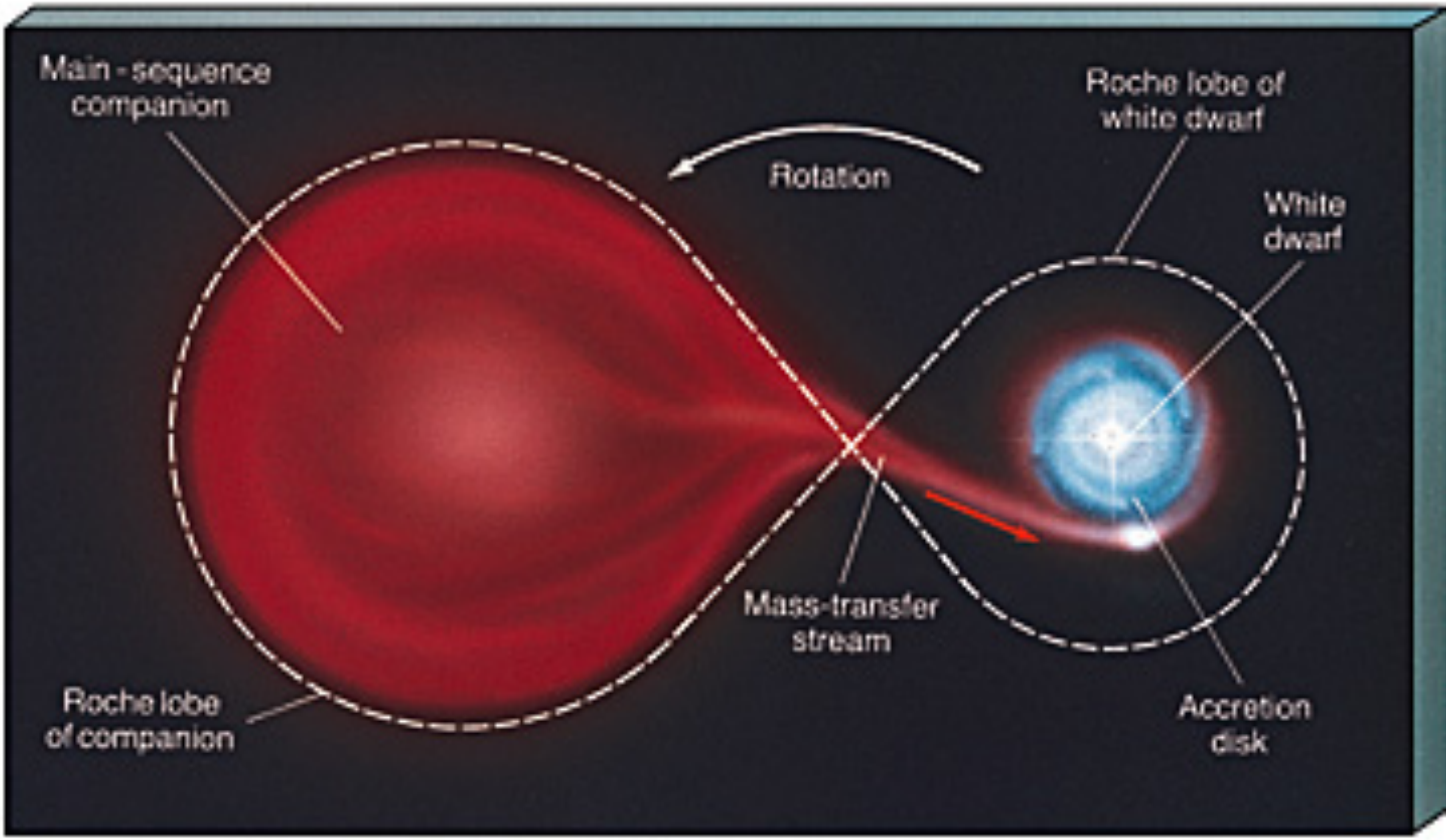
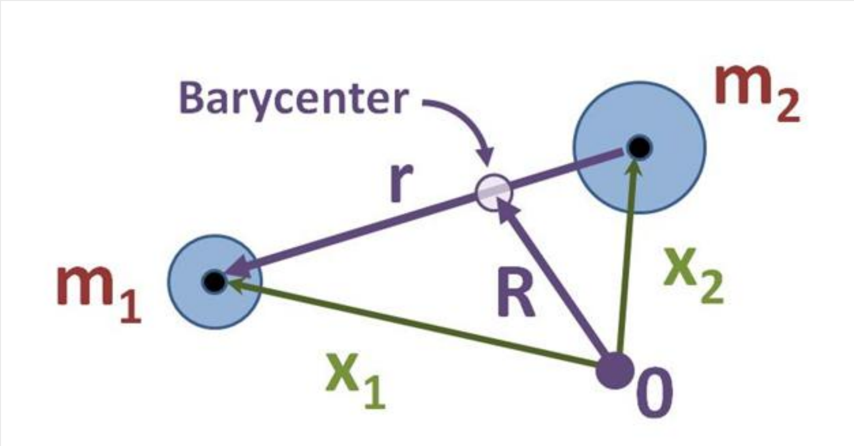
Roche Potential and its Astrophysical Implications



- Two Body Problem
- Roche Potential
  - Explanation
  - Lagrange Points
    - What Are They
    - Stability
- Applications
  - Trojan Satellites
  - Satellite Placement
- Accretion
  - Accretion Disks
  - Formation
  - Type 1A Supernova

Two Body Problem

- Take two masses, m_1 and m_2 and put them at r_1 and r_2
- Define $r = x_1 - x_2$
- To the center of mass frame
- Define a reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$
- Define the angular momentum $L = \mu r^2 d\theta/dt$ which for a gravitating system is a constant since r x F is zero
- Write down the Lagrangian of the system



$$\boldsymbol{R} = \frac{m_1}{M}\boldsymbol{x}_1 + \frac{m_2}{M}\boldsymbol{x}_2$$

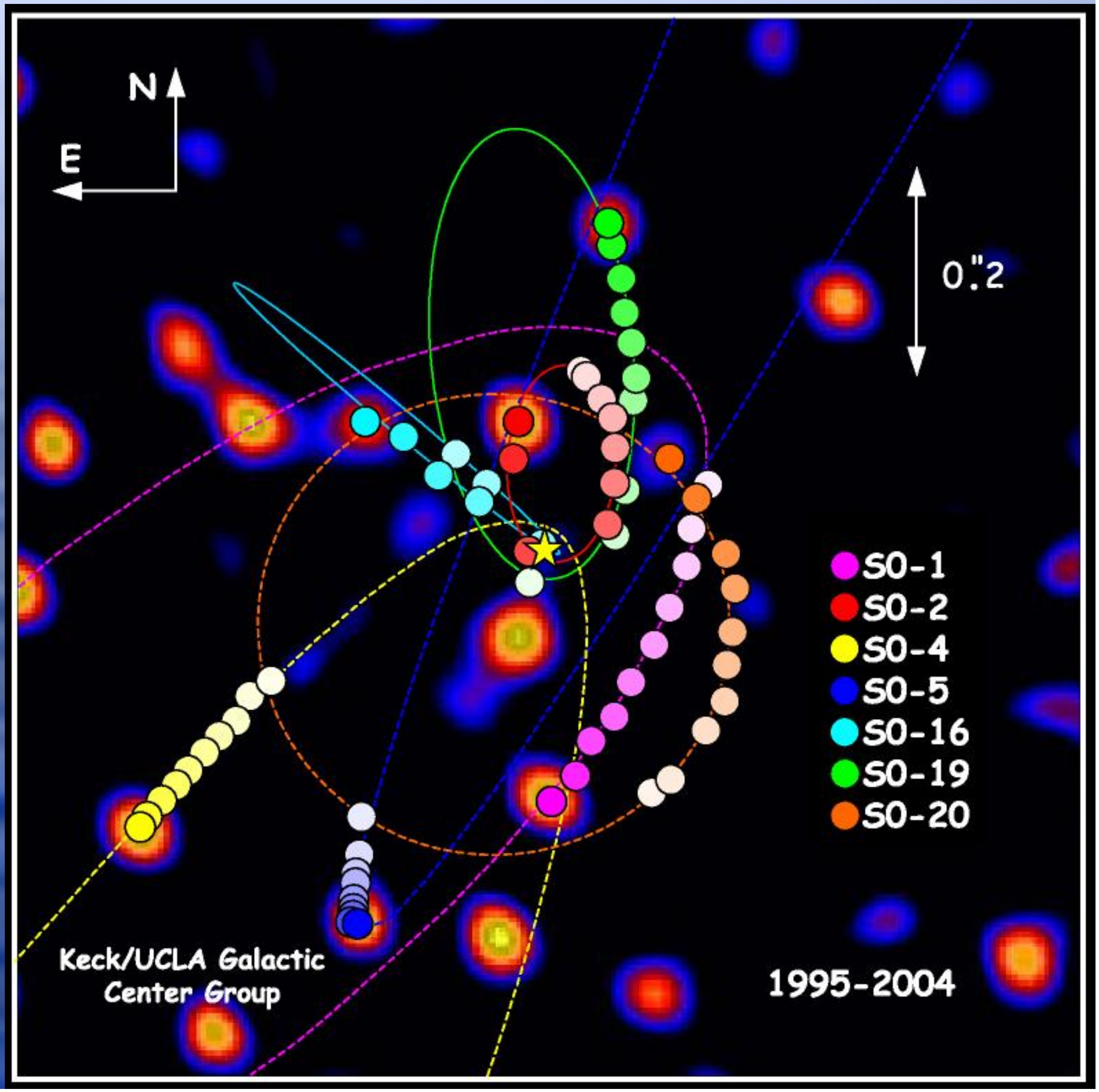
$$\mathcal{L} = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r),$$

$$\mathbf{x}_1(t) = \mathbf{R}(t) + \frac{m_2}{m_1 + m_2}\mathbf{r}(t)$$

$$\mathbf{x}_2(t) = \mathbf{R}(t) - \frac{m_1}{m_1 + m_2}\mathbf{r}(t)$$

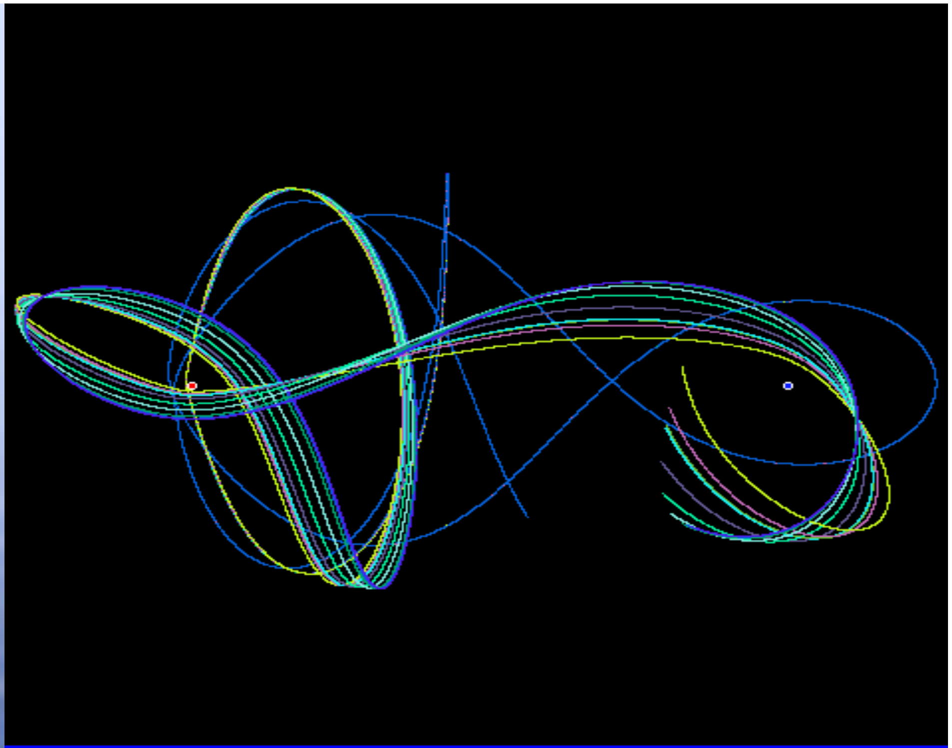
Two Body Continued

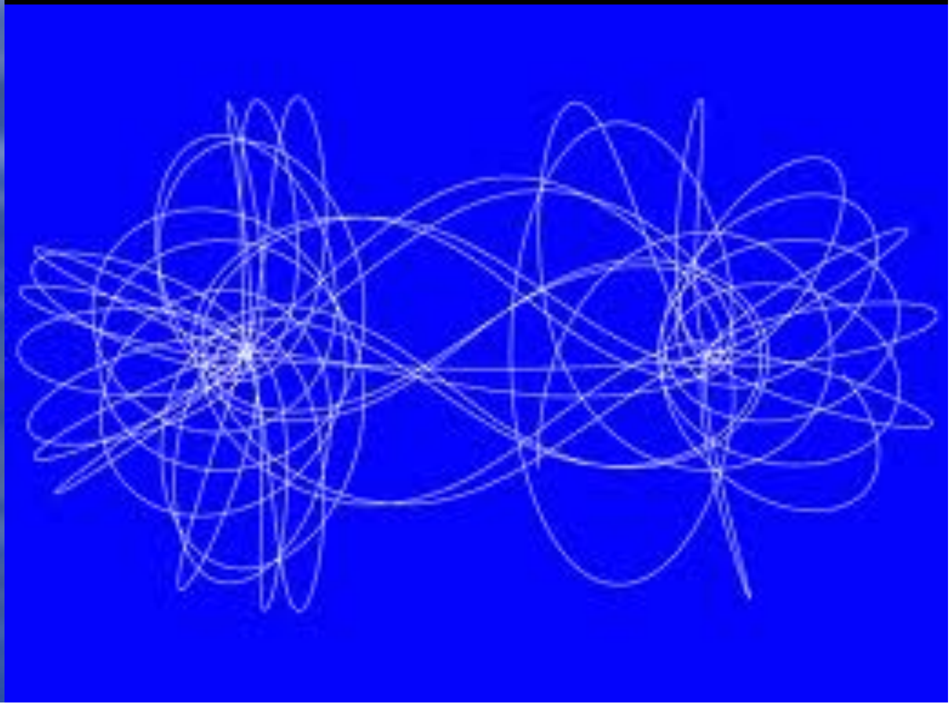
- From this
  - math happens, and you get analytic solutions for the motion Kepler's laws, etc.
  - We measure masses in astronomy this way
  - We find black holes with this



A Third Mass?

