Newton's Law

#1. $\vec{F} = 0 \rightarrow \vec{a} = 0 \quad \vec{a} = \dot{\vec{v}}$ so $\frac{d\vec{v}}{dt} = 0$ so $\vec{v} = $ Constant

Something has to happen for something to happen

#2. $\vec{F}_{net} = \frac{d(\vec{P})}{dt}$ cause = effect $\vec{P} = m\vec{v}$

$$\rightarrow \vec{F}_{net} = \Sigma \vec{F}_i = m\frac{d\vec{v}}{dt} + \underbrace{\vec{v}\frac{dm}{dt}}_{\text{generally } 0}$$

#3. $\vec{F}_{12} = -\vec{F}_{21} \quad \frac{d\vec{P}_1}{dt} = -\frac{dP_2}{dt}$ or, why movie physics is nonsense

For $\frac{dm}{dt} = 0 \quad m_1\frac{dv_1}{dt} = -m_2\frac{dv_2}{dt} \quad \left[\frac{m_1}{m_2} = -\frac{\vec{a}_1}{\vec{a}_2}\right]$

Tangent → System → enclose a group of interacting objects within a circle which encompasses anything that can exert an influence

$m_1 \; m_2$ ↙ System

$m_1 \; m_2$ not a System

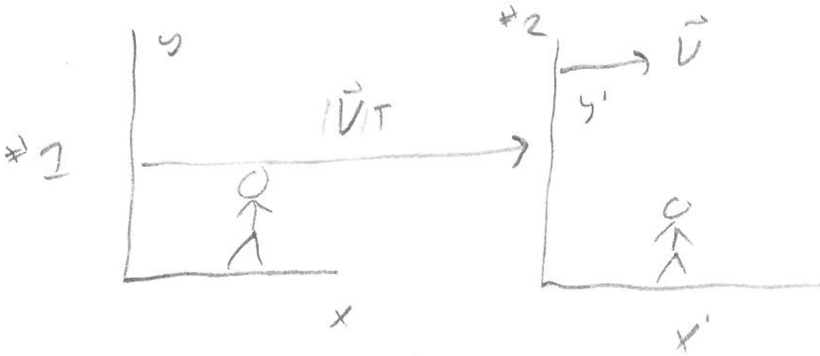
Then $\frac{d}{dt}(\vec{p}_1+\vec{p}_2) = 0$ or $\vec{p}_1+\vec{p}_2 =$ Constant

more generically $\sum \vec{p}_i =$ Constant

Inertial Reference Frames

Laws of physics are the same in 'inertial (non-accelerating) reference frames

Galilean invariance



$x + |\vec{v}|t = x'$

$y = y'$ #1 Watching a Train

$z = z'$ #2. On a Train

$t = t'$

is Earth inertial frame?

Locally "ok"

globally "not ok"

Equations of Motion

1. Coordinate System
2. Freebody
3. Newton's 2nd Law
4. Math

Uniform → constant $\vec{a}$

$\vec{F} = m\ddot{\vec{r}} \qquad \ddot{\vec{r}} = (a_x, a_y, a_z)$

$$\int \vec{F}\, dT = m \int \frac{d\vec{v}}{dT}\, dT = m \int (a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k})\, dT$$

$$\int d\vec{v} = a_xT\hat{\imath} + a_yT\hat{\jmath} + a_zT\hat{k} + C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k}$$

$$\vec{V} = [(V_{0x} + a_xT)\hat{\imath} + (V_{0y} + a_yT)\hat{\jmath} + (V_{0z} + a_zT)\hat{k}]$$

just 1d Now, you get the point

$$\vec{V} = \frac{d\vec{r}}{dT} \qquad \int \vec{V}\, dT = \int d\vec{r}$$

$$\int (V_{0x} + a_xT)\, dT = \int dx$$

$$x_0 + V_{0x}T + \frac{1}{2}a_xT^2 = x(T)$$

Can I make this a perfect differential

$$\underbrace{2\dot{x}}\ddot{x} = \underbrace{\underbrace{|\vec{g}|[\sin\theta - \mu_k\cos\theta]}_{\alpha}\,2\dot{x}}$$

Integrating factor

$$\frac{d(\dot{x}^2)}{dt} = 2\dot{x}\ddot{x} \checkmark$$

$$\frac{d(\dot{x}^2)}{dt} = 2\dot{x}\alpha$$

$$d(\dot{x}^2) = 2\frac{dx}{dt}\alpha\, dt = 2dx\alpha$$

$$\int d(\dot{x}^2) = \int 2\alpha dx \rightarrow \dot{x}^2 + C = 2\alpha x$$

$$\dot{x}^2(0) = 0$$

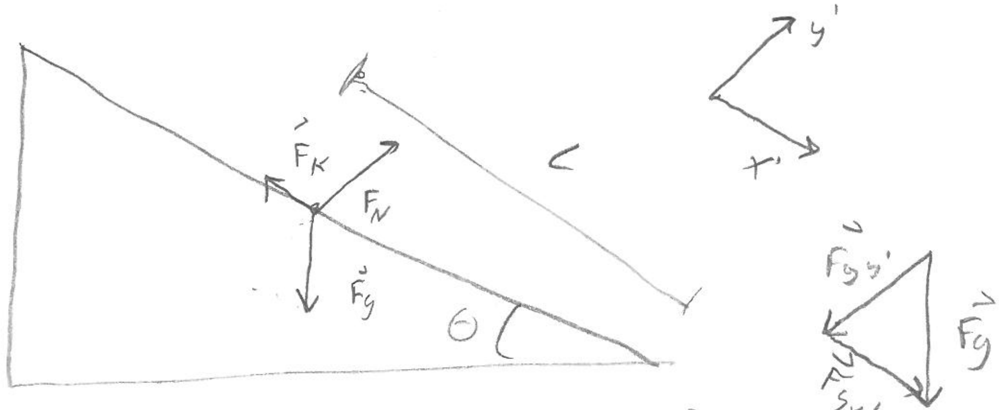
$$\dot{x}^2 = 2\alpha x \rightarrow \text{more generally } C = \dot{x}^2 - (\dot{x}(0))^2 = 2\alpha x$$

$$\boxed{V_f^2 = V_i^2 + 2a\Delta x\ ?}$$

So $\dot{x}(x) = \sqrt{2|\vec{g}|(\sin\theta - \mu_k\cos\theta)}$

Ex.

$$\vec{F} = \vec{N} + \vec{F_g} + \vec{F_k}$$



$$\vec{F} = \left[|\vec{F_g}| \sin\theta - |\vec{F_k}| \right] \hat{i}' + \left[|\vec{N}| - |\vec{F_g}| \cos\theta \right] \hat{j}'$$

$x'(0) = 0 \quad \vec{V}(0) = 0$

$y(0) = H = L \sin\theta$ but $y'(0) = 0$

$|\vec{N}| - m|\vec{g}|\cos\theta = 0 \rightarrow a_y = 0 \quad V_y(T) = 0$

$$m|\vec{g}|\sin\theta - \mu_k m|\vec{g}|\cos\theta = m\ddot{x}$$

$$\underbrace{|\vec{g}|\left[\sin\theta - \mu_k \cos\theta\right]}_{\vec{a}_x = \alpha} = \ddot{x}$$

#1.

$$\int \alpha \, dT = \int \frac{dV_x}{dT} dT \rightarrow V_x = \alpha T = \dot{x}$$

$$\int \alpha T \, dT = \int \frac{dx}{dT} dT = x(T) = \frac{1}{2}\alpha T^2 = \frac{1}{2}\left[|\vec{g}|\right](\sin\theta - \mu_k \cos\theta)T^2$$

#2. $V(x)$?

$$\frac{d\dot{x}}{dT} = \ddot{x}$$

$$2\dot{x}\ddot{x} = 2\frac{dx}{dT}\vec{a}_x \qquad \text{For } \ddot{x} = c$$

$$\frac{d(\dot{x}^2)}{dT} = 2\vec{a}_x\frac{dx}{dT} \qquad \text{For } \vec{a}_x = \ddot{x} = c$$

$$\int d(\dot{x}^2) = 2\vec{a}_x\int dx$$

Else

$$\int d(\dot{x}^2) = \int 2\vec{a}_x\,dx \rightarrow \text{Looks Like } \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \oint \vec{F}\cdot d$$

