Reading Question

What is the difference between speed and velocity?

A. Velocity contains both speed and direction
B. Velocity typically has more significant figures
C. Velocity takes into account things like air resistance
D. Velocity is valid for a higher range of values
E. Velocity is simply the scientific name for speed

This lecture

Conservation Laws

Energy

Newton

Forces

Gravity

Orbits

Tides

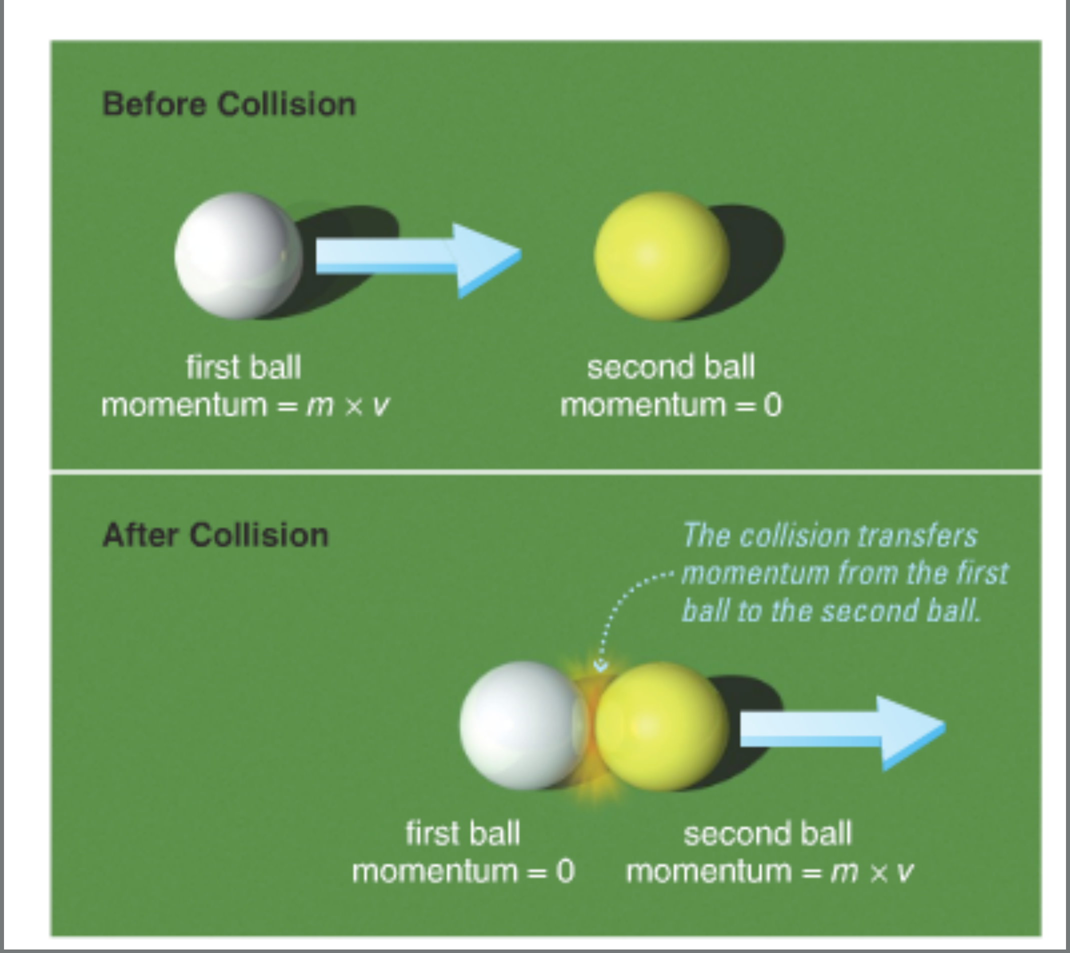
Orbits again

- Action at a distance? Weird
- Infinite energy or something else?
- What is energy?

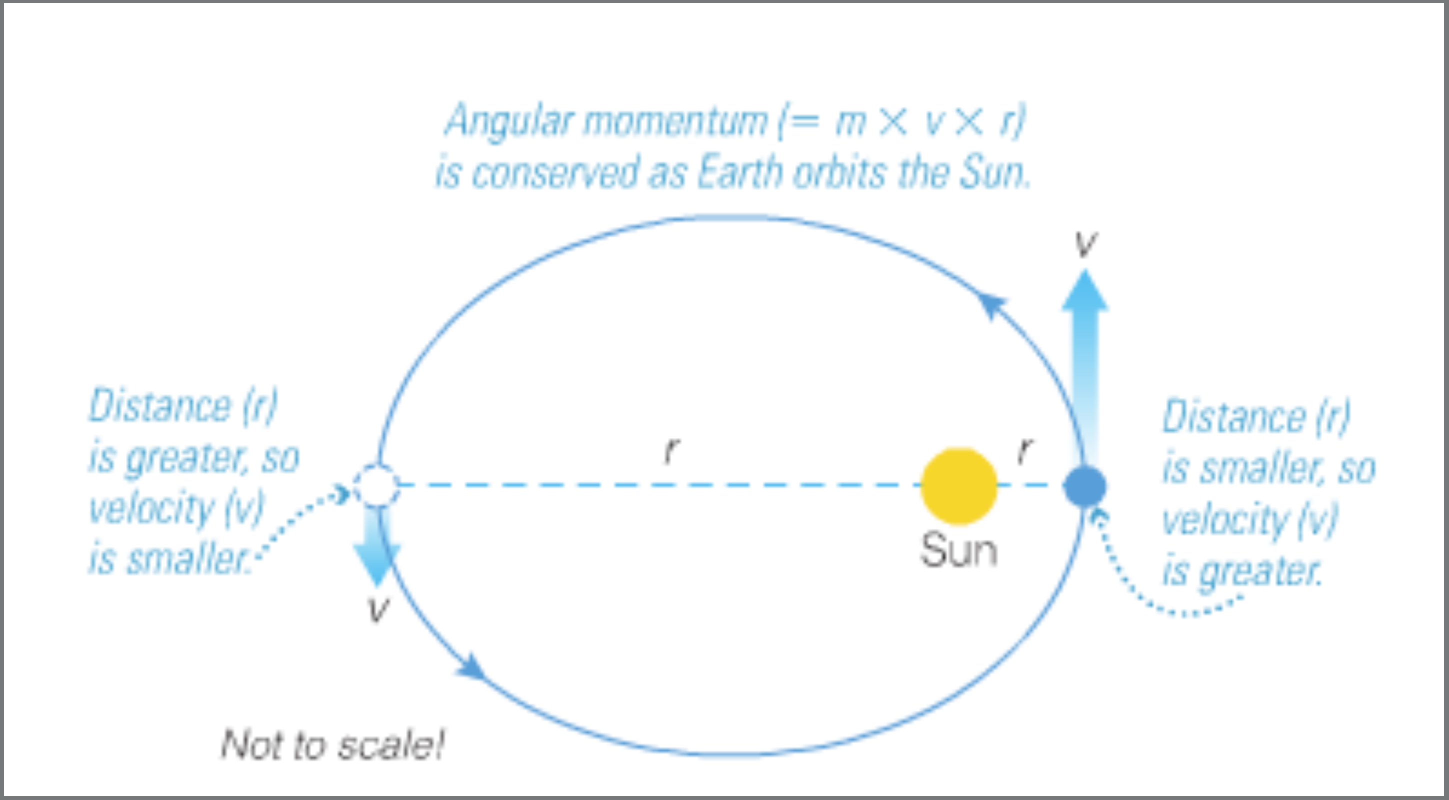
Conservation Laws

- Quantities that do not change unless acted upon by an external influence
- Linear Momentum
- Angular Momentum (orbitting and spinning)
- Energy