- And everything goes crazy!!
- CHAOTIC SYSTEM - extreme sensitivy to initial conditions, no analytic solution most of the time





Except for

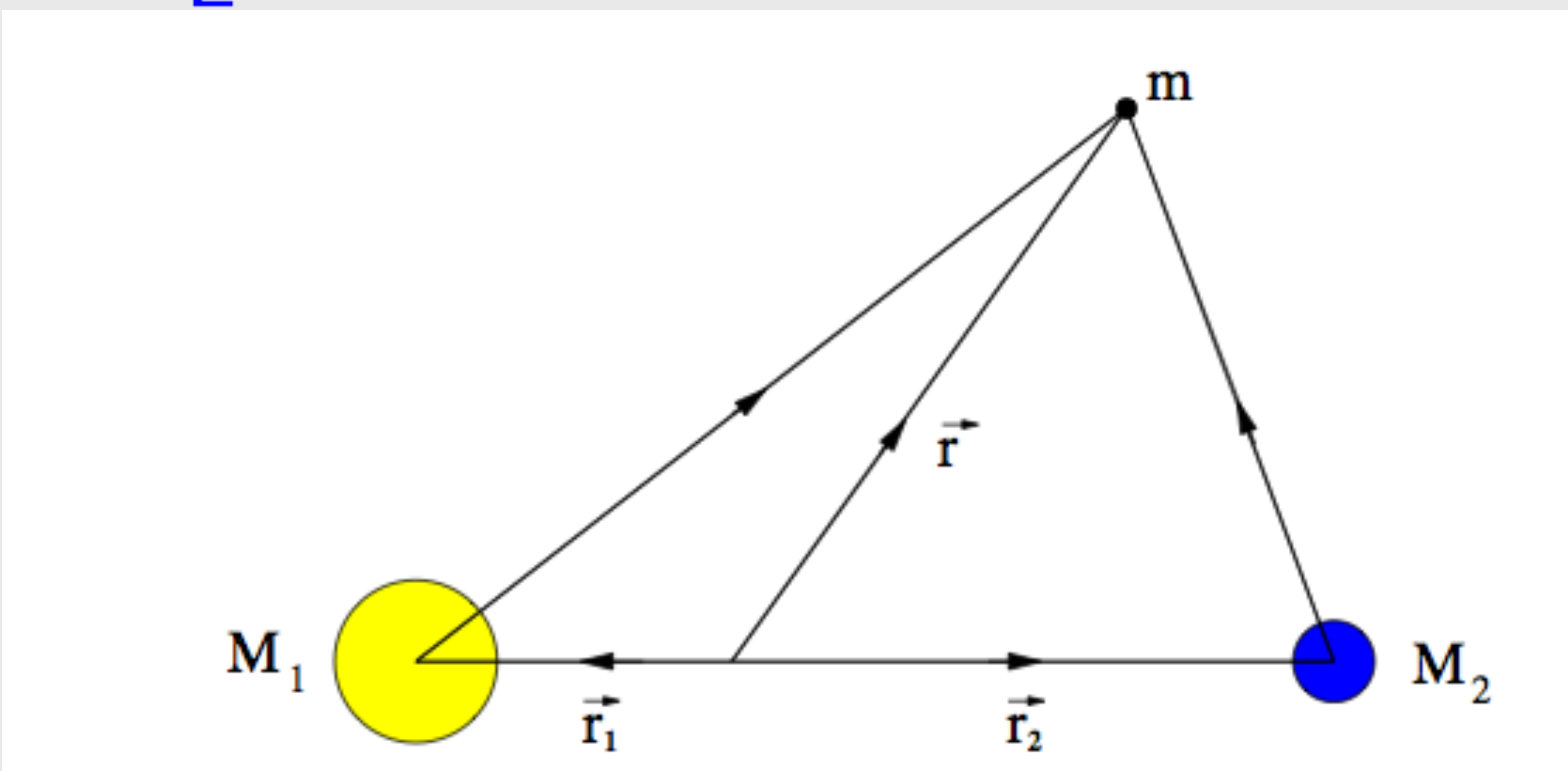
- The restricted three body system
- m_3 is negligible
- We get the Jacobi Integral
- Coriolois effect is not important for equilibrium points
- Only conserved quantity of motion

$$C_J = n^2(x^2 + y^2) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

Roche Potential

$$\Phi = -\frac{GM_1}{(x-\mu a)^2+y^2+z^2} - \frac{GM_2}{(x+(1-\mu)a)^2+y^2+z^2}$$

$$-\frac{1}{2}\Omega^2\left(x^2+y^2\right) \qquad (4.13)$$

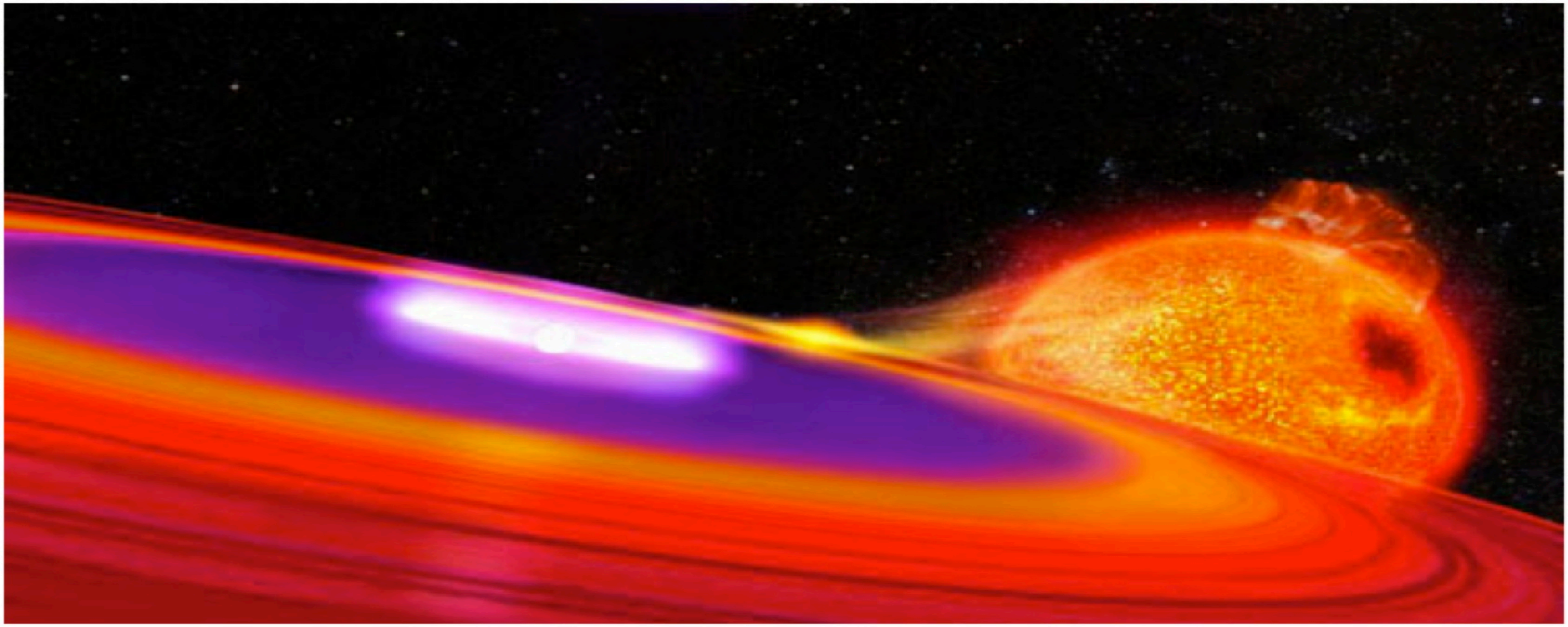


Now It Is Analytically Tractable

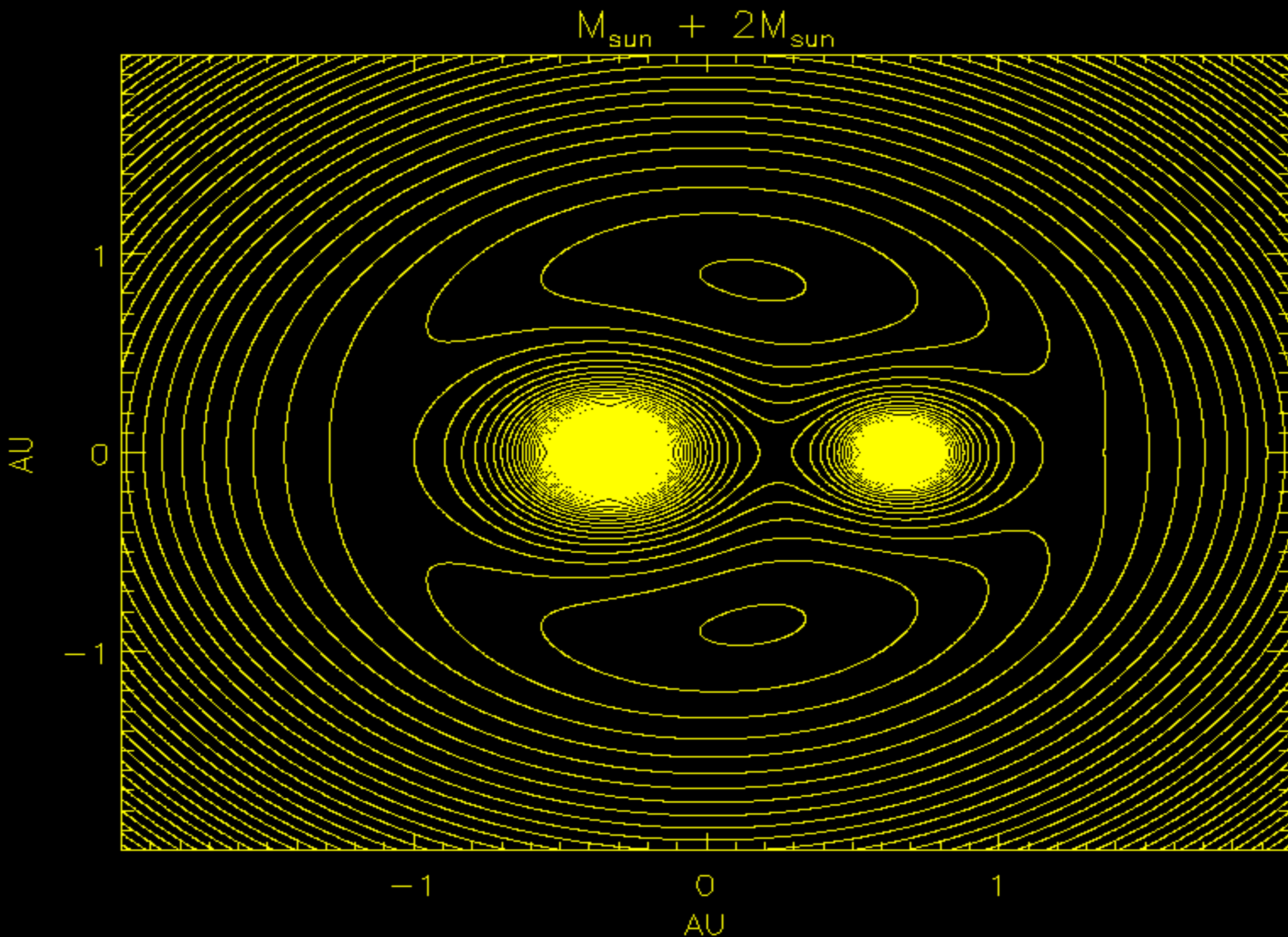
- The restricted three body problem gives the force on a third body whose mass is negligible compared to the other two
- Computers make calculating the potential and orbits a snap.
- So What?