What if $V = V(x)$? $\quad \frac{dV}{dT} = \frac{dV}{dx}\frac{dx}{dT} = \dot{x}\frac{dV}{dx}$

What is the tipping angle?

$$\vec{a}_x: |g|[\sin\theta - \mu_k\cos\theta] \geq 0$$

$$\tan\theta \geq \mu_k \quad \text{or} \quad \theta = \tan^{-1}(\mu_k)$$

Retarding Forces

$\vec{F} = \vec{F_g} + \vec{F_r} = m\vec{g} + \vec{F_r}(v)$

$\vec{F_r}(v) = mkv^n \frac{\vec{v}}{|v|}$

For $v \sim 25\frac{m}{s}$ or less and small $n \sim 1$

v $25\frac{m}{s}$ to Sound Speed $330\frac{m}{s}$ $n = 2$

One D ✓ ok

$m\vec{g} - mk(v_y)^2 \frac{\vec{v_y}\hat{j}}{|\vec{v_y}|} = m\vec{g} - mk|v_y|\vec{v_y}\hat{j}$

Two D? Nope $m\vec{g} - mk(v_x^2 + v_y^2)^{1/2}[v_y\hat{j} + v_x\hat{i}]$

often we say $\vec{F}_{drag} = \frac{1}{2} \underset{\text{drag coefficient}}{C_w} \overset{\text{density}}{\rho} A|\vec{v}|^2 \frac{\vec{v}}{|v|}$

Ex riding a bike? Pedal then stop so $v_0 \neq 0$

$$\vec{F} = m\frac{d\vec{v}}{dT} = -km|v|\hat{i}$$

$$\frac{d\vec{v}}{dT} = -kv$$

$$\int \frac{dv}{v} = -\int k\,dT$$

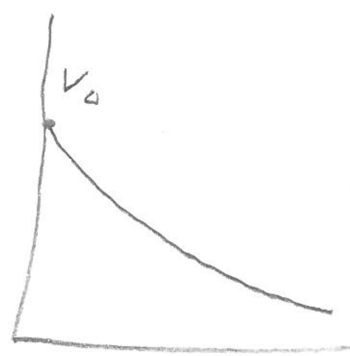
$$\ln(v)\Big|_{v_0}^{v(T)} = -kT$$

$$\ln(v(T)) - \ln(v_0) = -kT$$

$$\ln\left(\frac{v(T)}{v_0}\right) = -kT$$

$$\frac{v(T)}{v_0} = e^{-kT}$$

$$v(T) = v_0 e^{-kT}$$



$$T = \frac{1}{k} \quad v(T) = v_0 e^{-1} \sim \frac{v_0}{3}$$

e Folding time $k = \frac{1}{T}$

$v(T)$?

$V = \frac{dx}{dT} = V_0 e^{-kT}$

$x = V_0 \int e^{-kT} dT \rightarrow x(T) = \left(-\frac{1}{k} e^{-kT} + C\right) V_0$

Let $x(0) = 0$ Then $C = +\frac{V_0}{k}$

So $x(T) = \frac{V_0}{k}(1 - e^{-kT})$

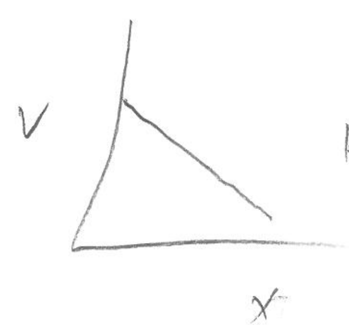
$x(\infty)? \rightarrow \frac{V_0}{k}(1 - \underset{0}{e^{-\infty}}) = \frac{V_0}{k}$

$\frac{dV}{dx}$? $V(x)$? $\frac{dV}{dx} = \frac{dV}{dT}\frac{dT}{dx} = \frac{dV}{dT} \cdot \frac{1}{V}$ So

$V\frac{dV}{dx} = \frac{dV}{dT} = -kV(T)$ or

$\frac{dV}{dx} = -k$

So $V = V_0 - kx$



$kx = V_0$

$V = 0$

$\downarrow m\vec{g} \quad \nearrow \vec{F}_d$

$$m\frac{d\vec{v}}{dT} = -m|\vec{g}| + km\vec{v}$$

$$\vec{v} = -|v|\hat{j}$$

$$m\frac{d\vec{v}}{dT} = -(m|\vec{g}| + km|v|)$$

$$\frac{dv}{|\vec{g}| + k|v|} = -dT \qquad u = |\vec{g}| + k|v| \qquad du = k\,dv$$

$$\frac{du}{u} = -k\,dT \rightarrow \ln(u) = -kT + C$$

$$u = e^{C} e^{-kT}$$

$$|\vec{g}| + k|v| = e^{-kT} e^{C}$$

$$v(T) = \frac{e^{-kT} e^{C} - |\vec{g}|}{k}$$

Now $v(0) = \dfrac{e^{C} - |g|}{k} = v_0 \qquad kv_0 + |g| = e^{C}$

$$\frac{e^{-kT}[kv_0 + |g|]}{k} - \frac{|g|}{k} = v(T).$$

positive

negative

Now $V = \frac{dz}{d\tau}$ or $V d\tau = dz$

$V d\tau = dz$

$$\int -\frac{g}{k} + \underbrace{\left(\frac{kV_0 + g}{k}\right)}_{\alpha} e^{-k\tau}\, d\tau = \int dz$$

$$-\frac{g}{k}\tau + \alpha \int e^{-k\tau}\, d\tau = z + C$$

$$-\frac{g}{k}\tau - \frac{1}{k}\alpha e^{-k\tau} = z(\tau) + C$$

$z(0) = h \qquad -\frac{\alpha}{k} = z(0) + C$

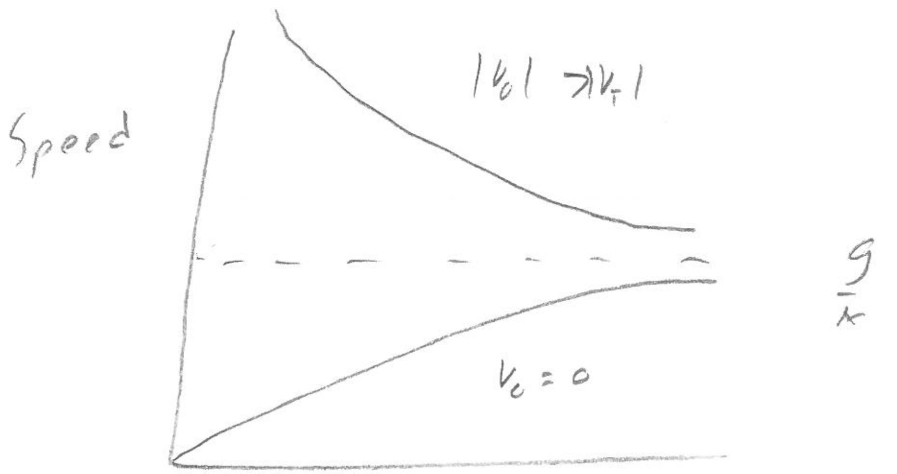
$$-h - \frac{\alpha}{k} = C \quad \text{or} \quad -C = h + \frac{\alpha}{k}$$

give $z = h + \frac{\alpha}{k} - \frac{\alpha}{k} e^{-k\tau}$

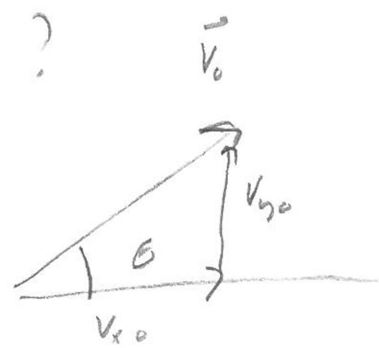
$$z(\tau) = h - \frac{g}{k}\tau + \left(\frac{kV_0 + g}{k^2}\right)\left(1 - e^{-k\tau}\right)$$

So $V(T) = -\frac{g}{k} + \left(\frac{kV_0 + g}{k}\right)e^{-kT}$

$V(T) = -\frac{g}{k}$



Projectile Motion?