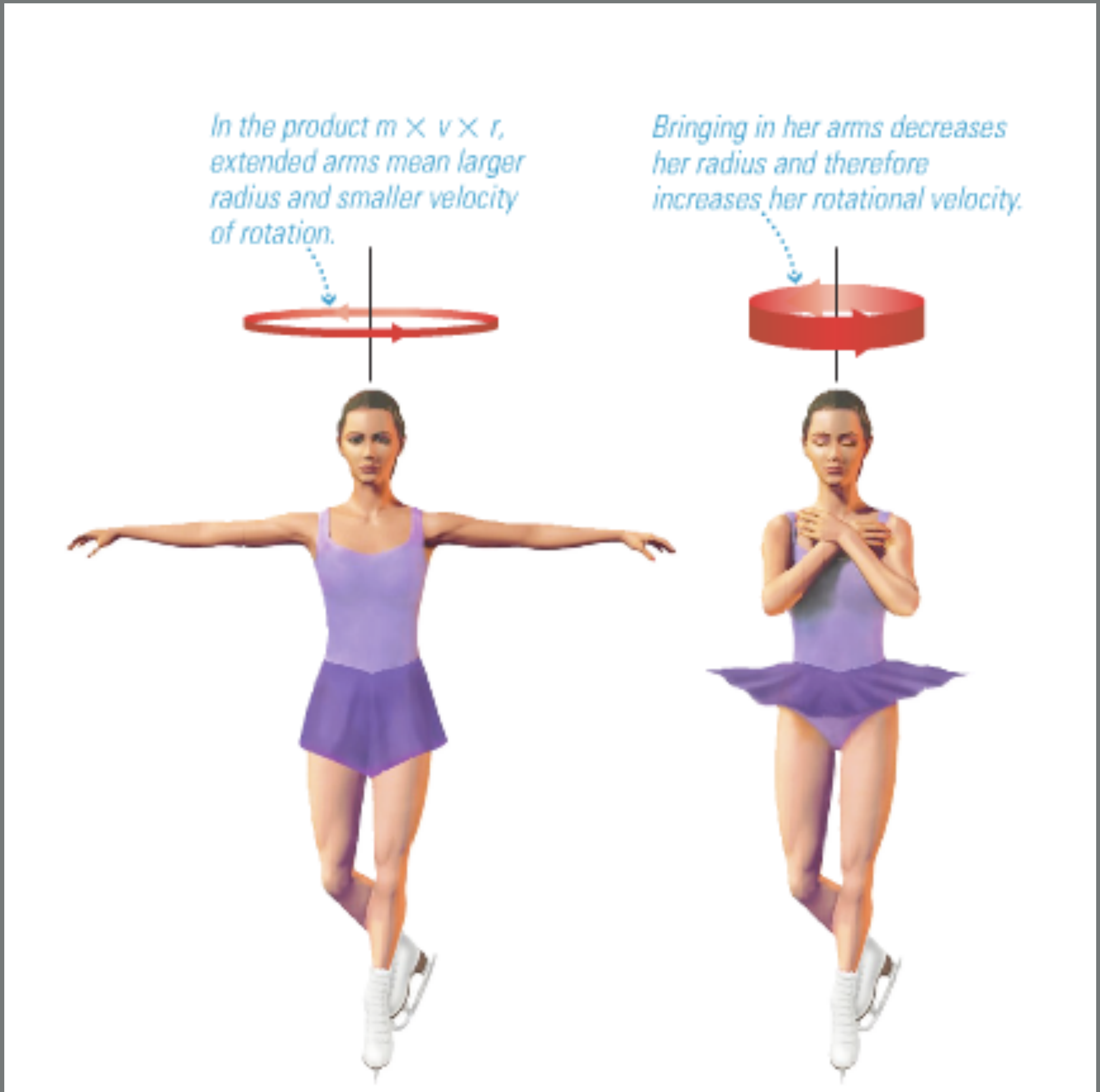
Linear Momentum



Orbital Angular Momentum



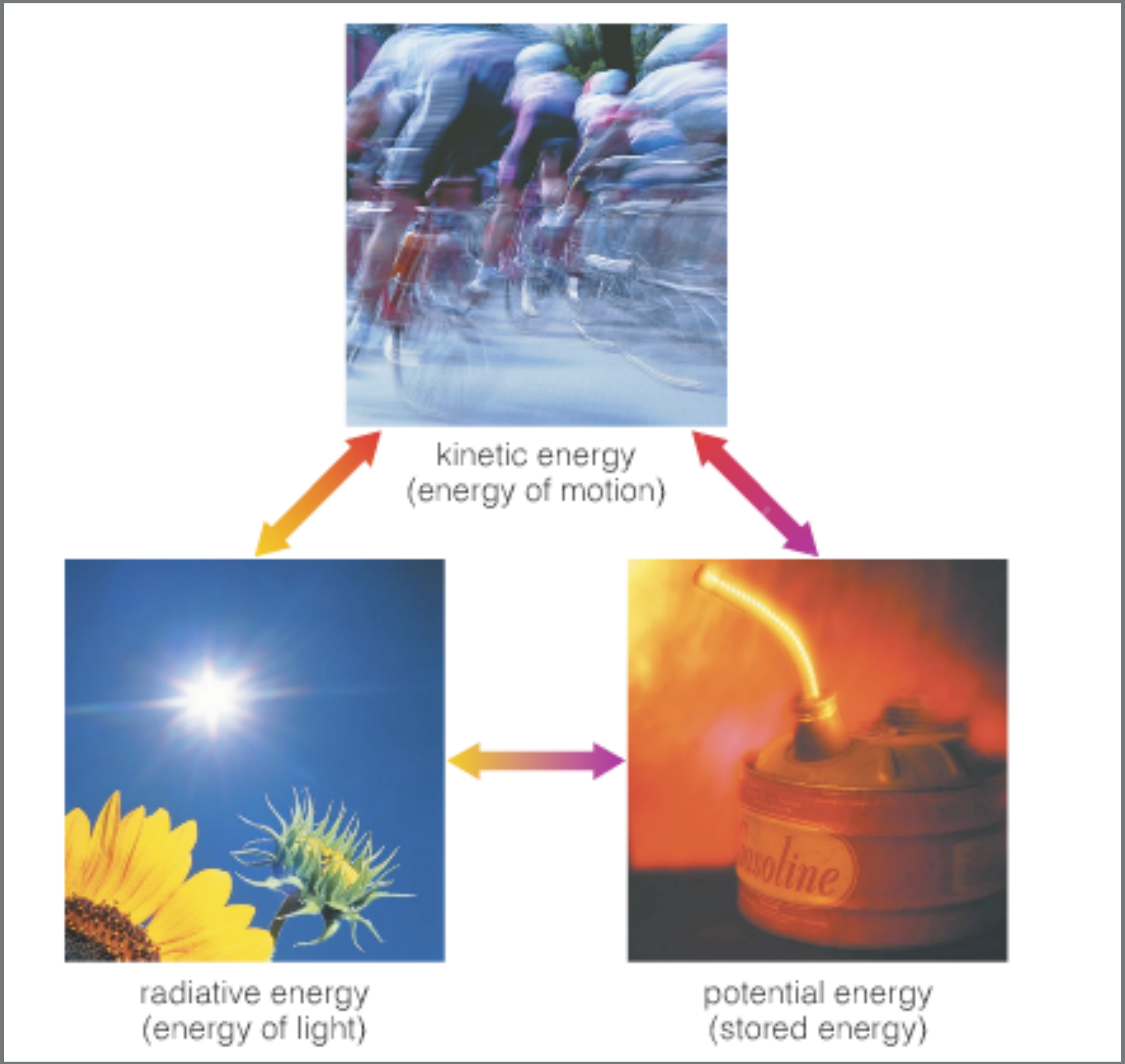
Spinning Angular Momentum