Well?

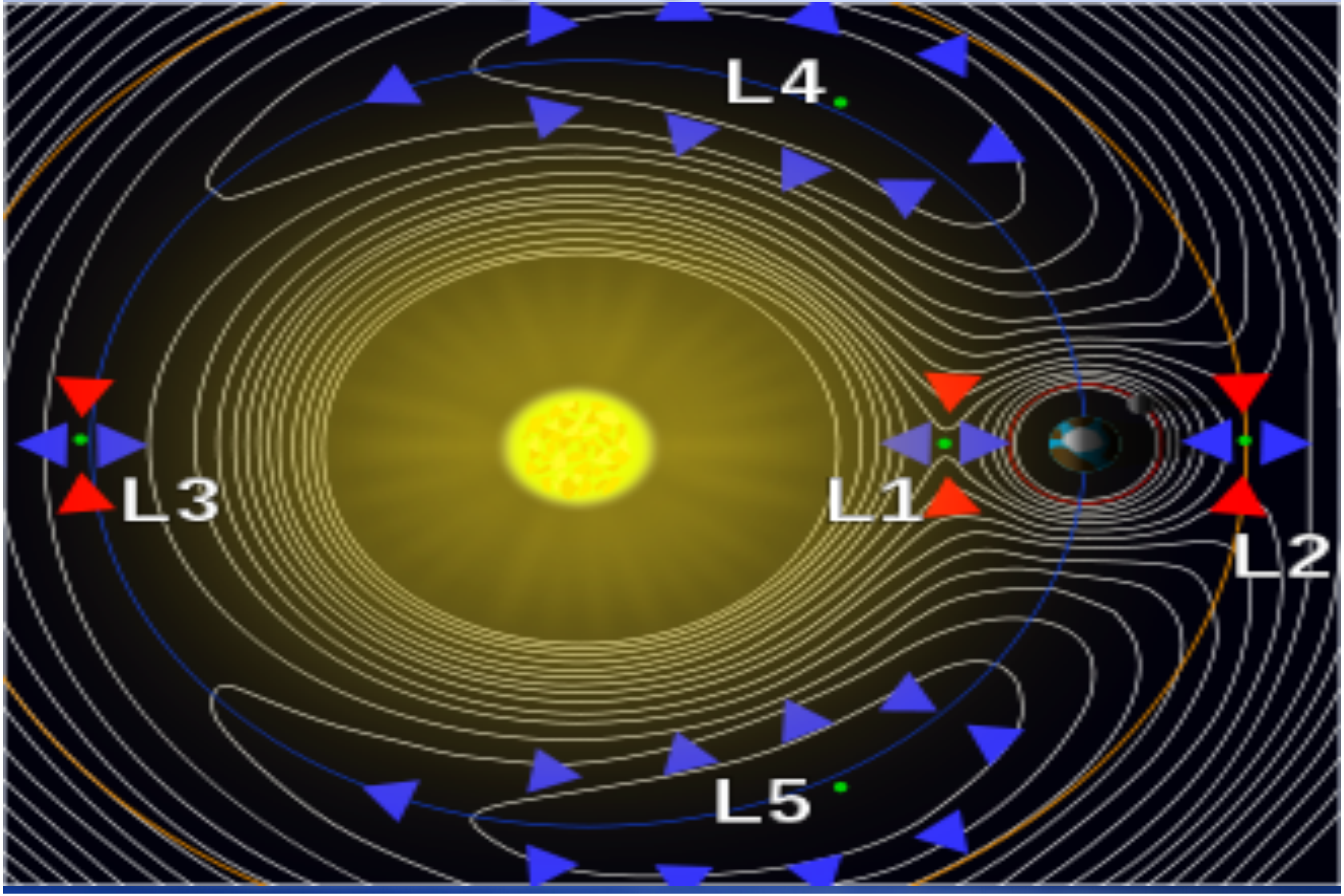
- Satellite are good examples
- So is the mass flowing from one object to another
- But Wait --- Doesn't the mass change the whole thing if the primary masses change?
  - Yes, binary systems dynamics depends critically on this
  - Type 1a Supernova, Cataclysmic Variables, etc.

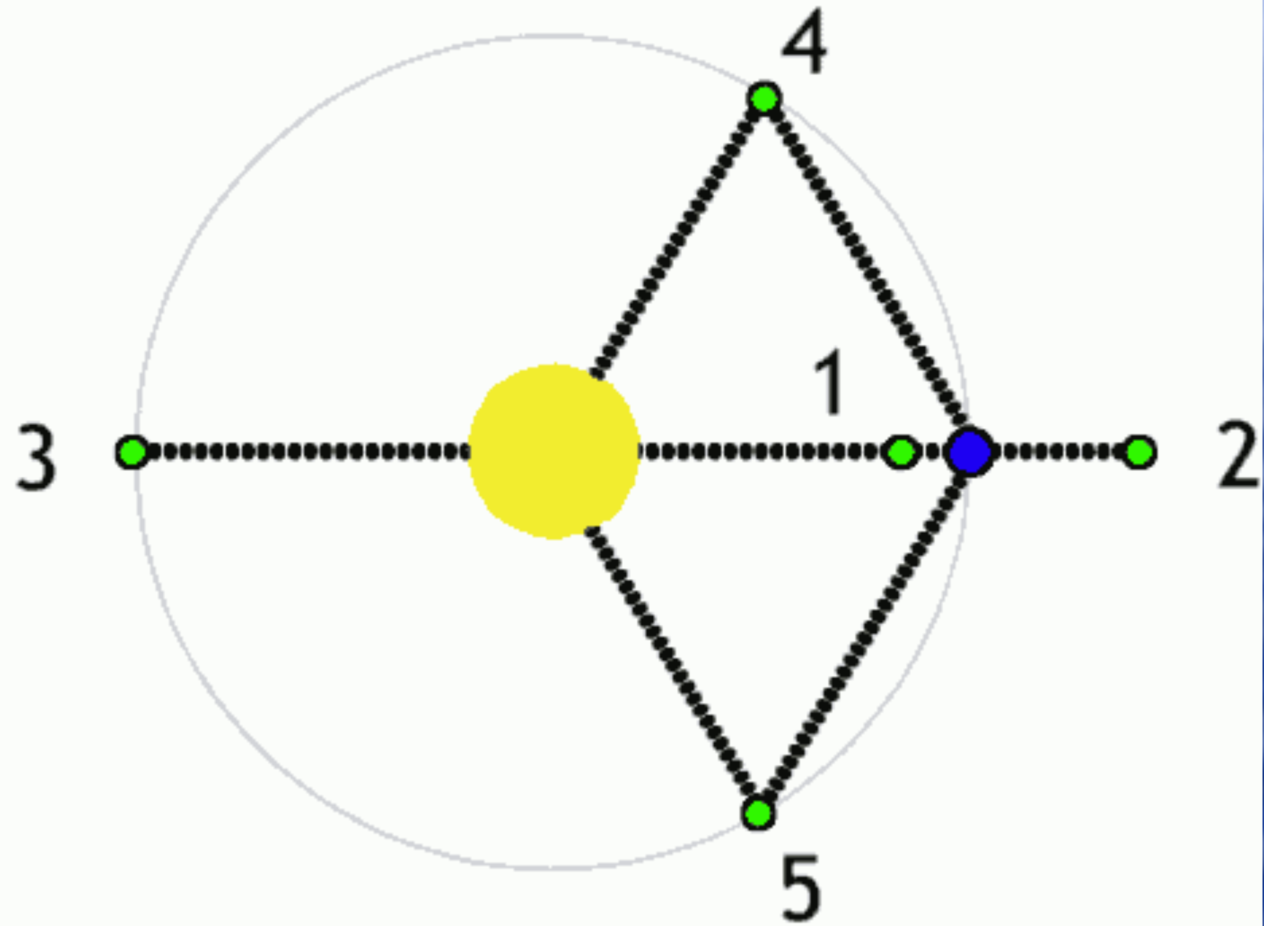


Lets Look At The Potential

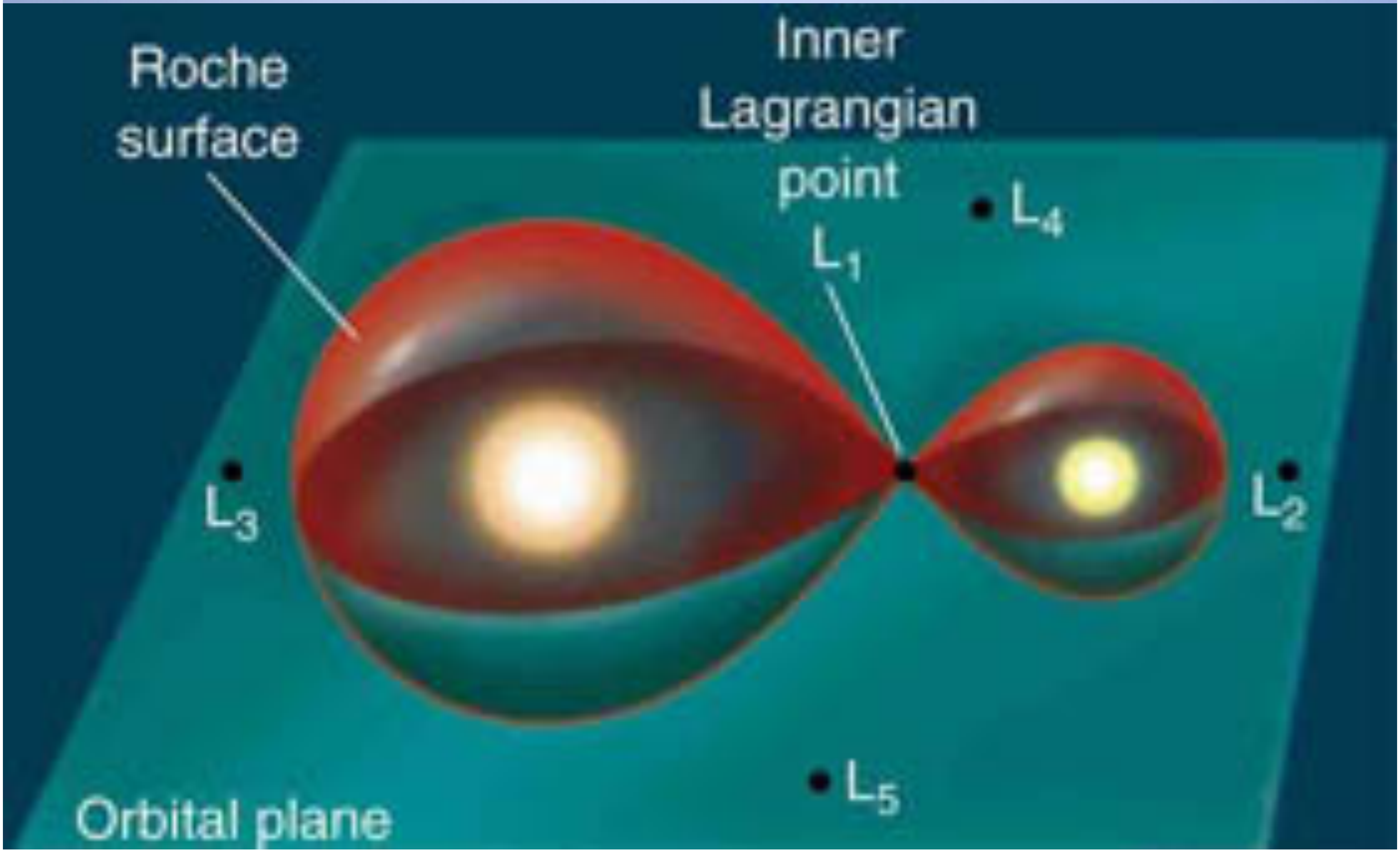


Lagrange Points




4
1
3
2
5

Roche Lobes


Roche surface
Inner Lagrangian point
L1
L4
L3
L2
L5
Orbital plane

Lagrange Points

- Stability determined by considering perturbations from L points, must include Coriolis forces in analysis
- L1,L2, L3 – Unstable
  - L1 & L2 Unstable on order 10s of days
  - L3 Unstable on order couple hundred years
- L4 and L5 - Stable