$\vec{F} = m\vec{g} = 0\hat{i} - m|\vec{g}|\hat{j}$

$m\ddot{x} = 0$

$\dot{x} = \text{Constant} = V_{x0}$ $\qquad m\ddot{y} = -m|\vec{g}| \quad \ddot{y} = -|\vec{g}|$

$V(T) = V_{x0}T + x_0$ $\qquad \dot{y} = V_{y0} - |\vec{g}|T$

$y = V_{y0}T - \frac{1}{2}|\vec{g}|T^2 + y_0$

$$|V|^2 = \dot{x}^2 + \dot{y}^2 = V_0^2\cos^2\theta + V_0^2\sin^2\theta + g^2T^2 - 2gTV_0\sin\theta$$

$$|V|^2 = V_0^2 + g^2T^2 - 2V_0 gT\sin\theta$$

$$|r|^2 = x^2 + y^2 = V_0^2 T^2 + \frac{g^2T^4}{4} - V_0 gT^3\sin\theta$$

R? T_{Flight}? $2T_{\uparrow}$? $V_y(T_\uparrow) = 0$ or $V_0\sin\theta - gT = 0$

$T_\uparrow$ To Top

$$T_\uparrow = \frac{V_0\sin\theta}{g} \qquad R \rightarrow 2T_\uparrow = \frac{2V_0\sin\theta}{g}$$

$$x(2T_\uparrow) = \underset{0}{\overset{\parallel}{x_0}} + V_0\cos\theta\left(\frac{2V_0\sin\theta}{g}\right) = \frac{2V_0^2\sin\theta\cos\theta}{g}$$

$$R = \frac{V_0^2}{g}\sin(2\theta) \qquad \theta = 45° = \max$$

Big Bertha $1450\ \frac{m}{s}$ $\theta = 55°$

$R = 202$ km

$Y_{max} = 72$ km

$T = 242$ Seconds

2D Projectile with $\vec{F} = m\vec{g} + \vec{F}_r$

$\vec{f} = (-km\dot{x}\,\hat{i} + (km\dot{y} + m|g|))\hat{j}$

$m\ddot{x} = -km\dot{x}$ $\qquad m\ddot{y} = -(km\dot{y} + m|g|)$

Solved $\quad x = \frac{U}{k}(1-e^{-kT}) \qquad y = -\frac{gT}{k} + \frac{kV+g}{k^2}(1-e^{-kT})$

with $V_{x_0} = U \quad V_{y_0} = V$

For $y=0$ $\quad T = \frac{kV+g}{gk}(1-e^{-kT}) \rightarrow$ put into

$x = \frac{U}{k}(1-e^{-kT})$ For range $\rightarrow$ Ignore $T=0$ Solutions

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2 Methods to Solve $\quad T = a(1-e^{-bT}) \rightarrow$ Transcendental

1. Perturbative $\rightarrow$ look at 2-9 $\rightarrow$ Stupid but we'll See here

2. Numerical $\rightarrow$ 21st century, do it this way.

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1. Expand $e^{-kT} \rightarrow e^{x} = \sum \frac{x^n}{n!}$ with $x = -kT$ $\qquad$ Expand Expand 1st order Sub

Then $\quad T = \frac{kV+g}{k^2}\left(1 - \left(1 - kT + \frac{1}{2}k^2T^2 - \frac{1}{6}k^3T^3 \ldots\right)\right)$

$T = \frac{kV+g}{k^2}\left(kT - \frac{1}{2}k^2T^2 + \frac{1}{6}k^3T^3 + \ldots\right)$

$$T = \frac{\frac{2V}{g}}{1+\frac{kV}{g}} + \frac{1}{3}kT^2 \rightarrow \text{To 3rd order}$$

Still too messy Expand $\frac{1}{1+\frac{kV}{g}} = 1 - \frac{kV}{g} + \left(\frac{kV}{g}\right)^2 \ldots$

To First order

$$T = \frac{2V}{g} + \left(\frac{T^2}{3} - \frac{2V^2}{g}\right)k + O(k^2) \equiv$$

If $k=0$ $T_0 = \frac{2V}{g} = \frac{2V_0 \sin\theta}{g}$ ✓ Put T_0 into rhs for T

Then Let $T = \frac{2V}{g} + \left(\frac{4V^2}{3g^2} - \frac{2V^2}{g^2}\right)k = \frac{2V}{g} + \left(\frac{V^2}{g^2}\left(\frac{4-6}{3}\right)\right)k \equiv$

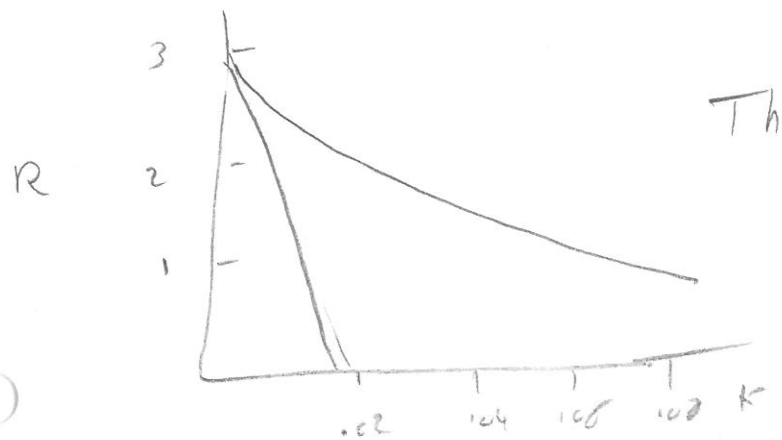
Then $T = \frac{2V}{g} - \frac{2}{3}\frac{V^2}{g^2}k = \boxed{\frac{2V}{g}\left(1 - \frac{V}{3g}k\right)} \equiv$

Next $X = \frac{U}{k}\left(1 - e^{-kT}\right) = \frac{U}{k}\left(kT - \frac{1}{2}k^2T^2 + \frac{1}{6}k^3T^3 + \ldots\right)$

$X(T=T) = R$ $R \approx U\left(T - \frac{1}{2}kT^2\right) \rightarrow$ keep only linear Terms in k

$$R' \cong \frac{2uv}{g}\left(1-\frac{4kv}{g}\right) = \underbrace{\frac{2V_0^2}{g}\sin\theta\cos\theta}_{R \text{ for } \vec{F}_r = 0}\left(1-\frac{4kv}{3g}\right)$$

$$R' \cong R\left(1-\frac{4kv}{3g}\right)$$



This is crap

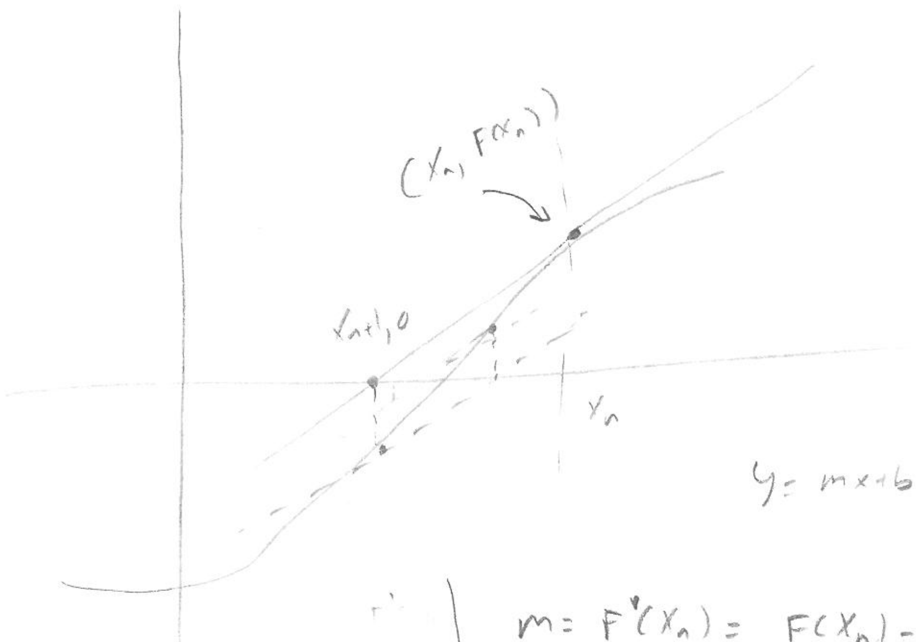
1. We kept only the 2nd order Term in e^{-kT}
2. We kept only the 2nd order Term in $\frac{1}{1+\frac{kv}{g}}$
3. We used the zeroth order approximation for $T = T_0$
4. We linearized $x(t)$ in k after keeping only 2nd order Terms