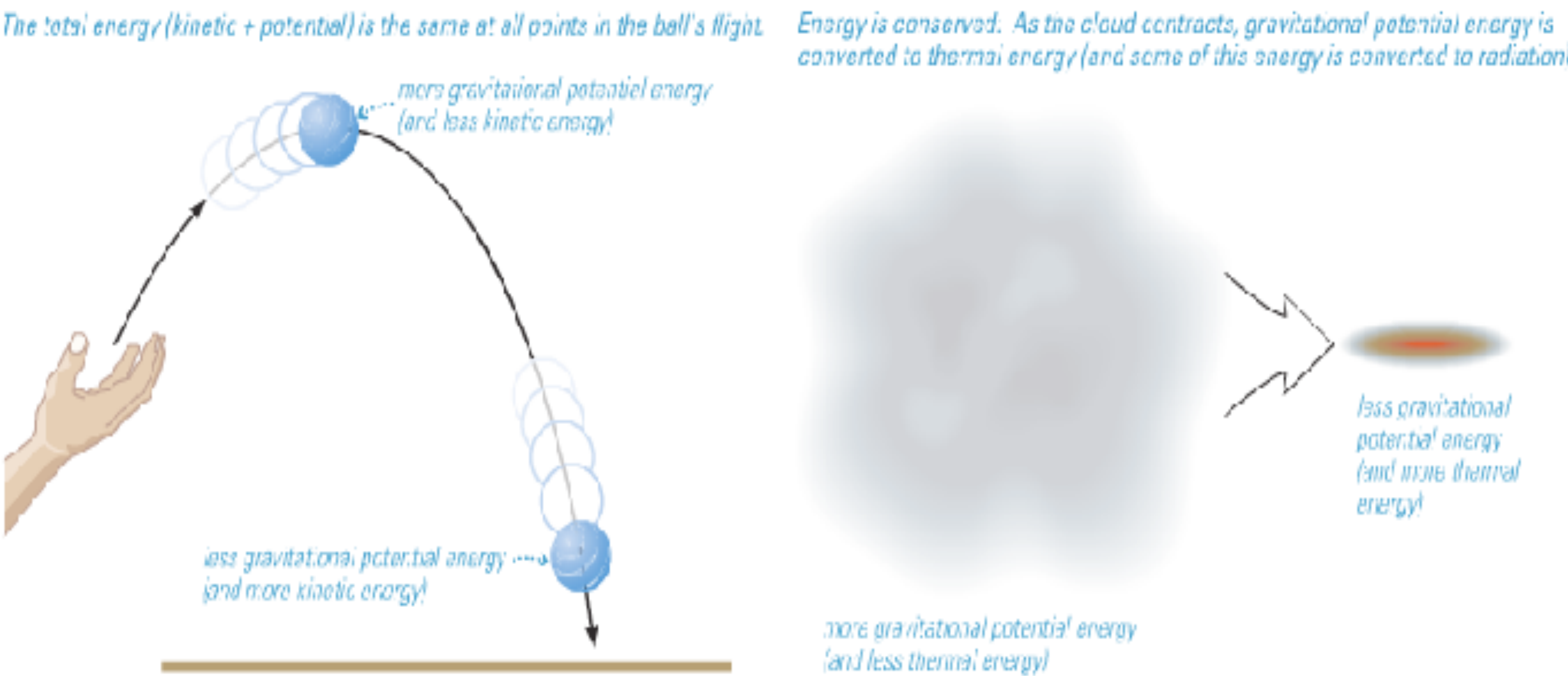


Energy

- Kinetic (moving energy)
- Potential (stored energy - gravitational, chemical, etc.)
- Radiative (energy carried by light)