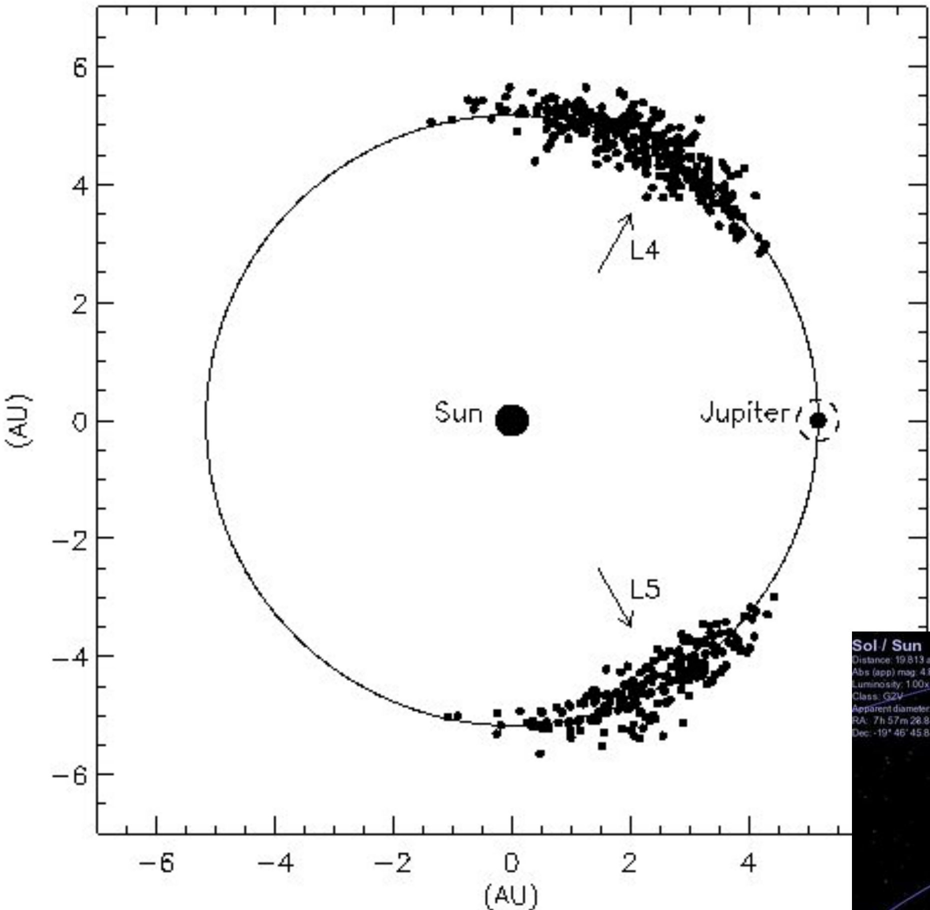
$$x = x_i + \delta x\,, \quad v_x = \delta v_x\,,$$
$$x = y_i + \delta y\,, \quad v_y = \delta v_y\,,$$

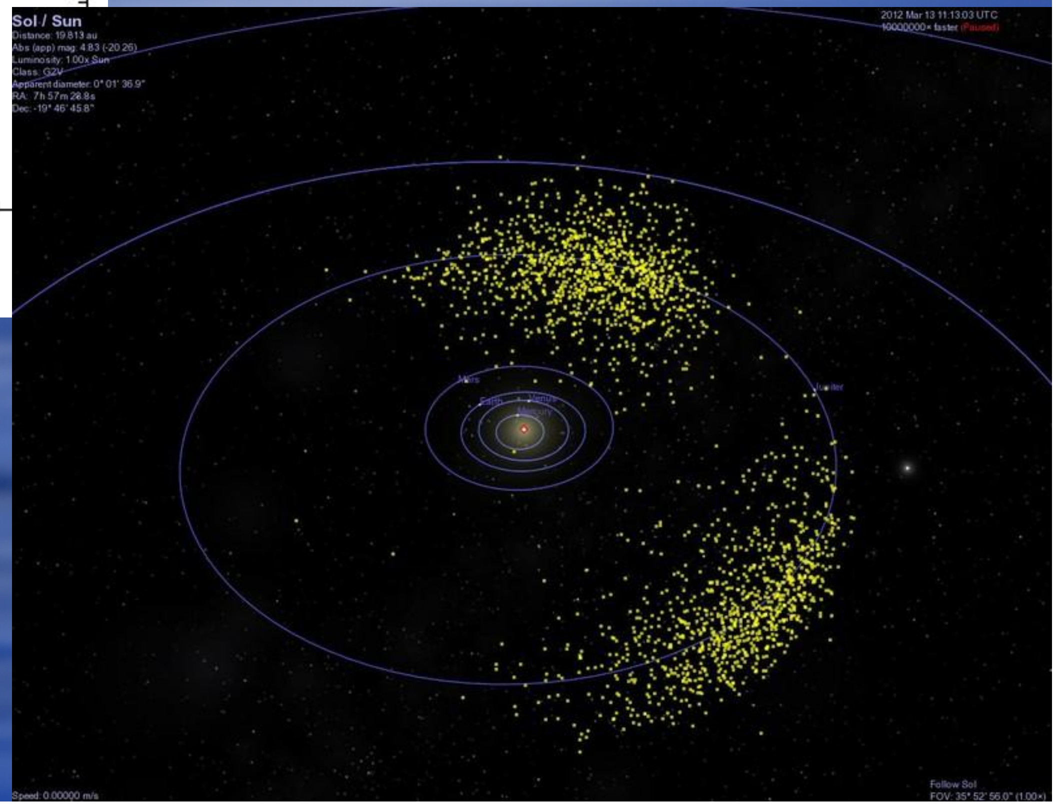
$$\frac{d}{dt}\begin{pmatrix} \delta x \\ \delta y \\ \delta v_x \\ \delta v_y \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{d^2 U_\Omega}{dx^2} & \frac{d^2 U_\Omega}{dxdy} & 0 & 2\Omega \\ \frac{d^2 U_\Omega}{dydx} & \frac{d^2 U_\Omega}{dy^2} & -2\Omega & 0 \end{pmatrix}\begin{pmatrix} \delta x \\ \delta y \\ \delta v_x \\ \delta v_y \end{pmatrix}.$$

Astrophysical Examples

- Trojans
- Sattelites
- Type 1A SN

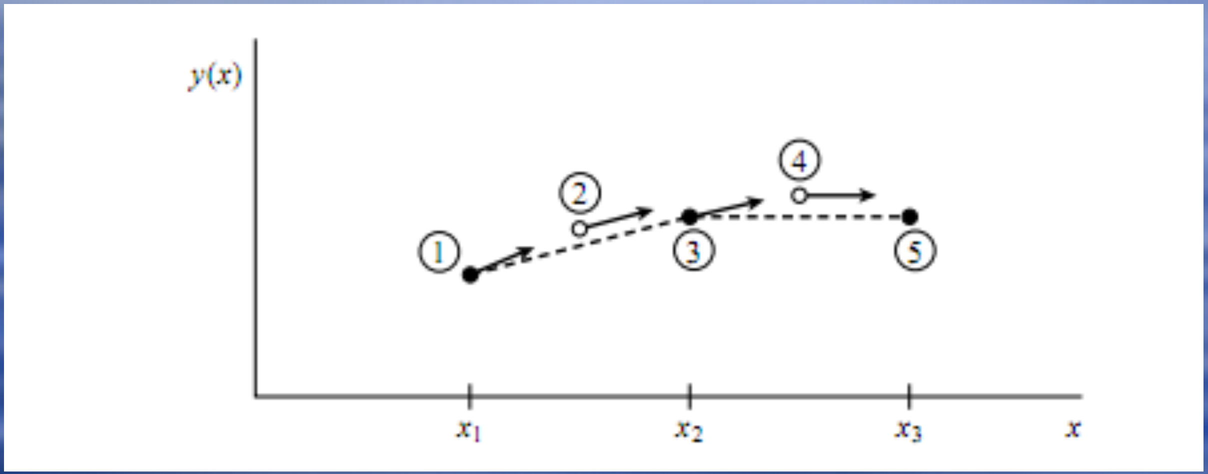
Jupiter' s Trojans





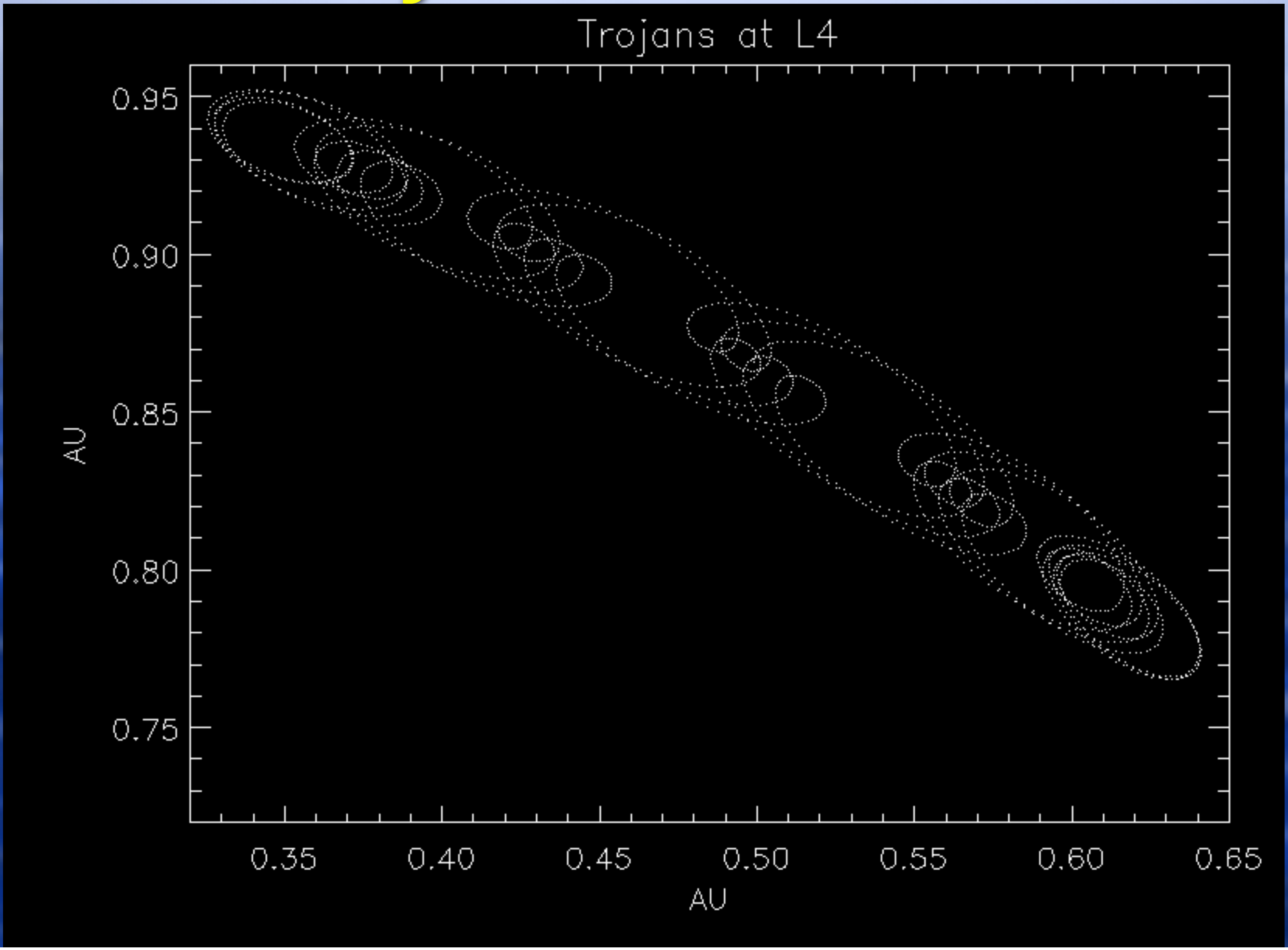
More Math Happens With Computers

- 4th Order Runge Kutta Used to Solve Orbits



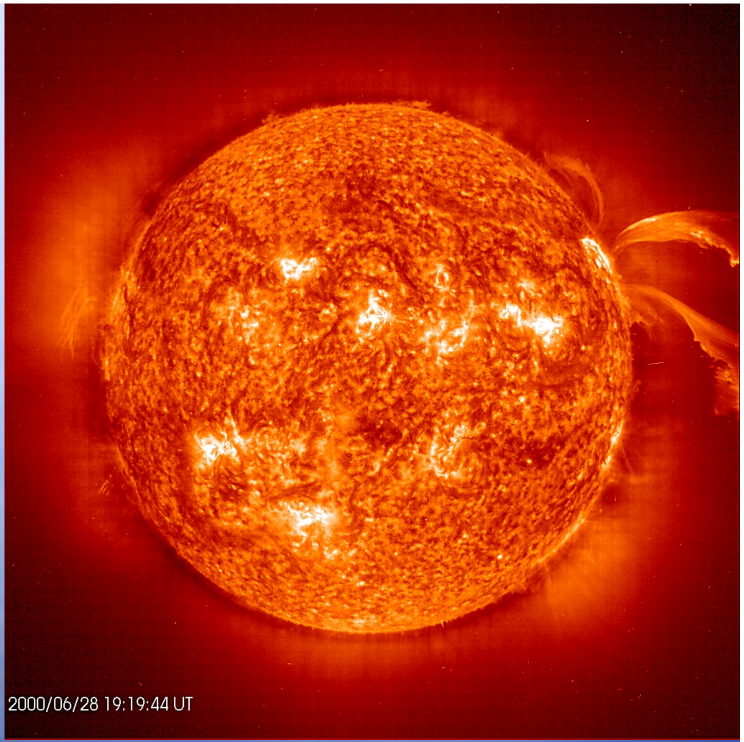
$$
\begin{aligned}
k_1 &= hf(x_n, y_n) \\
k_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \\
k_3 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}) \\
k_4 &= hf(x_n + h, y_n + k_3) \\
y_{n+1} &= y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5)
\end{aligned}
$$