Stop doing this

Take 2

$$T = \frac{kv+y}{gr}\left(1-e^{-kT}\right) \rightarrow \underbrace{a\left(1-e^{-bT}\right)-T=0}_{F(T)} \qquad \underbrace{abe^{-bT}-1}_{F'(T)}$$

$$y = F'(T_n)(T-T_n) + F(T_n)$$



$y = mx+b \qquad m = F'(x) \quad b = X \text{ st } y=0$

$$y = F'(X_n)(X-X_n)+F(X_n)$$

$$m = F'(X_n) = \frac{F(X_n)-0}{X_n - X_{n+1}}$$

$$X_n - X_{n+1} = \frac{F(X_n)}{F'(X_n)}$$

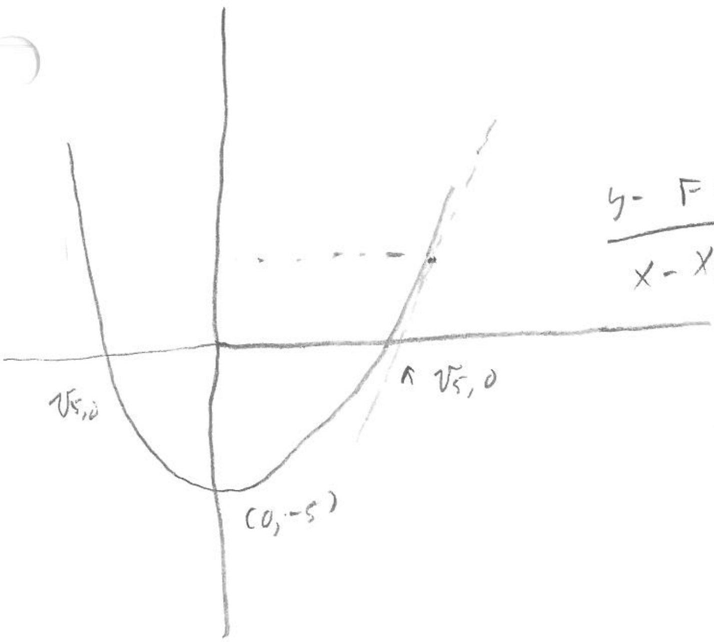
$$X_{n+1} = X_n - \frac{F(X_n)}{F'(X_n)}$$

1. Google This $\rightarrow$ Check 3-5 iterations

   get T use T

2. or just plot $\rightarrow$ use computer

Ex. $F(x) = x^2 - 5 = 0 \quad F'(x) = 2x$



$$\frac{y - F(x_n)}{x - x_n} = F'(x_n)$$

$$y = F'(x_n)(x - x_n) + F(x_n)$$

$$0 = F'(x_n)(x_{n+1} - x_n) + F(x_n)$$

$$x_n - x_{n+1} = \frac{F(x_n)}{F'(x_n)}$$

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

guess root $\sqrt{4} < \sqrt{5} < \sqrt{9}$

$2 < \sqrt{5} < 3$

2.5?

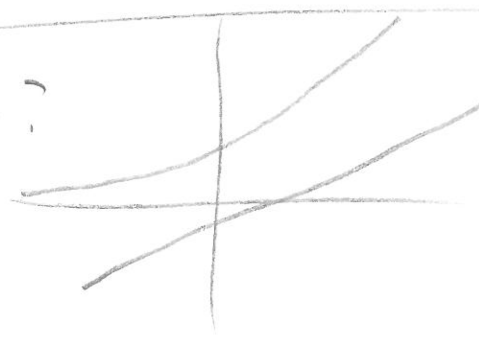
$x_{n+1} = 2.5 - .25 = 2.25 \rightarrow \frac{x_{n+1}}{real} = 1.00623$

$x_{n+1} = 2.25 - .013889 = 2.23611 \rightarrow \frac{x_{n+1}}{real} = 1.00002$

just get $\sqrt{5}$

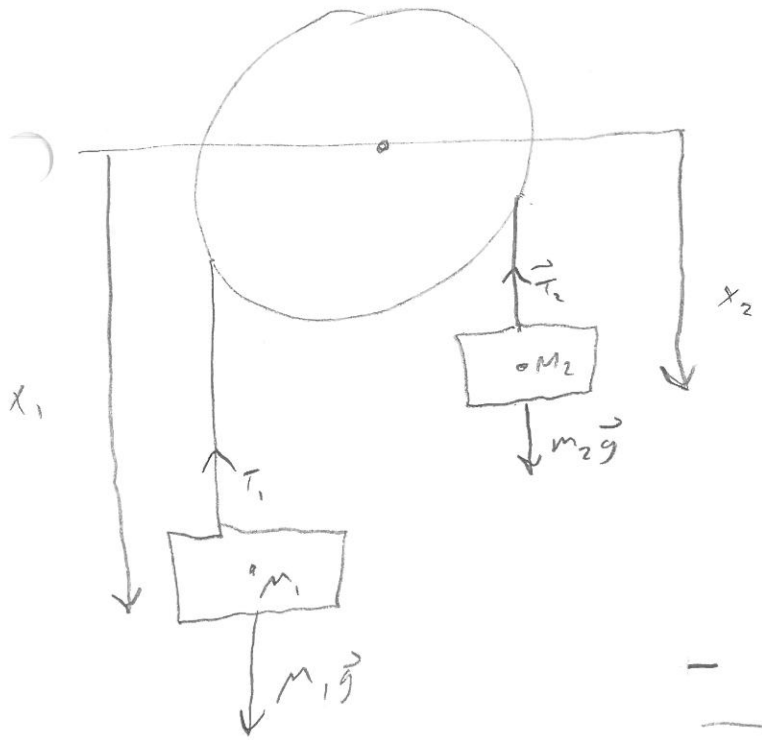
depends on sensible choice of x_n

Pathological Ex $e^x = 2x$?