Energy of an isolated system is conserved

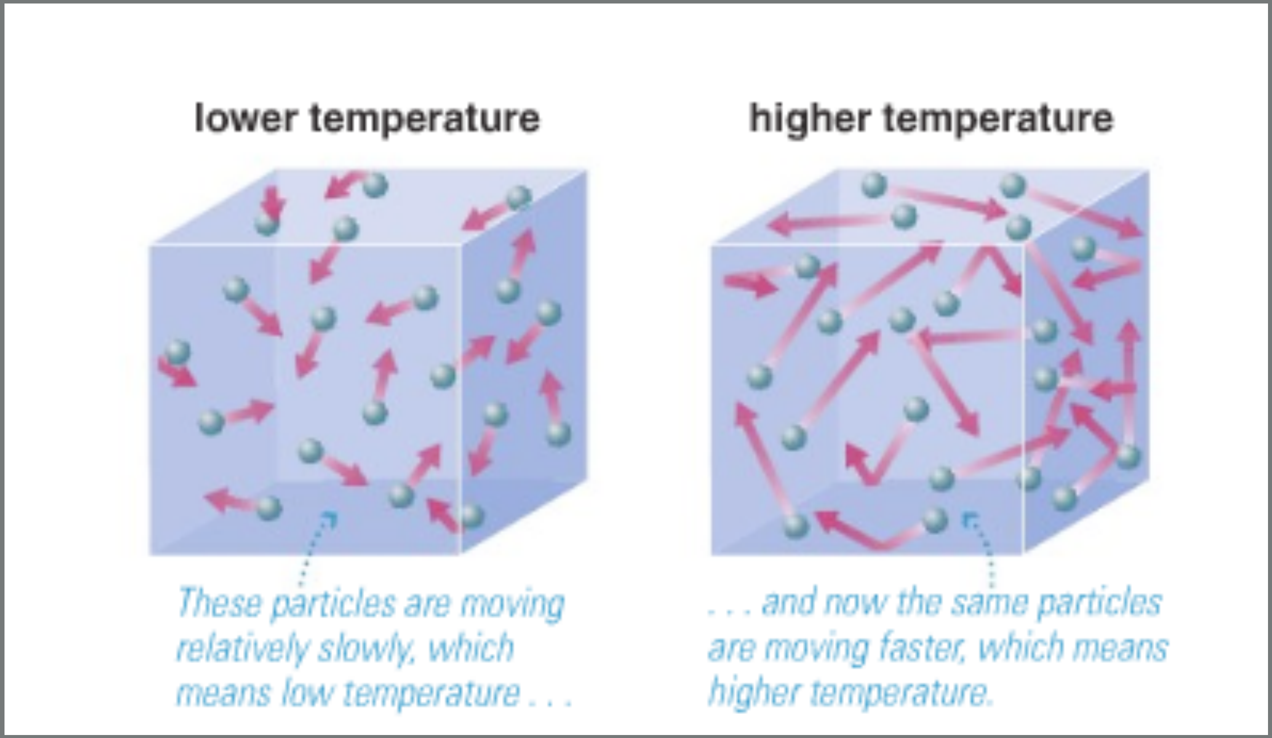




a The ball has more gravitational potential energy when it is high up than when it is near the ground.

b A cloud of interstellar gas can contract due to its own gravity. It has more gravitational potential energy when it is spread out than when it shrinks in size.

Temperature/Heat?

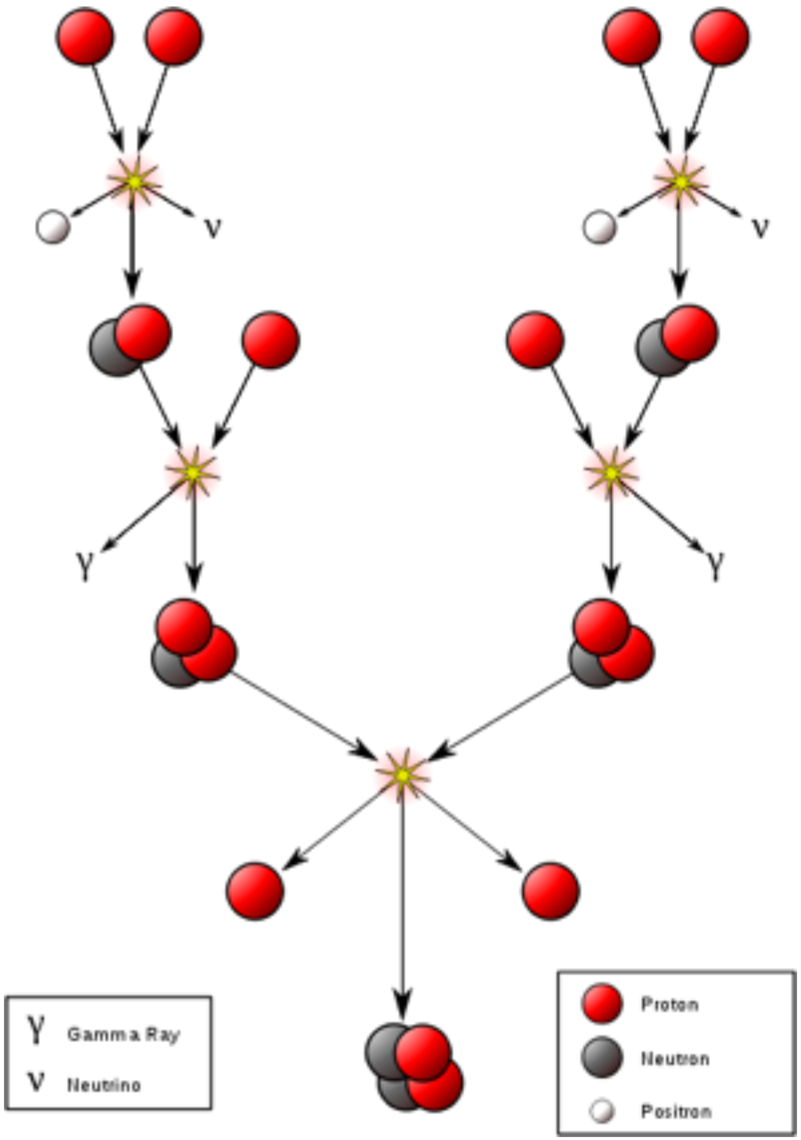


Nuclear Energy

- $E^2=p^2c^2+m^2c^4$
- What does the conversion of matter into energy mean?
- Fusion/Fission
- squish/smash