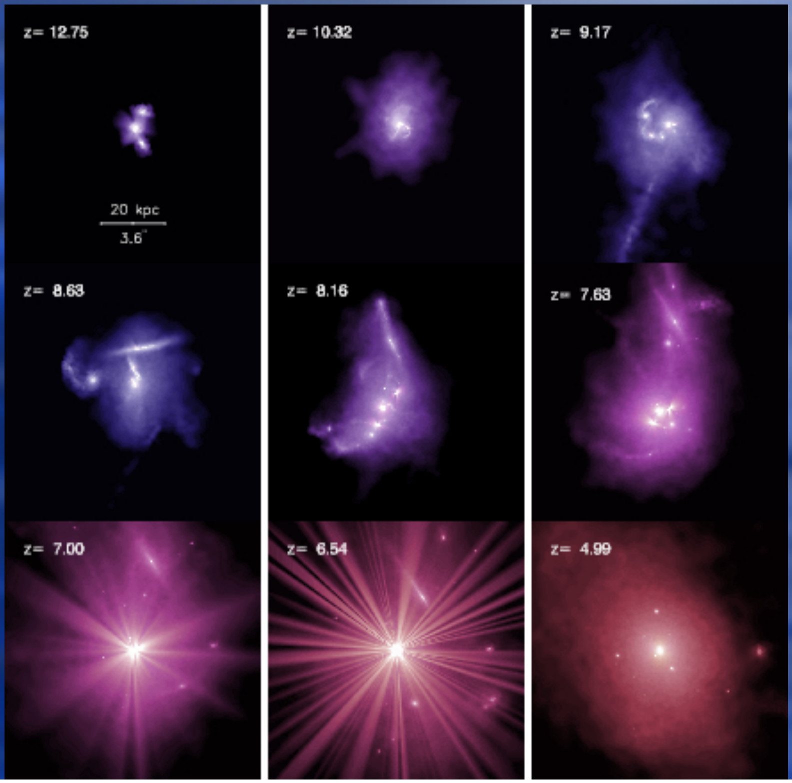
Trojan Orbits at L4

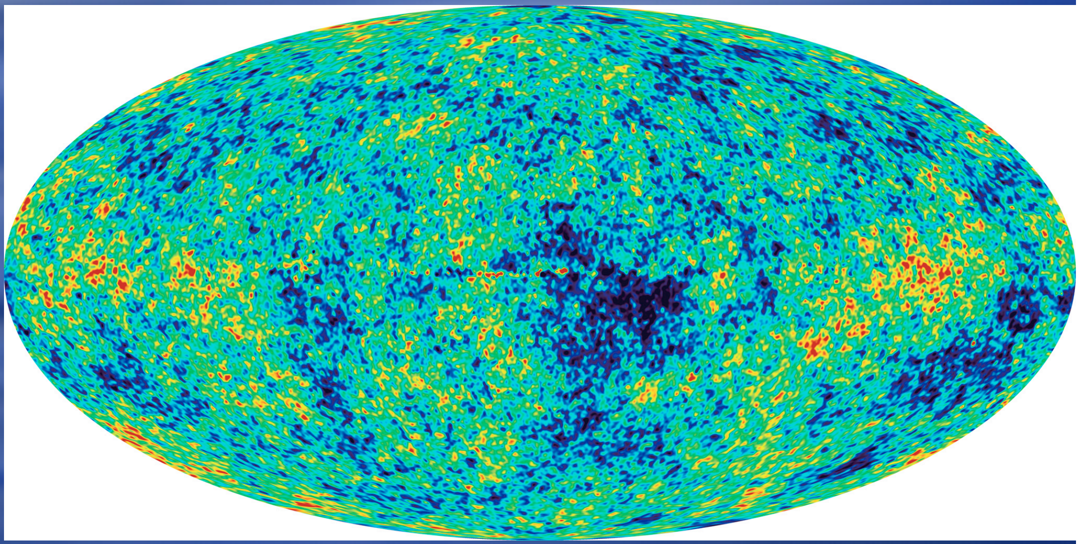


Satellites

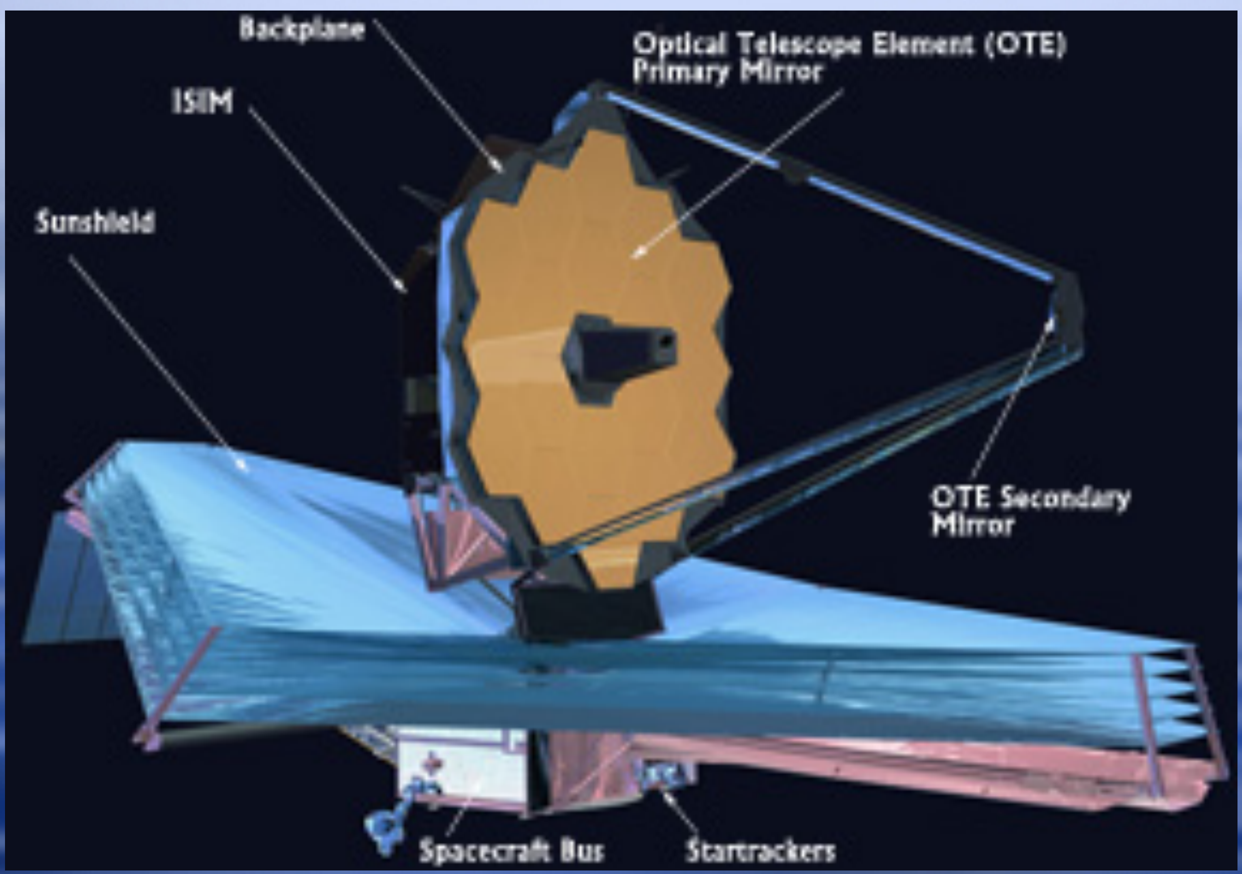
- SOHO
- Wilkinson Microwave Anisotropy Probe
- James Webb Space Telescope - Does the Renaissance end?

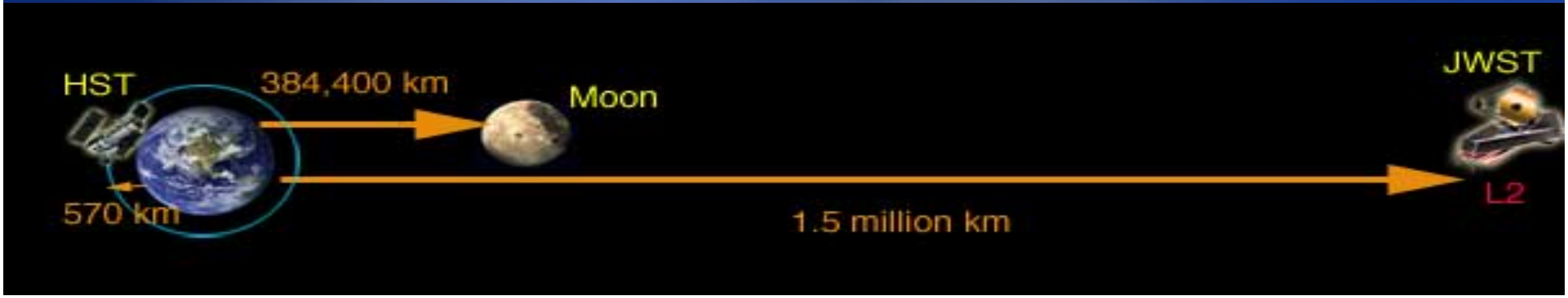






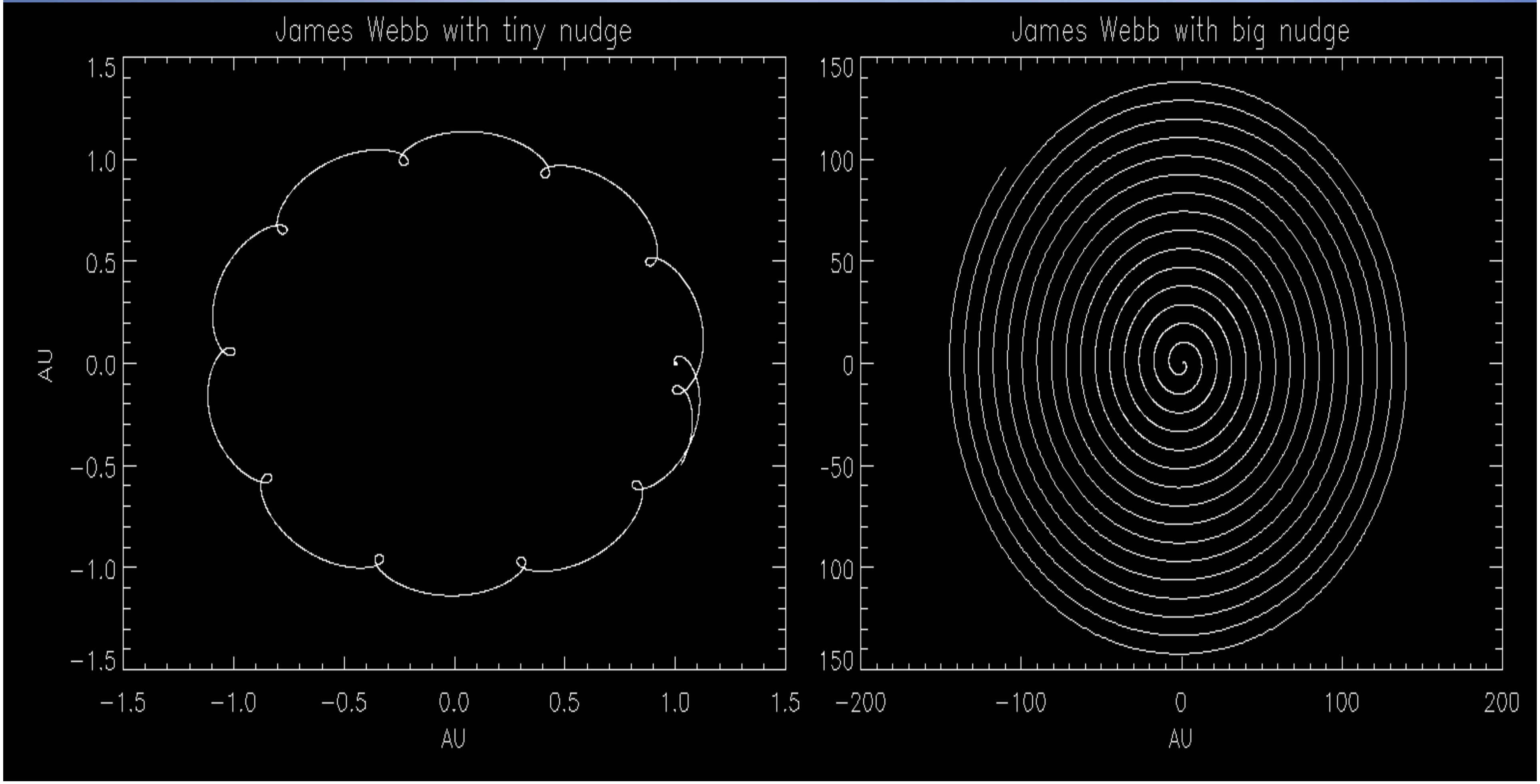
James Webb Space Telescope





But be Careful!!!