No intersection

$M_1 > M_2$



$$m_1 \ddot{x}_1 = |\vec{T}_1| - m_1 g$$

$$- \quad m_2 \ddot{x}_2 = |\vec{T}_2| - m_2 g$$

$|\vec{T}_1| = |\vec{T}_2|$ $\quad \ddot{x}_1 = -\ddot{x}_2$ $\quad m_1 \ddot{x}_1 + m_2 \ddot{x}_1 = -m_1 g + m_2 g$

$$(m_1 + m_2)\ddot{x}_1 = (m_2 - m_1) g$$

$$\ddot{x}_1 = \frac{(m_2 - m_1) g}{m_1 + m_2}$$

Note $m_2 - m_1 < 0$ So $\downarrow \ddot{x}_1 = \dfrac{-|(m_2 - m_1)|}{m_1 + m_2} g = -\ddot{x}_2 \uparrow$

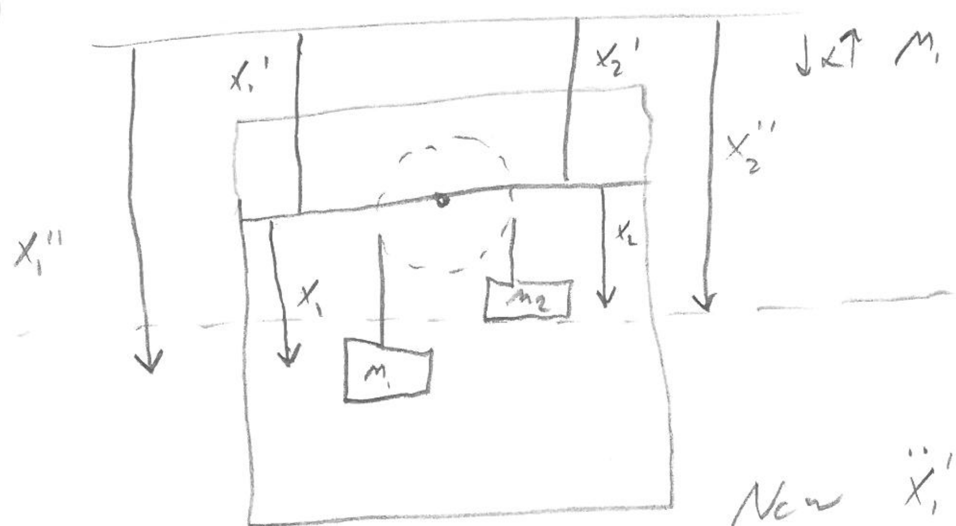
$$m_1 \ddot{x}_1 = |T_1| - m_1 g \Rightarrow T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

$$\frac{m_1}{m_1 + m_2}(m_2 - m_1 g) + m_1 g = T$$

$$\frac{m_1 m_2 - \cancel{m_1^2} g + \cancel{m_1^2} g + m_2 m_1 g}{m_1 + m_2} = T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Non-Inertial ?

Newton's Laws work WRT Inertial System



$\downarrow < \uparrow$ $m_1 > m_2$

$x_1'' = x_1 + x_1'$

$x_2'' = x_2 + x_2'$

Now $\ddot{x}_1' = \ddot{x}_2'$ either up or down

call this α - constant

and $\ddot{x}_1 = -\ddot{x}_2$

$m_1 \ddot{x}_1'' = m_1(\ddot{x}_1' + \ddot{x}_1) = |T| - m_1 g$

$m_2 \ddot{x}_2'' = m_2(\ddot{x}_2' + \ddot{x}_2) = |T| - m_1 g$

$m_1 \ddot{x}_1'' = m_1(\ddot{x}_1' + \ddot{x}_1) = |T| - m_1 g$ $\quad m_1 \ddot{x}_1 = |T| - m_1 |g| - m_1 \alpha$

$m_2 \ddot{x}_2'' = m_2(\ddot{x}_1' - \ddot{x}_1) = |T| - m_2 g$

$g = |\vec{g}|$

$m_1(\alpha + \ddot{x}_1) = |T| - m_1 g$

$m_2(\alpha - \ddot{x}_1) = |T| - m_2 g$

$m_1 \alpha + m_1 \ddot{x}_1 - m_2 \alpha + m_2 \ddot{x}_1 = m_2 g - m_1 g$

$m_1(\alpha + \ddot{x}_1) + m_2(\ddot{x}_1 - \alpha) = (m_2 - m_1) g$

$m_1 \alpha + m_1 \ddot{x}_1 + m_2 \ddot{x}_1 - m_2 \alpha = (m_1 + m_2)\ddot{x}_1 + (m_1 - m_2)\alpha = (m_2 - m_1) g$

$(m_1 + m_2)\ddot{x}_1 = (m_2 - m_1)\alpha + (m_2 - m_1) g$

ascending

$\ddot{x}_1 = \dfrac{(m_2 - m_1)(\alpha + g)}{m_1 + m_2}$ $\quad$ Now $m_2 < m_1$ & $\ddot{x}_1 = \dfrac{-|m_2 - m_1|(\alpha + g)}{m_1 + m_2}$

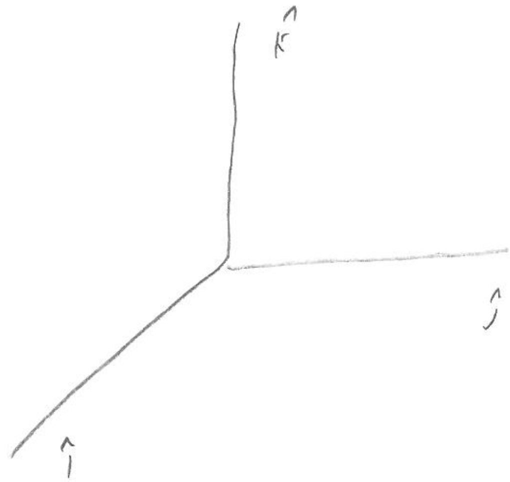
Lorentz Force

$\vec{F} = q\vec{v}\times\vec{B}$

$\vec{V} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$

$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

$\vec{B} = B_0\hat{j}$



$$m(\ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}) = q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \dot{x} & \dot{y} & \dot{z}\\ 0 & B_0 & 0\end{vmatrix} = qB_0(\dot{x}\hat{k} - \dot{z}\hat{i})$$

$m\ddot{x} = -qB_0 z \qquad m\ddot{y} = 0 \qquad m\ddot{z} = qB_0\dot{x}$

$\boxed{\dot{y} = C = V_{y_0} \qquad y = V_{y_0}t + y_0}$

$\ddot{x} = -\alpha\dot{z} \qquad \ddot{z} = \alpha\dot{x} \quad \rightarrow \quad \alpha = \frac{qB_0}{m}$

$\dddot{x} = -\alpha\ddot{z} \qquad \dddot{z} = \alpha\ddot{x}$

$\dddot{x} = -\alpha^2\dot{x} \qquad \dddot{z} = -\alpha^2\dot{z}$

Let $\dot{x} = u$ and $\dot{z} = v$

Then $\ddot{u} = -\alpha^2 u$ and $\ddot{v} = -\alpha^2 v$

Solve $\ddot{u} + \alpha^2 u = (D^2 u + \alpha^2 u) = 0$ or $(D^2 + \alpha^2) u = 0$

$(D + i\alpha)(D - i\alpha) u = 0 \rightarrow$ Integrate $\left(\frac{d}{d\tau} \pm i\alpha\right) u = 0$

$u = A_1 e^{i\alpha\tau} + B_2 e^{-i\alpha\tau} \rightarrow A_2 \cos(\alpha\tau) + B_2 \sin(\alpha\tau)$

or $\dot{x} = \frac{dx}{d\tau} = A_2 \cos(\alpha\tau) + B_2 \sin(\alpha\tau)$

$x = +\frac{A_2}{\alpha} \sin(\alpha\tau) - \frac{B_2}{\alpha} \cos(\alpha\tau) + x_0$

$$x(\tau) = \underset{\substack{\downarrow \\ -\frac{B_2}{\alpha}}}{A_3} \cos(\alpha\tau) + \underset{\substack{\downarrow \\ \frac{A_2}{\alpha}}}{B_3} \sin(\alpha\tau) + x_0$$

Similarly $z(\tau) = C \cos(\alpha\tau) + D \sin(\alpha\tau) + z_0$

Or $(X - X_0) = \overset{A_3}{\overset{\downarrow}{A}} \cos(\alpha T) + \overset{B_3}{\overset{\downarrow}{B}} \sin(\alpha T)$