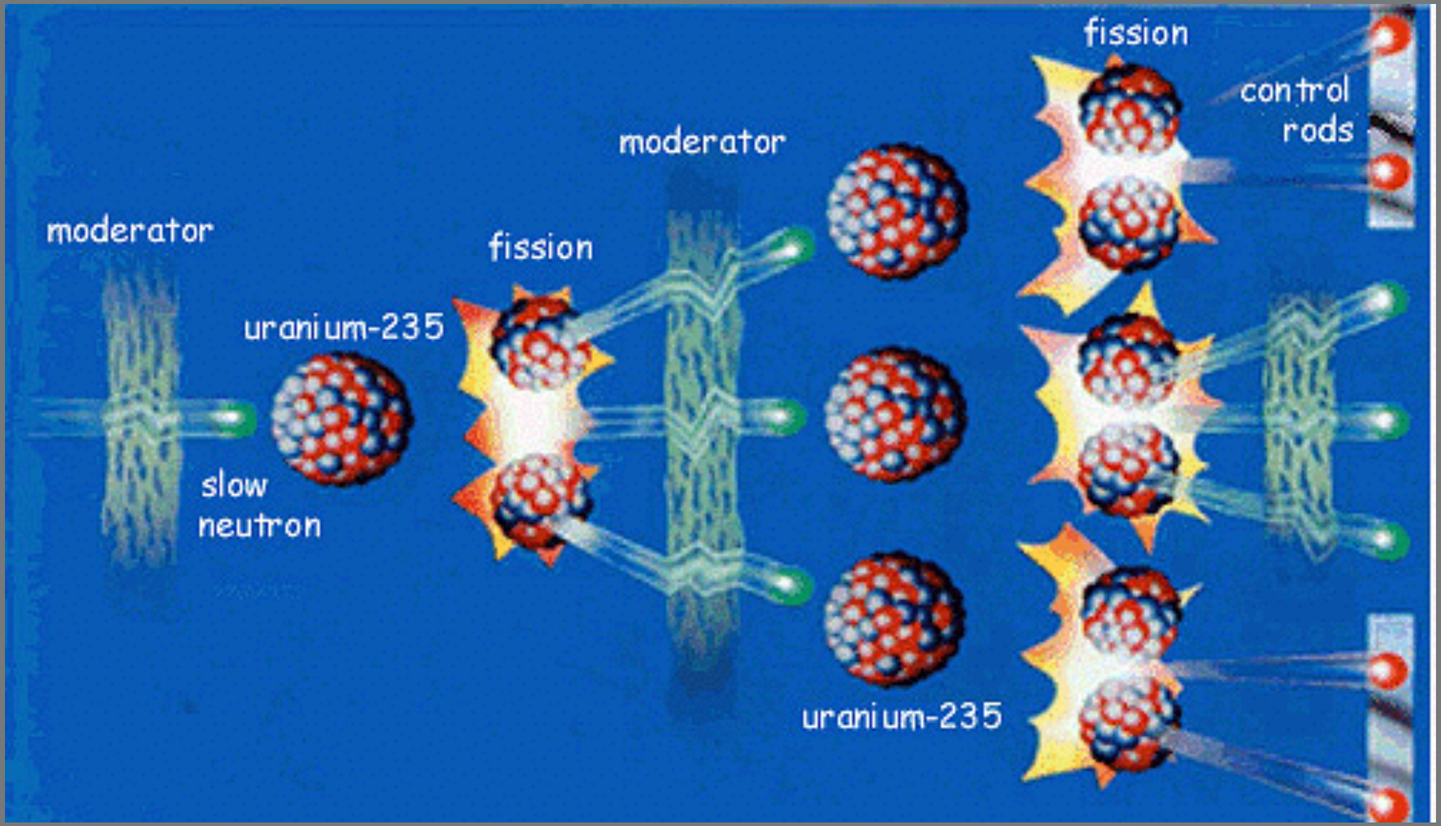
Fusion

- The Proton-Proton Chain is the principal set of reactions for solar-type stars to transform hydrogen to helium
- But what kind of energy is made?
- Kinetic and radiative



Fission

- Splitting but energy is still radiative and kinetic



Item	*Energy (joules)*
Energy of sunlight at Earth (per m^2 per second)	$1.3 * 10^{3}$
Energy from metabolism of a candy bar	$1 * 10^{6}$
Energy needed to walk for 1 hour	$1 * 10^{6}$
Kinetic energy of a car going 60 mi/hr	$1 * 10^{6}$
Daily food energy need of average adult	$1 * 10^{7}$
Energy released by burning 1 liter of oil	$1.2 * 10^{7}$
Thermal energy of parked car	$1 * 10^{8}$
Energy released by fission of 1 kilogram of uranium-235	$5.6 * 10^{13}$
Energy released by fusion of hydrogen in 1 liter of water	$7 * 10^{13}$
Energy released by 1-megaton H-bomb	4×10^{15}
Energy released by major earthquake (magnitude 8.0)	$2.5 * 10^{16}$
Annual U.S. energy consumption	10^{20}
Annual energy generation of Sun	10^{34}
Energy released by a supernova	$10^{44}–10^{46}$

This week: How did Newton change our view of the Universe?

- Motion
  - Stating the obvious, but doing it scientifically
    - Including, how to launch yourself into space
- Gravity
  - It's not just a good idea, it's the Law

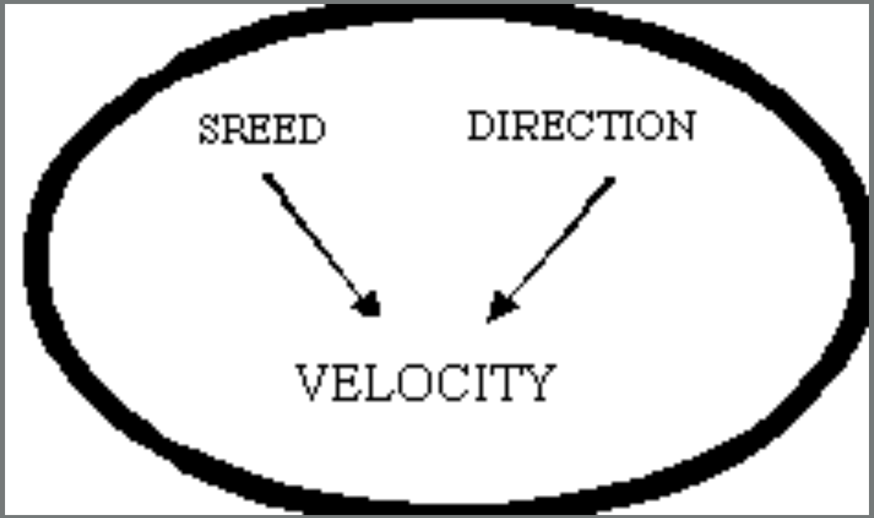
I derive from the celestial phenomena the forces of gravity with which bodies tend to the Sun and the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the Moon and the sea.

--Isaac Newton

"If I have seen far, it is because I have stood on the shoulders of giants" - the giant he meant was <u>Galileo</u>

Newton's 3 Laws of Motion

1. An object moves at constant velocity if there is no net force acting on it
2. When a force, F, acts on a body of mass, M, it produces in it an acceleration, A, equal to the force divided by the mass. Or $A = F/M$
3. For any force, there is always an equal and opposite reaction force.