- Webb will need a bit of thrust
  - L2 is Unstable

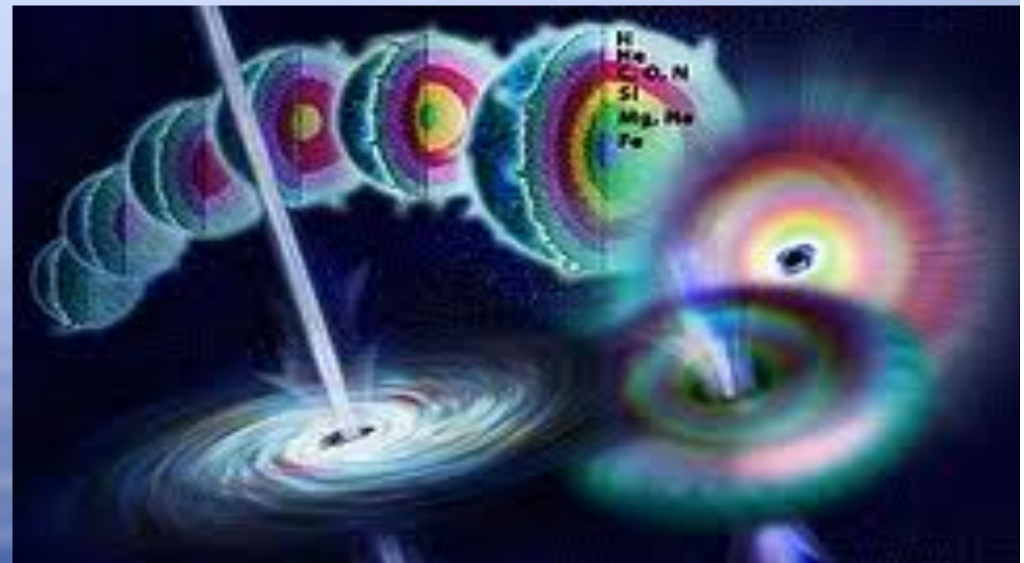


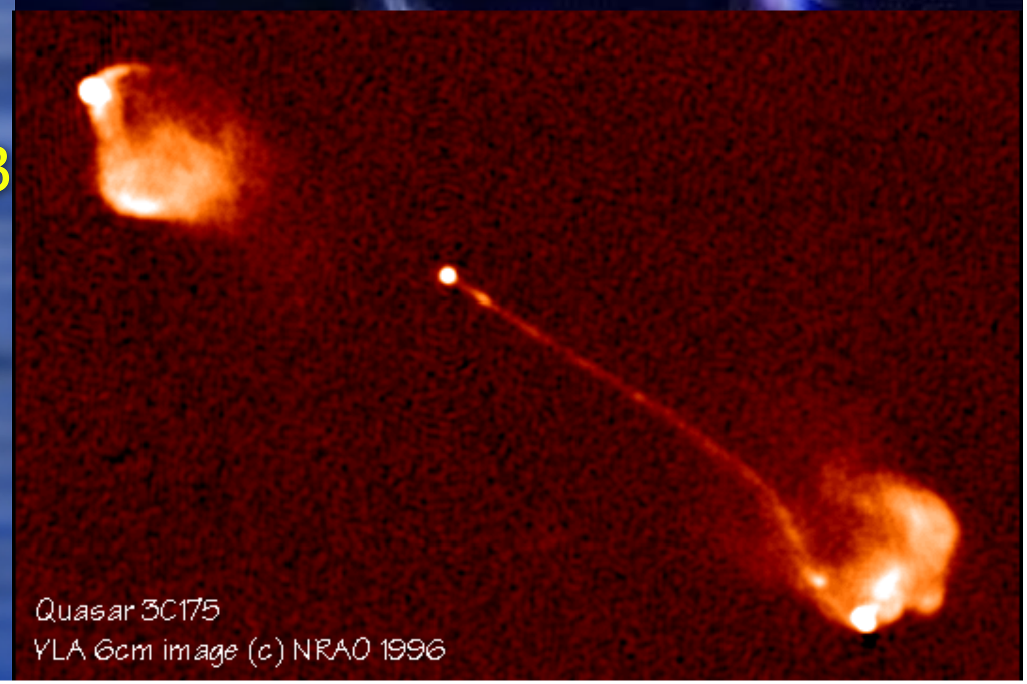
Now On to the Cool Stuff

- Cataclysmic Variables & Type 1A SN
  - Binaries where a lot of energetic stuff is going on
  - Form when one star overfills its Roche Lobe
  - But First How Do Disks Form?

Accretion Disks

- Much of my graduate work was on instabilities which allow material to accrete
- Disks form due to mass transfer
- Found in Binaries
- Form around gravitating masses
- Give rise to the most energetic phenomenon in universe
  - Type 1a supernovae, GRB Quasars,etc.





Accretion

- Best way to convert mass to energy short of pair annihilation - drop material onto a compact object
  - Efficiency of P-P chain is .7%
  - Luminosity is given by

$$L = \frac{GM(1)\dot{M}}{2r_{\rm in}}\left\{1 - \frac{r_{\rm in}}{r_{\rm out}}\left[3 - 2\left(\frac{r_{\rm in}}{r_{\rm out}}\right)^{1/2}\right]\right\}. \qquad (10)$$

  - Temperature is given by

$$T(R) = \left(\frac{3GM_*\dot{M}}{8\pi\sigma}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right)^{1/4} \qquad (2)$$

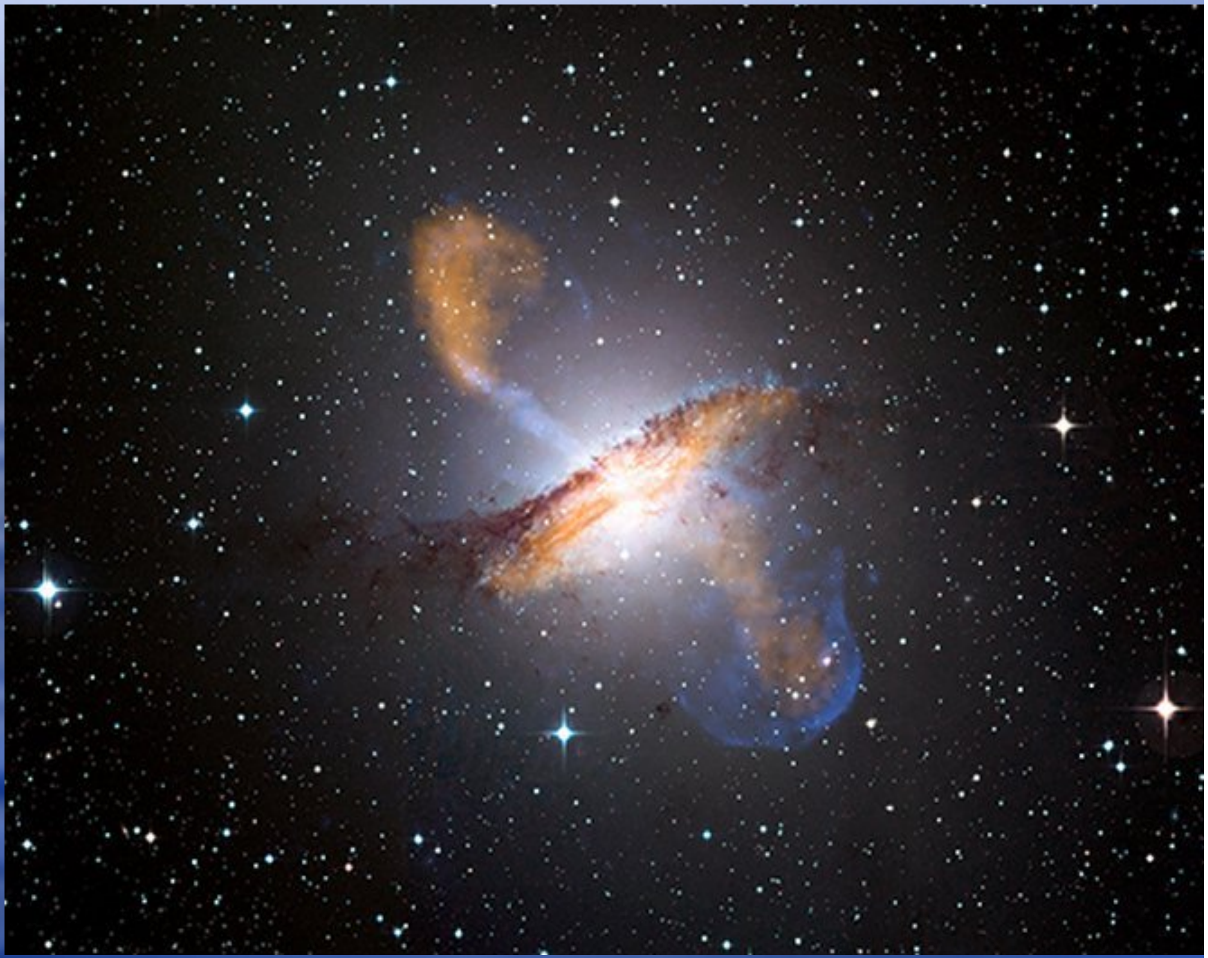
  - ~10%-20% efficient, compare this with .7% for P-P chain in the sun