$(y - y_0) = V_{y_0} T$

$(z - z_0) = \underset{\underset{C}{\downarrow}}{A'} \cos(\alpha T) + B' \sin(\alpha T)$

but $\ddot{x} = -\alpha \dot{z}$

$\ddot{z} = \alpha \dot{x}$

$\ddot{x} = -\alpha^2 A \cos(\alpha T) - \alpha^2 B \sin(\alpha T)$

$-\alpha \dot{z} = \alpha^2 A' \sin(\alpha T) - \alpha^2 B' \cos(\alpha T)$

Or $A = B'$

$B = -A'$

$(X - X_0) = A \cos(\alpha T) + B \sin(\alpha T)$

$(y - y_0) = V_{y_0} T$

$(z - z_0) = -B \cos(\alpha T) + A \sin(\alpha T)$

Need IC'S for $T = 0$ $\dot{z} = V_{z_0}$ $\dot{x} = 0$

$\dot{z}\Big|_{T=0} = B\alpha \underset{\underset{0}{\uparrow}}{\sin(\alpha T)}\Big|_{T=0} + A\alpha \cos(\alpha T)\Big|_{T=0} = V_{z_0}$ $\alpha A = V_{z_0}$

$$\dot{X}\Big|_{T=0} = -A\alpha\,\text{SEN}(\alpha T)\Big|_{T=0} + B\alpha\cos(\alpha T) = 0$$

$$\underbrace{-A\alpha\,\text{SEN}(\alpha T)\Big|_{T=0}}_{0} \qquad B\alpha = 0$$

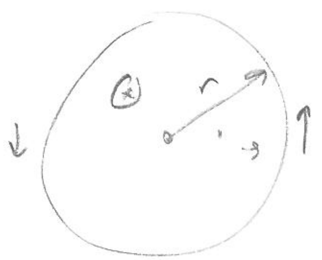
$\underline{\underline{B = 0}}$ $\quad A = \dfrac{V_{Z0}}{\alpha}$

Now $\quad (X - X_0) = \dfrac{V_{Z0}}{\alpha}\cos(\alpha T)$

$$(Y - Y_0) = V_{Y0} T$$

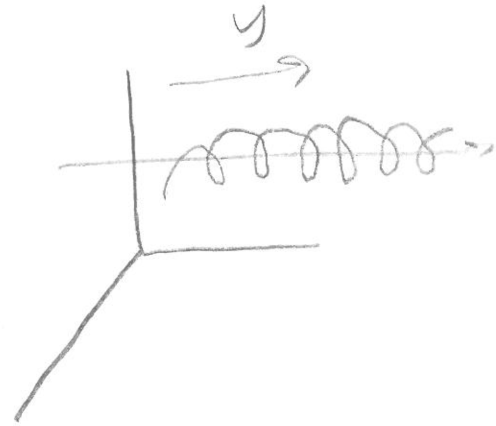
$$(Z - Z_0) = \frac{V_{Z0}}{\alpha}\,\text{SIN}(\alpha T)$$

Hmm meaning? $\quad \alpha = \dfrac{q B_0}{m}$



$\rightarrow \quad qvB = \dfrac{mv^2}{r} \qquad \dfrac{qB}{m} = \dfrac{v}{r} \rightarrow \quad r = \dfrac{vm}{qB} \rightarrow \dfrac{V_{Z0}}{\alpha} = \dfrac{vm}{qB}$

Larmor radius $= R_L$

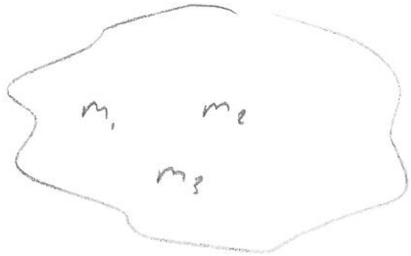


$$(X - X_0)(T) = R_L\cos(\alpha T)$$

$$(Y - Y_0)(T) = V_{Y0} T$$

$$(Z - Z_0)(T) = R_L\,\text{SIN}(\alpha T)$$

Conservation Theorems



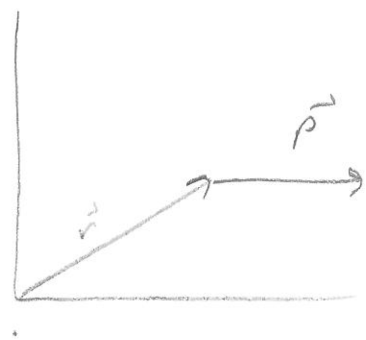
$F_{12} + F_{21} + F_{13} + F_{31}$

$+ F_{32} + F_{23} = 0$

If $\vec{F}_{net} = 0$

So $\dot{P} = 0$ or $P_{net} = \text{Constant}$

$\vec{L} = \vec{r} \times \vec{p}$



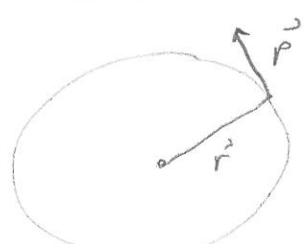
$\vec{N} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$

$= \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt}$ $\qquad \frac{d\vec{r}}{dt} \parallel \vec{p}$

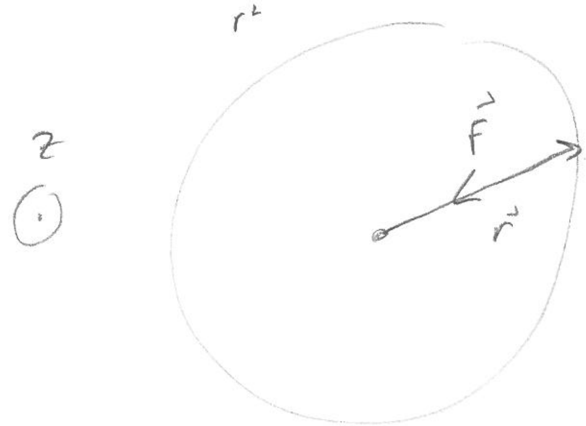
$\downarrow 0$

if $\vec{N}_{net} = 0$ $\quad \dot{L} = \vec{r} \times \dot{\vec{p}} = N$ Then $\vec{L} = \text{Constant}$

Why do planets orbit?



$$\dot{\vec{P}} = \vec{F} = \frac{GmM_s}{r^2}\hat{r}$$



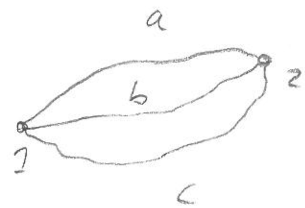
$-\vec{F} \parallel \vec{r}$

So $|r|\hat{r} \times \frac{GmM_s}{r^2}\hat{r} = \vec{N} = 0$

So $\vec{L} =$ constant $= mr^2\hat{z}$

$=$ orbit

$$W_{12} = \oint_1^2 \vec{F}\cdot d\vec{r}$$



$$\vec{F}\cdot d\vec{r} = m\frac{d\vec{v}}{dt}\cdot d\vec{r} = m\frac{d\vec{v}}{dt}\frac{d\vec{r}}{dt}dt = m\frac{d\vec{v}}{dt}\vec{v}\,dt$$

$$\vec{F}\cdot d\vec{r} = \frac{m}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v})\,dt = \frac{m}{2}\frac{d}{dt}(v^2)\,dt = \frac{m}{2}d(v^2)$$

$$\vec{F}\cdot d\vec{r} = d\left(\frac{mv^2}{2}\right)$$

$$W_{12} = \frac{mv^2}{2}\Big|_1^2 = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = T_2 - T_1 \rightarrow \Delta KE_{12}$$

For $\vec{F} \cdot d\vec{r}$ independent of path

$$\oint \vec{F} \cdot d\vec{r} = U_1 - U_2 \quad \text{where} \quad \vec{F} = -\vec{\nabla} U$$

$$\int_1^2 \vec{F} \cdot d\vec{r} = -\int_1^2 (\vec{\nabla} U) \cdot d\vec{r} = \underbrace{-\int_1^2 dU = U_1 - U_2}_{\text{definition of Potential energy}}$$