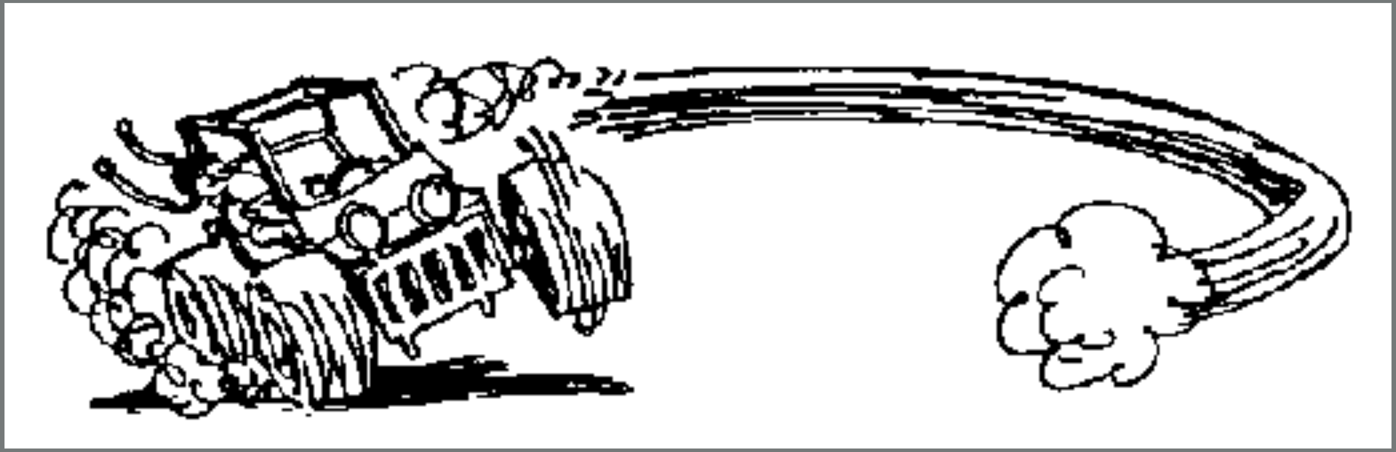


Acceleration is a change of <u>velocity</u>

= change in <u>speed</u>

= change in <u>direction</u>

= change in <u>both</u>



Newton's 3 Laws of Motion

1. An object at rest tends to stay at rest. An object in motion tends to continue in motion at constant speed in a straight line.

That's it!

Newton's 3 Laws of Motion

2. When a force, F, acts on a body of mass, M, it produces in it an acceleration, A, equal to the force divided by the mass.

$$F = MA$$

$$A = F/M$$

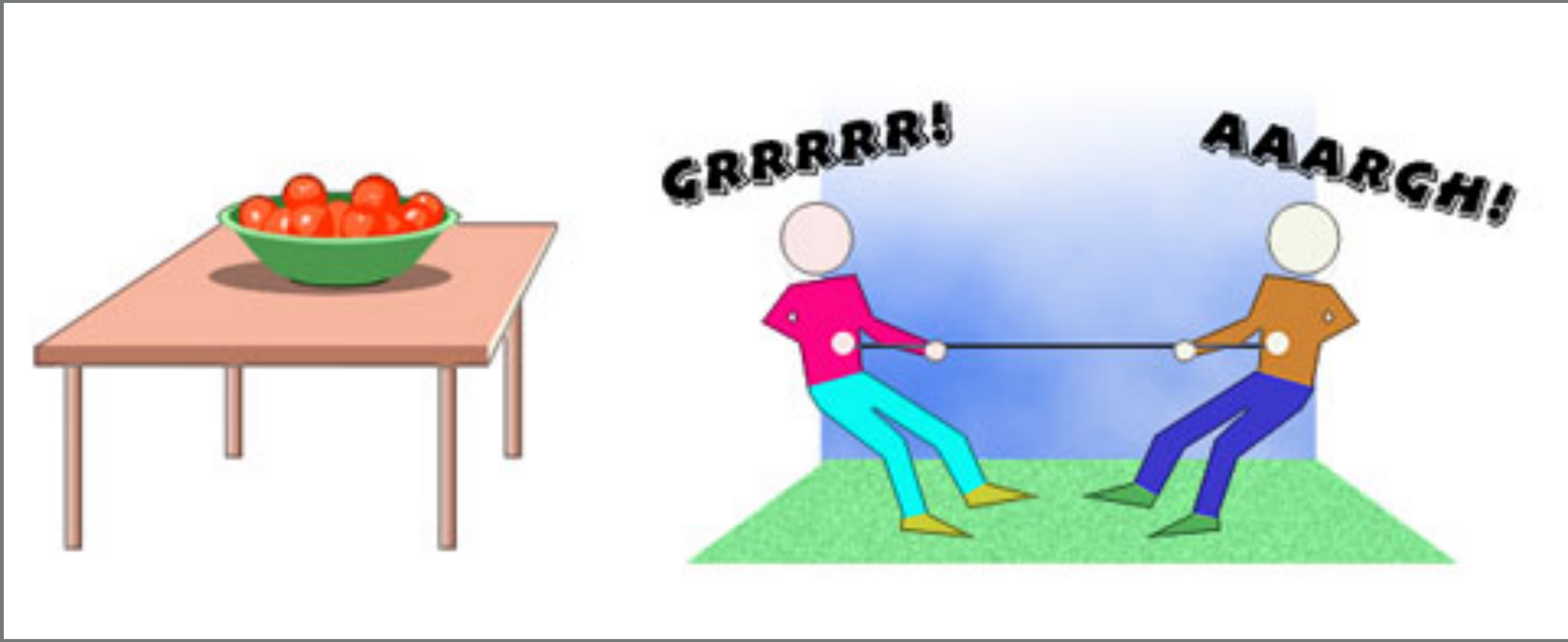
Newton's 3 Laws of Motion

F = MA   A=F/M

The more force on an object, the more it accelerates. But the more massive it is, the more it resists acceleration.

Acceleration = Force / Mass

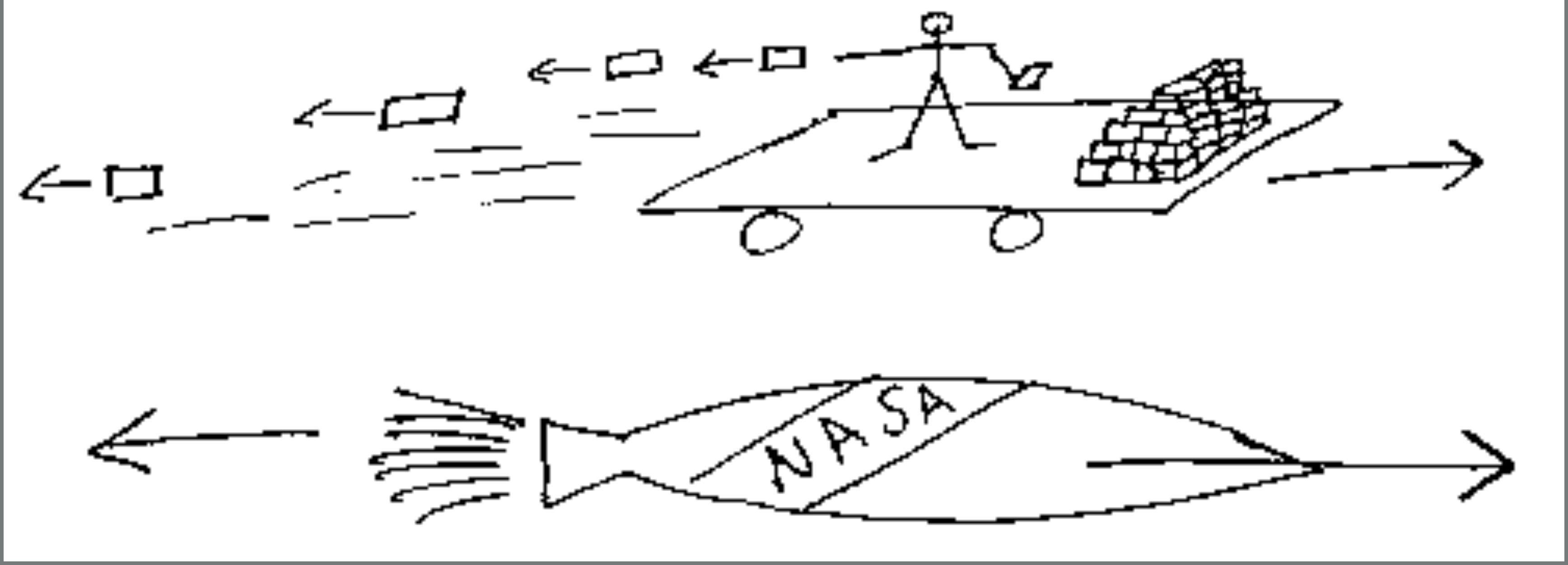
Nothing Happens If the Forces Are Balanced