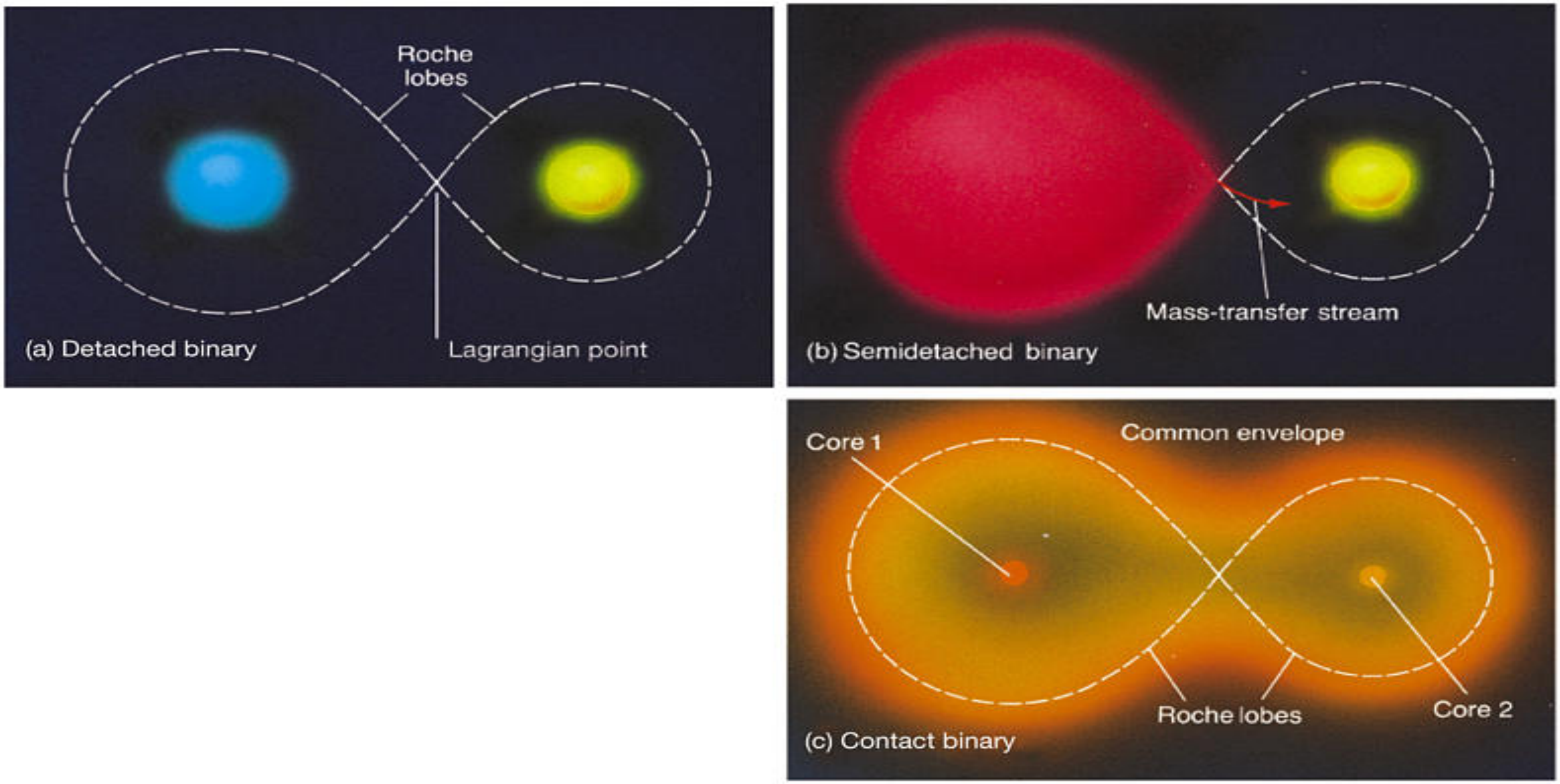
Some Examples

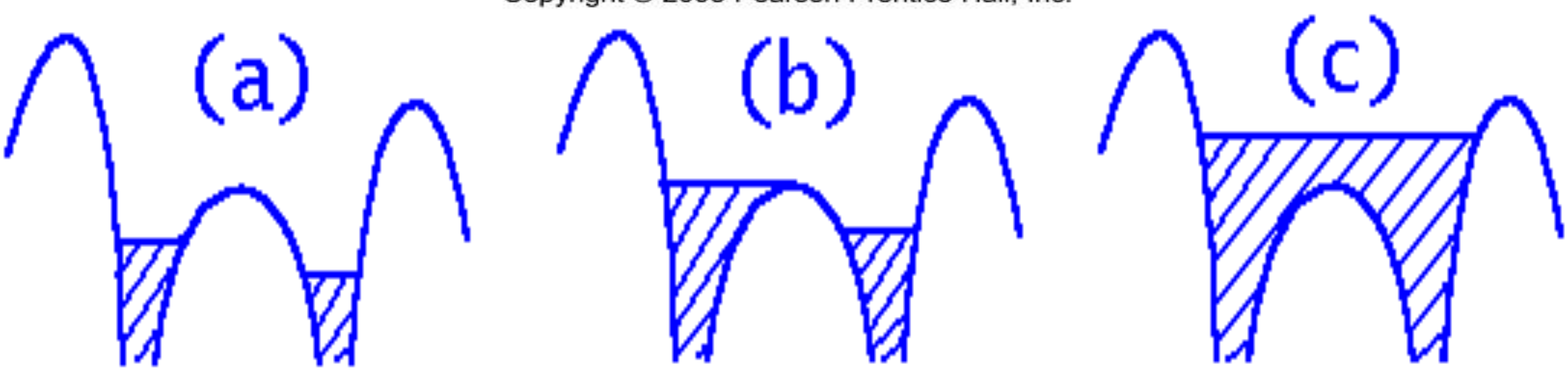
- For a 1000 km white dwarf
  - L is ~ 10^{33} ergs/s
  - Blackbody Temperature is ~500,000 Kelvin
  - photon temperature $h\nu$ is 6eV - 100keV
    - Optical through soft X-ray
- For a 10 km neutron star
  - L is ~ 10^{36} ergs/s
  - Blackbody temperature is ~10 million Kelvin
  - photon temperature is 1keV - 50 MeV
    - Hard X-ray through gamma ray
- The Sun's luminosity is $4*10^{33}$ ergs/s
- 1 keV is ~ 10,000 K
- Assumes feeding 10^{16} grams/second or 10^{-9} solar masses a year
  - BUT FEEDING RATE IS NOT CONSTANT - outbursts

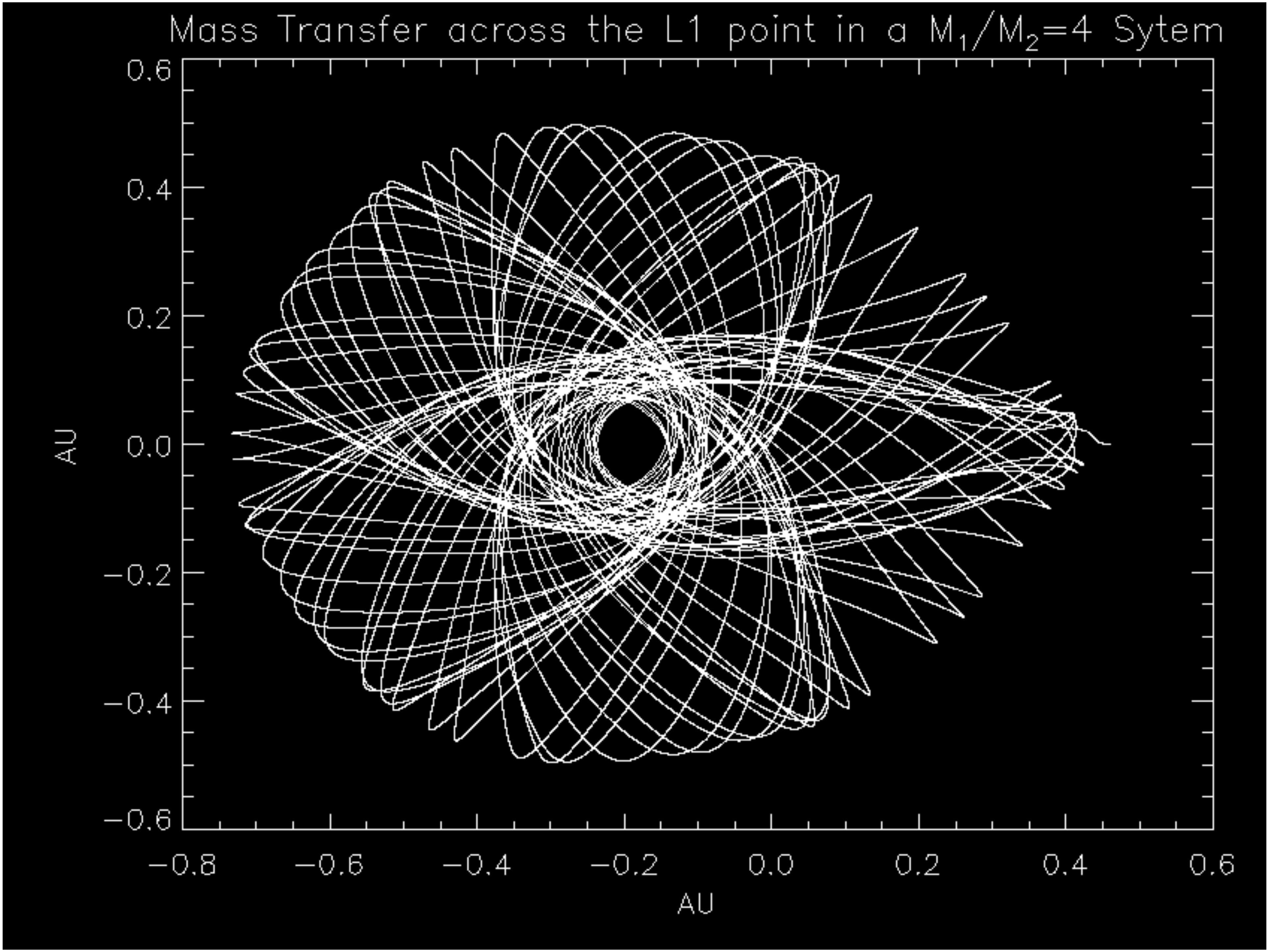
How Do Accretion Disks Form In Binary Systems?

- Note this method is distinct from accretion disks at galactic centers (GC)
- GC accretion disks give us Active Galactic Nuclei in young galaxies, this would be a good idea for another talk




Roche lobes
(a) Detached binary
Lagrangian point
Mass-transfer stream
(b) Semidetached binary
Core 1
Common envelope
Roche lobes
Core 2
(c) Contact binary


(a)
(b)
(c)



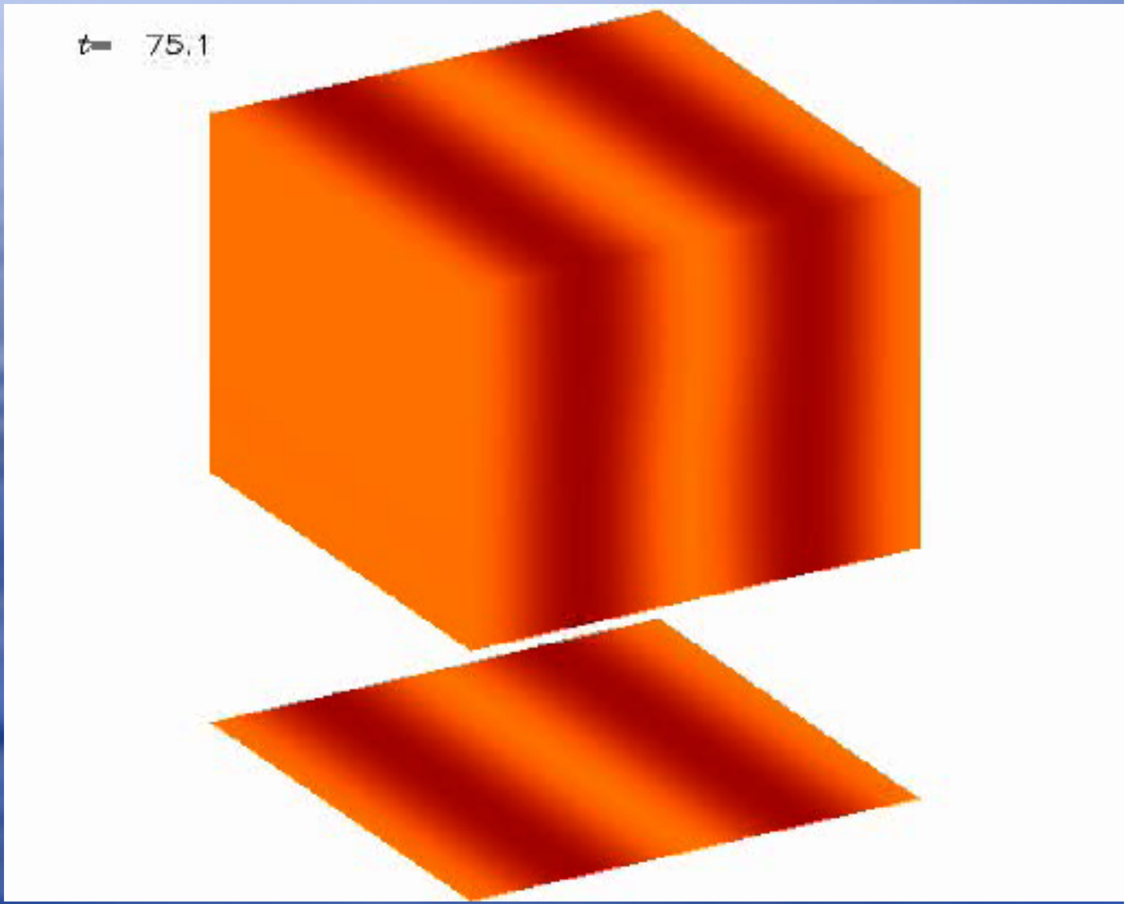
Mass Transfer across the L1 point in a $M_1/M_2=4$ Sytem
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
AU
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
AU

Binaries

- All stars expand after leaving the main sequence when they no longer have hydrogen cores (Giant Phase)
- When they expand beyond there L1 point (Roche Lobe surface) they spill out onto their companion
- Material still has specific angular momentum
- Can't accrete directly onto companion any more than we spiral into the sun
- Material settles into circulation radius sets by its keplerian velocity $(GM/r)^{(1/2)}$
- A disk forms as different parcels of gas collide
- Separation evolves as masses change, must be solved iteratively, mass transfer rate changes.

How material accretes in the disk

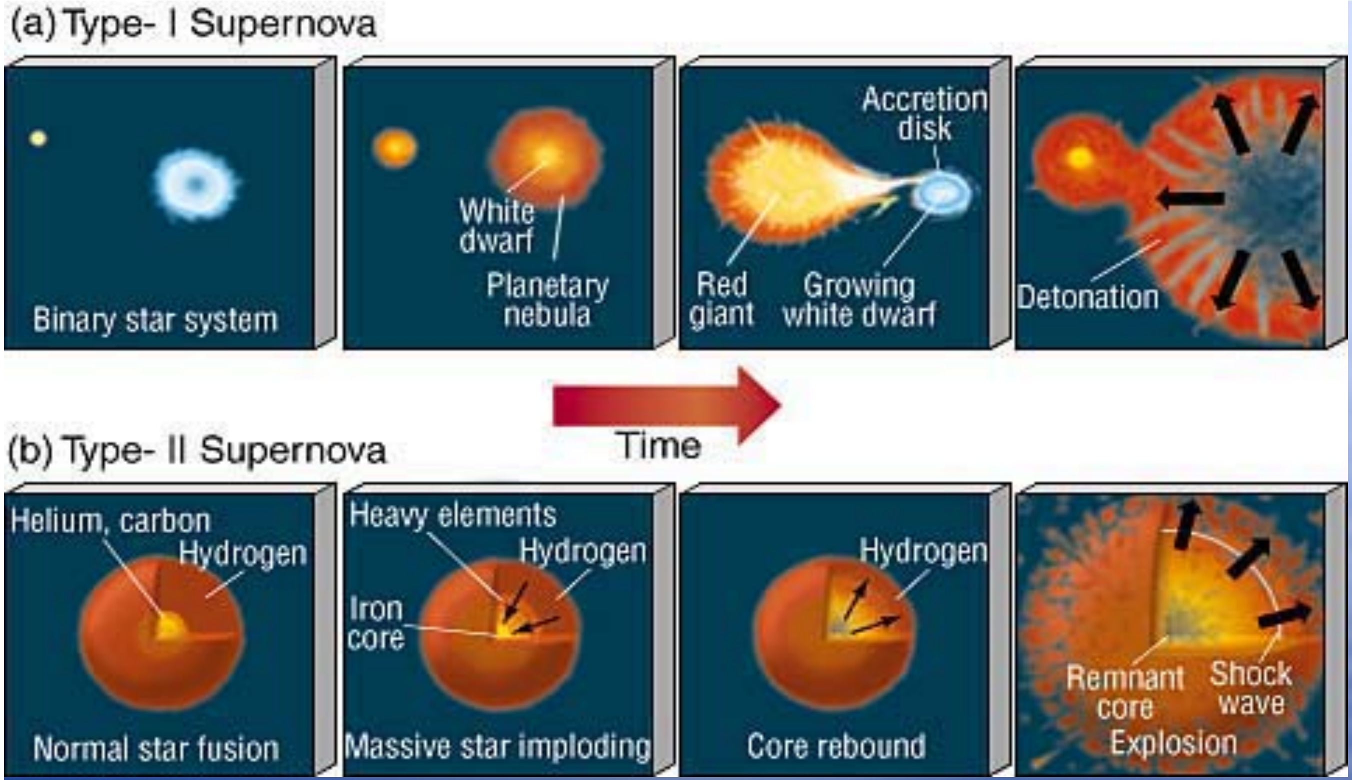
- Material has specific angular momentum
- Material slowly sheds orbital angular momentum through instabilities and accretes