$\vec{E}$, $\vec{g}$

Total Energy $= E = T + U$

$$\frac{dE}{dt} = \frac{dT}{dt} + \frac{dU}{dt}$$

$$\vec{F} \cdot d\vec{r} = d\left(\frac{1}{2} m v^2\right) = dT$$

$$\frac{dT}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v} = \vec{F} \cdot \dot{\vec{r}}$$

$$\frac{dU}{dt} = \frac{d\,U(x,y,z,t)}{dt} = \frac{\partial U}{\partial x}\frac{dx}{dt} + \frac{\partial U}{\partial y}\frac{dy}{dt} + \frac{\partial U}{\partial z}\frac{dz}{dt} + \frac{\partial U}{\partial t}$$

$$\frac{dU}{dt} = (\vec{\nabla} U) \cdot \dot{\vec{r}} + \frac{\partial U}{dt}$$

$$\frac{dE}{dt} = \vec{F}\cdot\dot{\vec{r}} + (\nabla u)\cdot\dot{\vec{r}} + \frac{\partial u}{\partial t} \qquad \text{For } \vec{F} = -\vec{\nabla} u$$

$$\frac{dE}{dt} = (\vec{F} + \nabla u)\cdot\dot{\vec{r}} + \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}$$

If u is independent of time

Energy is conserved

Ex: 2D collision

$\uparrow m_1 v_1 \quad \leftarrow m_2 v_2$

Ex: mouse mass m jumps on edge of record with

Moment of inertia I, radius R $\quad \omega_{new} = ?$

$$\vec{L}_0 = I\omega_0 \qquad \vec{L} = (I + MR^2)\omega$$

$$\vec{\omega} = \left(\frac{I}{I + MR^2}\right)\vec{\omega}_0$$

Ex. Orbits $\quad E < 0$

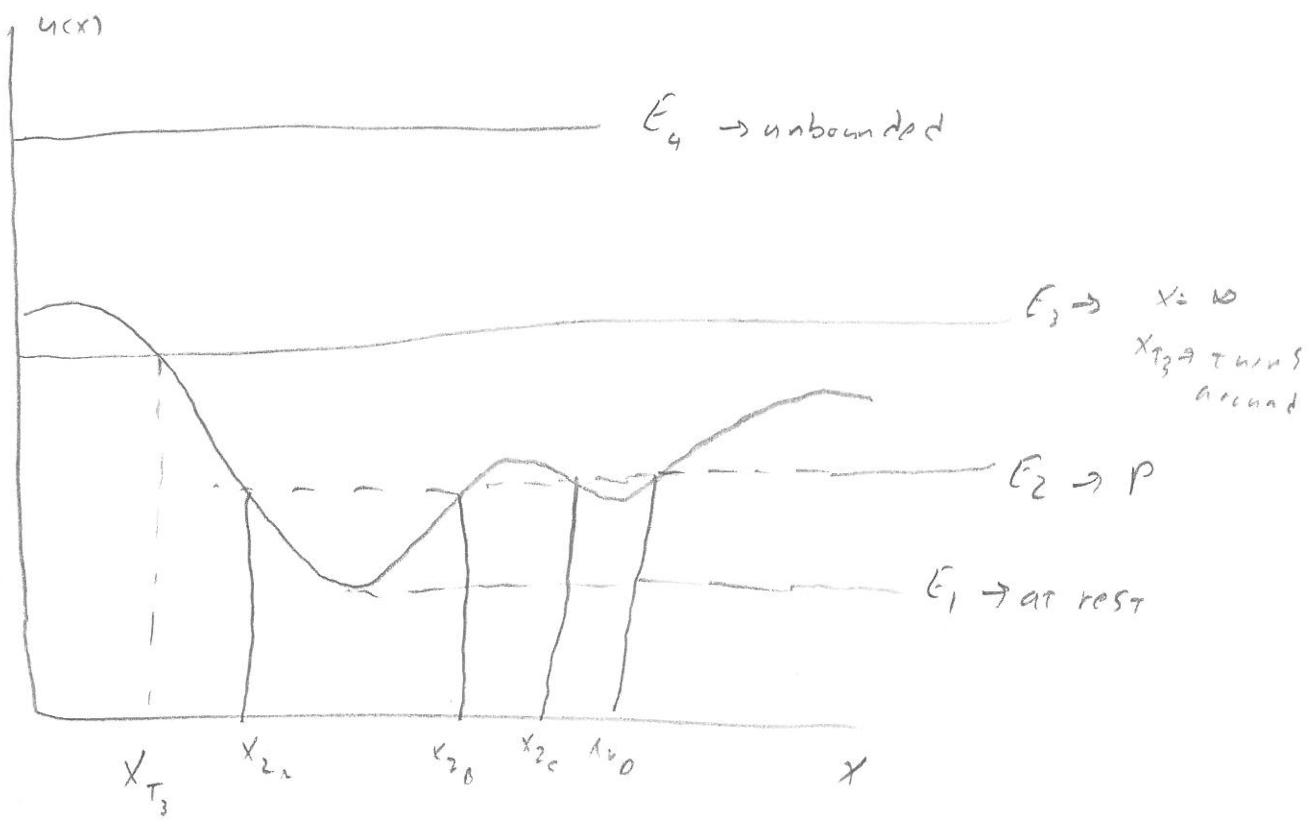
$E > 0$

Energy

$$E = T + U = \frac{1}{2}mv^2 + U(x)$$

$$v(t) = \left(\frac{2}{m}(E - U(x))\right)^{1/2} = \frac{dx}{dt}$$

$$t - t_0 = \int_{x_0}^{x} \frac{\pm dx}{\sqrt{\frac{2}{m}(E - U(x))}} \rightarrow E - U(x) \geq 0 \quad T = \frac{1}{2}mv^2$$



$x_{2A,2B,2C,2D} \rightarrow$ Turning Points, periodic

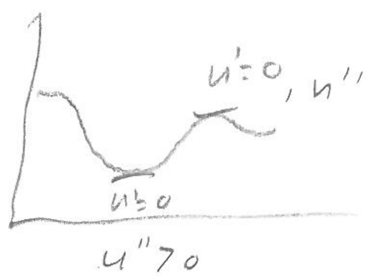
$\vec{F} = -\frac{du}{dx}\hat{i}$

$u(x) = u_0 + x\,u'\big|_0 + \frac{x^2}{2!}x''\big|_0 + \frac{x^3}{3!}x'''\big|_0$

$u'\big|_{x=0} \rightarrow$ equilibrium point also place where $\vec{F} = 0$

Then $u(x) \approx \frac{x^2}{2!}u''\big|_{x=0}$ if $u'' > 0$ Stable

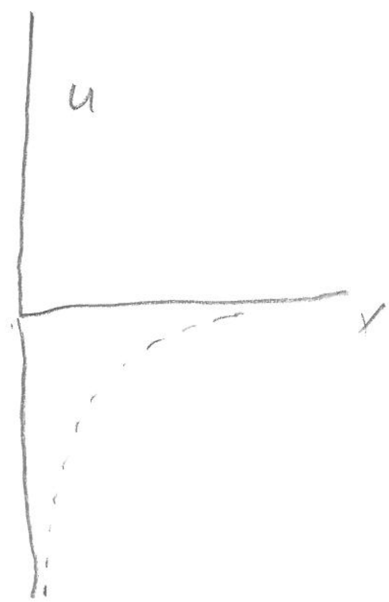
$u'' < 0$ unstable



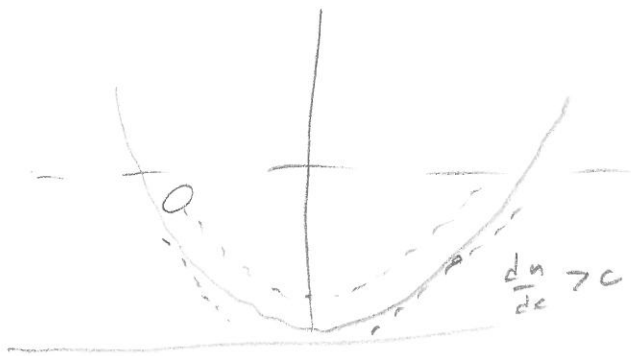
Ex $\vec{F} = -\frac{Gm_1m_2}{x^2}\hat{i}$