A body accelerates only when an <u>unbalanced force</u> is applied

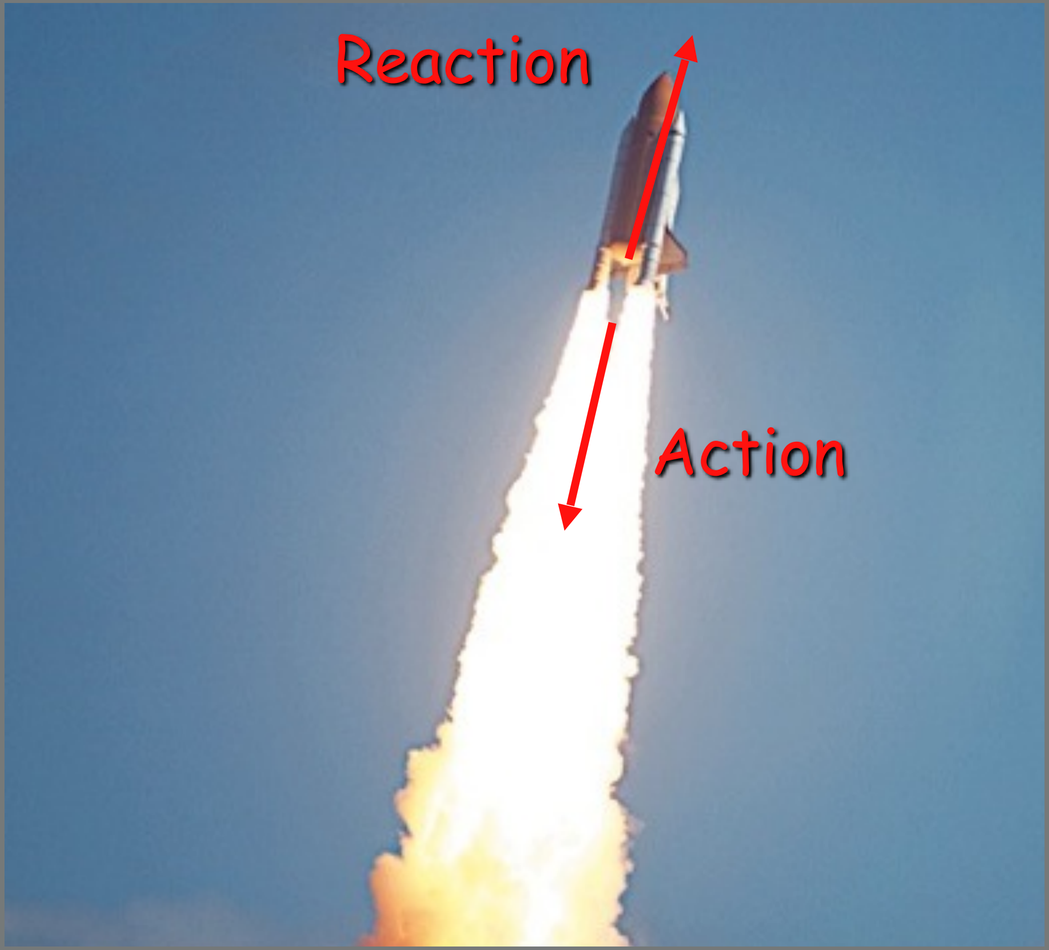
Newton's 3 Laws of Motion

3. For every action, there is always an <u>equal</u> and opposite reaction.

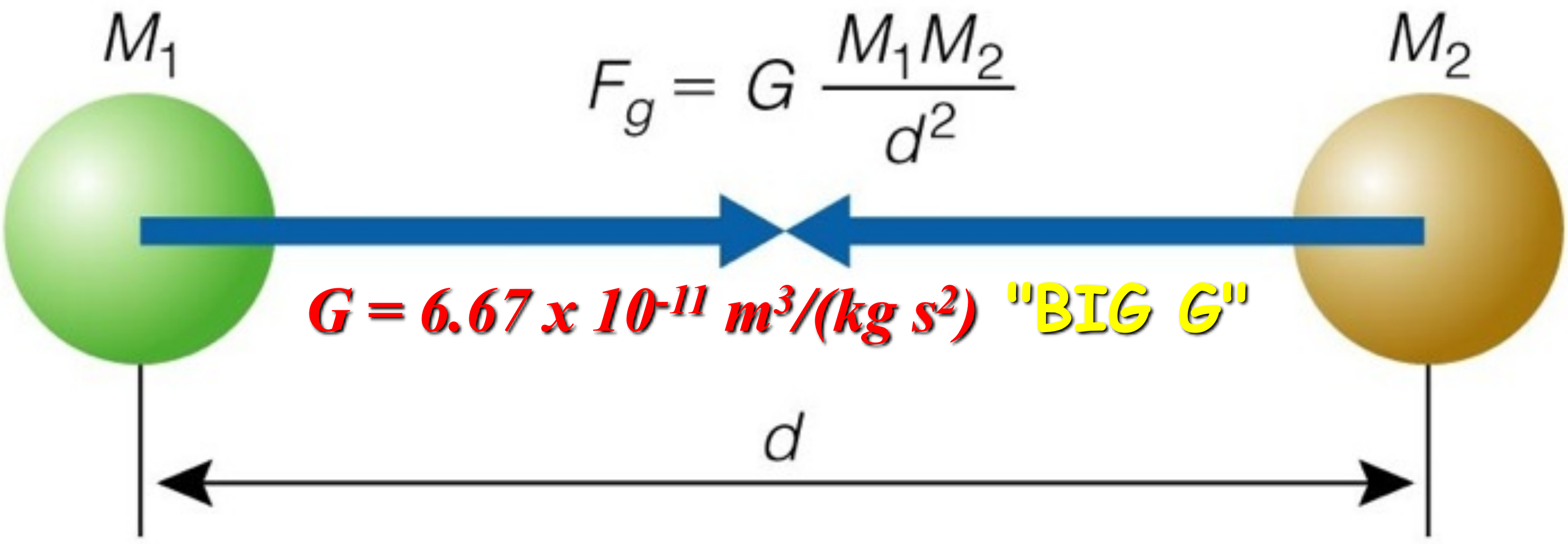


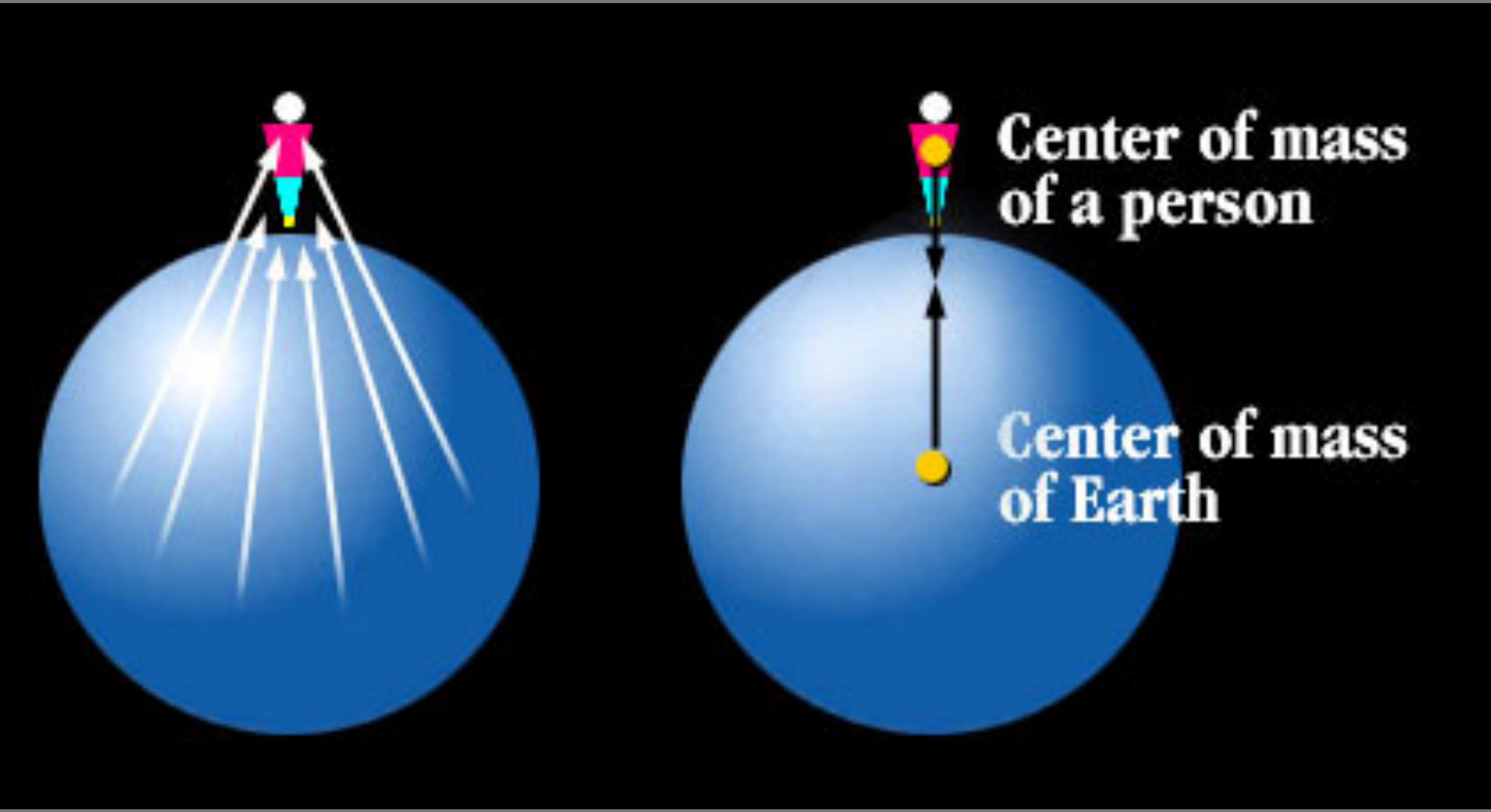
Newton's 3 Laws of Motion

To every action there is an equal and opposite reaction.



Universal Law of Gravity