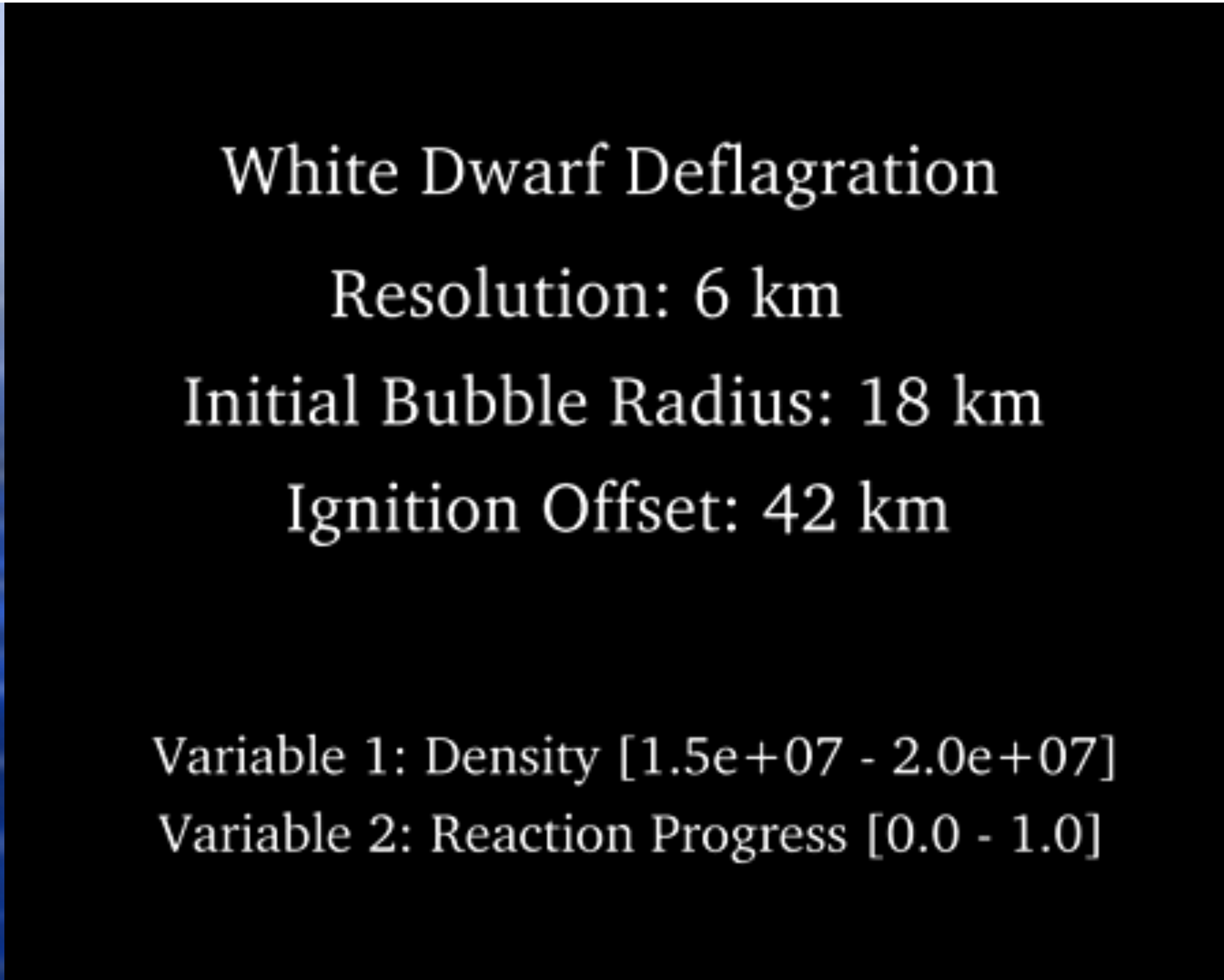
Cataclysmic Variables

- Binary star systems where one is DEAD
- Disk Forms and time dependent mass transfer from companion feeds disk
- Material in disk spirals inwards and heats
- Nova, Supernova, X-Ray Bursters
- Hydrogen and Helium fusion on disk
- For type 1A this gives us standard candles
- TYPE 1A

Type 1A Supernovae



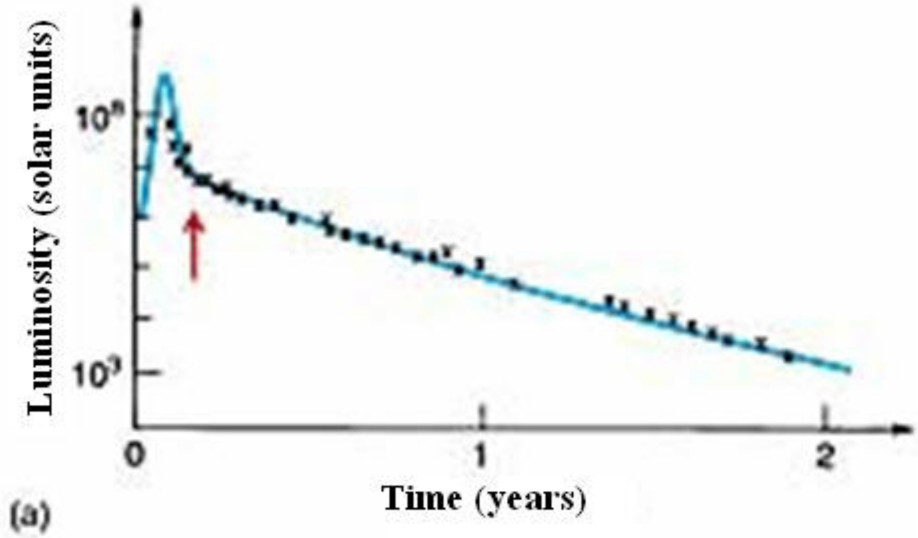
- White Dwarf Accretes more than Chandreskhar Mass
  - Blows up when electron degeneracy pressure is exceeded by gravitational force and contraction raises the temperature to carbon fusion limit
  - Two Mechanisms
    - Accretion
    - Binary separation changes due to gravity wave emission
      - One star reaches L1, shredded by gravity, dumps mass onto neighbor



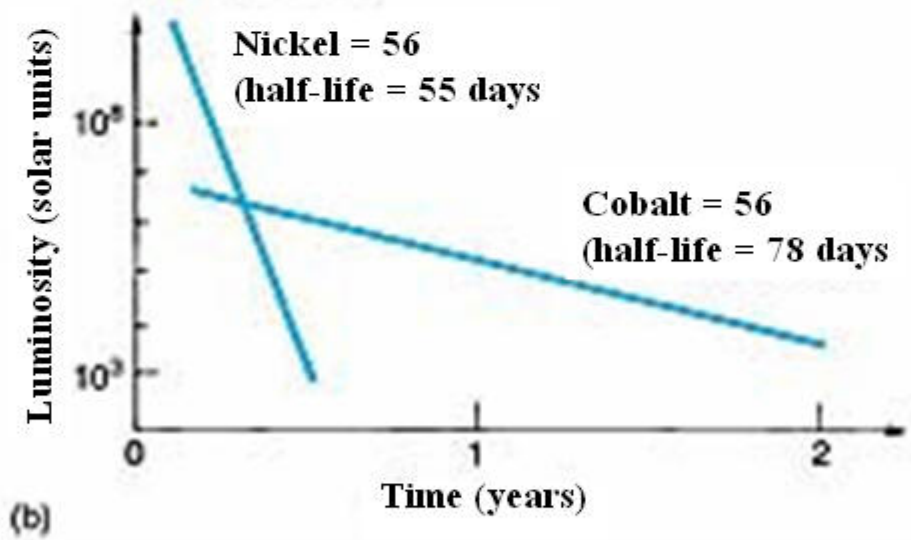
Video From The Flash Center for Computational Science

Standard Candles

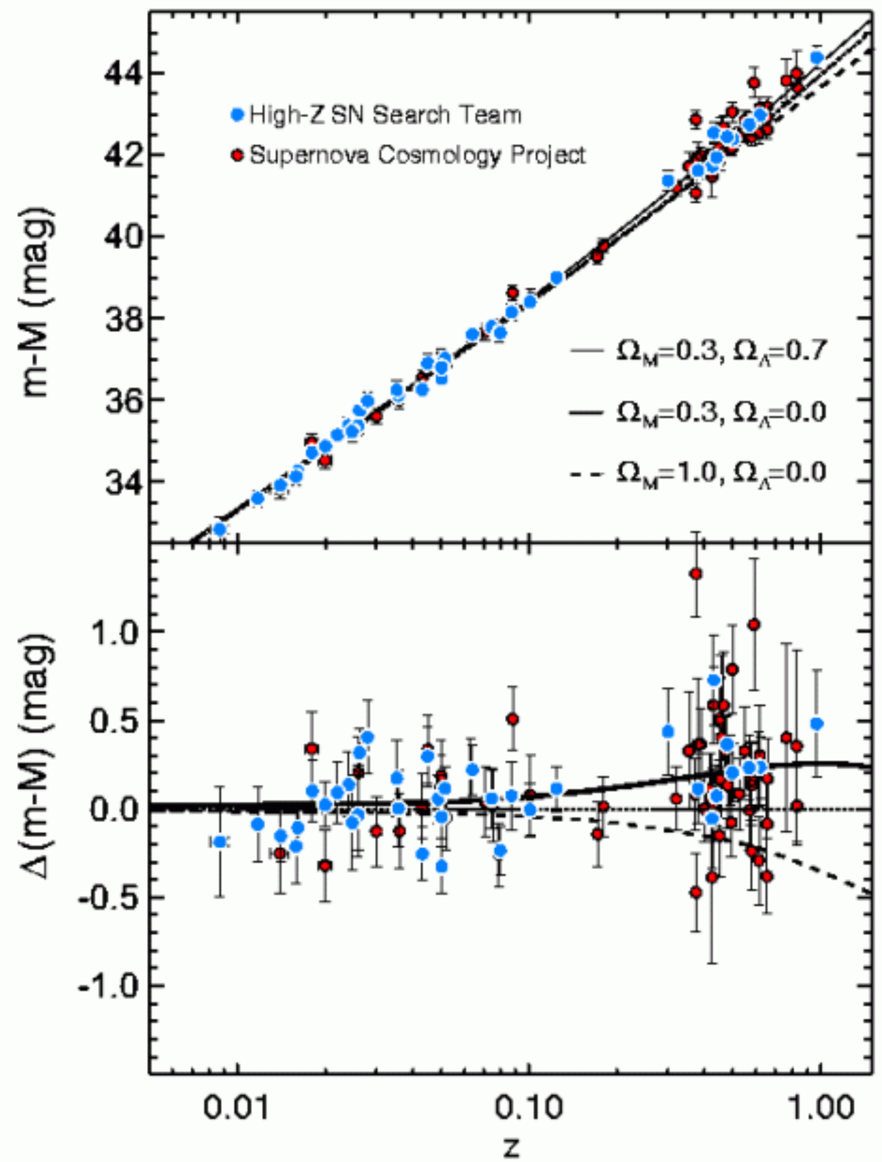
- However it blows up it blows up with ~ 1.4 Solar Masses
- Radiates primarily due to Nickel 56 decay
- Allow determination of distance to far away galaxies
  - Bright 10^^{51} ergs
  - ~5 Billion times brighter than the sun
  - Really Good Flood Light
- Allowed us to recognize the universe's expansion is accelerating
- DARK ENERGY



The light curve of a Type I supernova, showing not only the dramatic increase and slow decrease in luminosity but also the characteristic change in the rate of decay about 2 months after the explosion (after the time indicated by the arrow).



All This Started with Astrophysicists understanding the implication of the Restricted Body



Take Away

- Relatively simple physics goes a long way!!