$u = -\int F\,dx = Gm_1m_2\int \frac{1}{x^2}\,dx$

$u(x) = -\frac{Gm_1m_2}{x}$



$F = -kx \qquad x \in [-a, a]$



$-F \cdot dx = du$

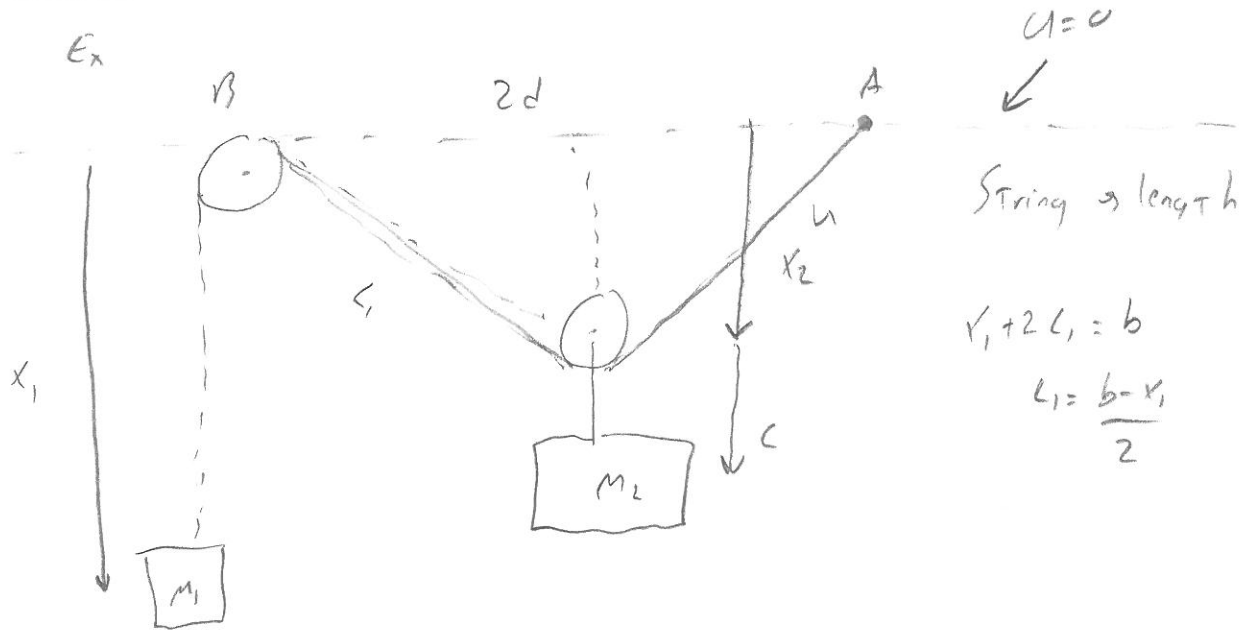
$u = \frac{1}{2} kx^2$

$\frac{du}{dx} > 0 \qquad -\frac{du}{dx} < 0 \quad \vec{F} \leftarrow$

$\frac{du}{dx} < 0$

$-\frac{du}{dx} > 0 \quad \vec{F} \rightarrow$

Ex

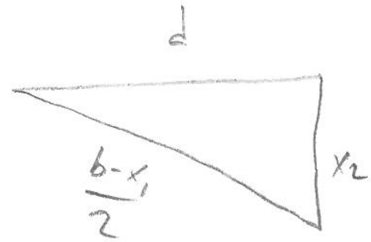


String → length b,

$x_1 + 2l_1 = b$

$l_1 = \frac{b - x_1}{2}$

$U = -m_1 g x_1 - m_2 g (x_2 + c)$



$\frac{(b - x_1)^2}{4} - d^2 = x_2^2$

Equilibrium?

$$U = -m_1 g x_1 - m_2 g \sqrt{(b-x_1)^2/4 - d^2} - m_2 g c$$

$$\frac{dU}{dx_1} = 0 \quad -m_1 g - \frac{m_2 g}{2\sqrt{(b-x_1)^2/4 - d^2}} \cdot \frac{2(b-x_1)}{4}(-1) = 0$$

$$-m_1 g + \frac{m_2 g (b-x_1)}{4\sqrt{(b-x_1)^2/4 - d^2}} = 0$$

$$\frac{4 m_1 g}{m_2 g} = \frac{b-x_1}{((b-x_1)^2/4 - d^2)^{1/2}}$$

$$\frac{16 m_1^2}{m_2^2} = \frac{(b-x_1)^2}{\frac{(b-x_1)^2}{4} - d^2}$$

$$16 m_1^2 \frac{(b-x_1)^2}{4} - 16 m_1^2 d^2 = m_2^2 (b-x_1)^2$$

$$(4m_1^2 - m_2^2)(b-x_1)^2 = 16 m_1^2 d^2$$

$$(b-x_1)^2 = \frac{16 m_1^2 d^2}{4m_1^2 - m_2^2}$$

$$b - x_1 = \pm \frac{4 m_1 d}{\sqrt{4m_1^2 - m_2^2}}$$

$$x_1 = b \mp \frac{4 m_1 d}{\sqrt{4m_1^2 - m_2^2}}$$

not physical

Only good $4m_1^2 - m_2^2 > 0$

$$u'' = \frac{g(4m_1^2 - m_2^2)^{3/2}}{4m_1^2 d} \quad \text{but } 4m_1^2 - m_2^2 > 0$$

So $u'' > 0$ and equilibrium is stable

Example: $u(x) = \dfrac{-Wd^2(x^2+d^2)}{x^4+8d^4}$

let $y = \dfrac{x}{d} \to u(x) = \dfrac{-Wd^2 \cdot d^2\left(\frac{x^2}{d^2}+1\right)}{d^4\left(\frac{x^4}{d^4}+8\right)}$

$$\frac{u(x)}{W} = -\frac{y^2+1}{y^4+8} = z(y) \qquad \frac{dz}{dy} = \frac{d}{dy}(-1)(y^2+1)(y^4+8)^{-1}$$

$$\frac{dz}{dy} = (-1)\left[2y(y^4+8)^{-1} - (y^2+1)(y^4+8)^{-2}4y^3\right]$$

$$\frac{dz}{dy} = (-1)\left[\frac{2y}{y^4+8} - \frac{4y^3(y^2+1)}{(y^4+8)^2}\right] = 0$$

$2y(y^4+8) = 4y^3(y^2+1) \to y=0$ is one

$y^4+8 = 2y^2(y^2+1)$

$y^4 + 8 = 2y^4 + 2y^2$

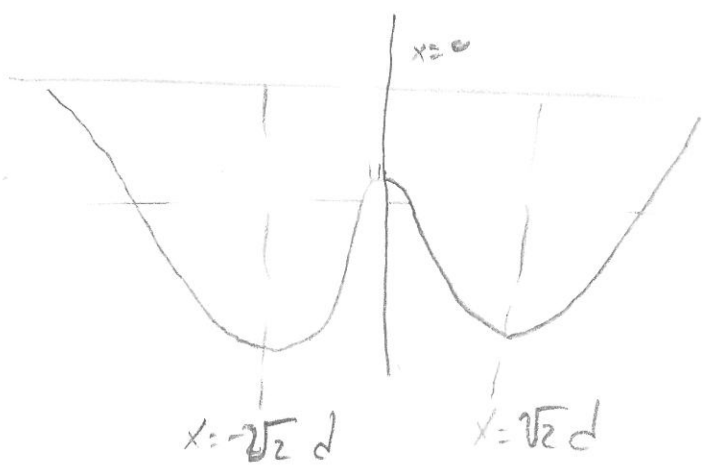
$y^4 + 2y^2 - 8 = 0 \quad (y^2+4)(y^2-2) = 0$

$\uparrow$ imaginary

$y = \pm\sqrt{2}, 0 \rightarrow$ equilibrium

$y = \frac{x}{d}$ $\quad x = \pm(2)^{1/2} d$ Stable

$x = 0$ unstable



$E = U(x) = U(\pm\sqrt{2}d, 0)$

$-\frac{W}{8} = U(\pm\sqrt{2}d, 0)$

$x = -2\sqrt{2}d, +2\sqrt{2}d, 0$

Done Ch. #2

Read 6.1–6.4