force, a centrifugal force, and a gravitational force. Free body the gravitational versus centrifugal force to determine what the pressure force MUST look like. Recall $\vec{F}_p + m\vec{g} + \vec{F}_{centripetal} = 0$ More specifically, determine the angle between mg and $mr\omega^2$ and notice that the tangent of this angle is $\frac{dz}{dr}$. Either that or google equilibrium height of rotating fluid. What curve represents this shape? This is actually how people spin mirror surface when making telescopes using parabolic mirrors.