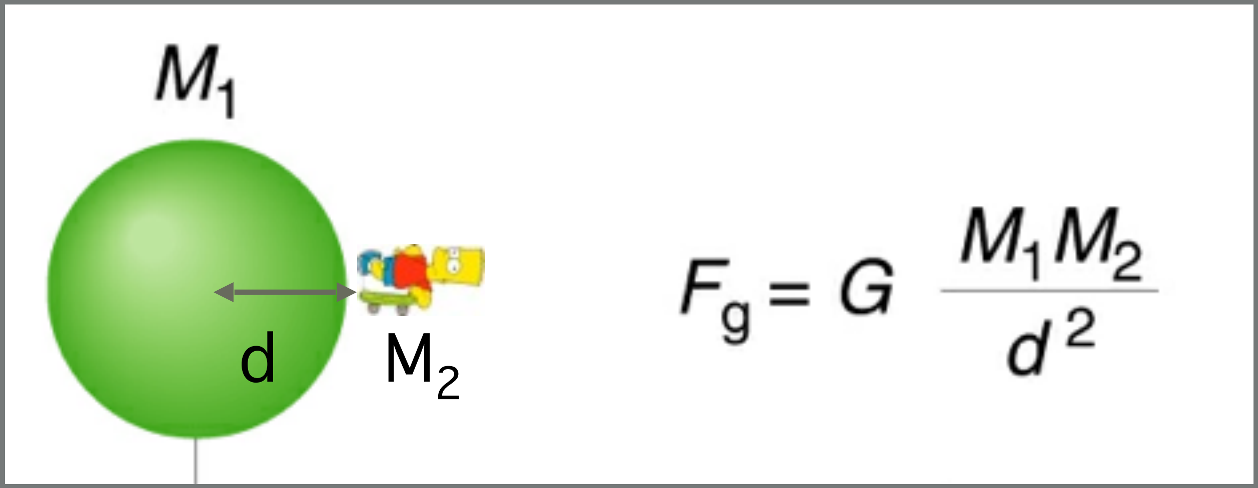
Net effect equivalent to force between centers of mass

Thought Question

Is the force the Earth exerts on you larger, smaller, or the same as the force you exert on it?

A. Earth exerts a larger force on you.
B. You exert a larger force on Earth.
C. Earth and you exert the same force on each other.
D. It depends on how far above the Earth's surface you are.

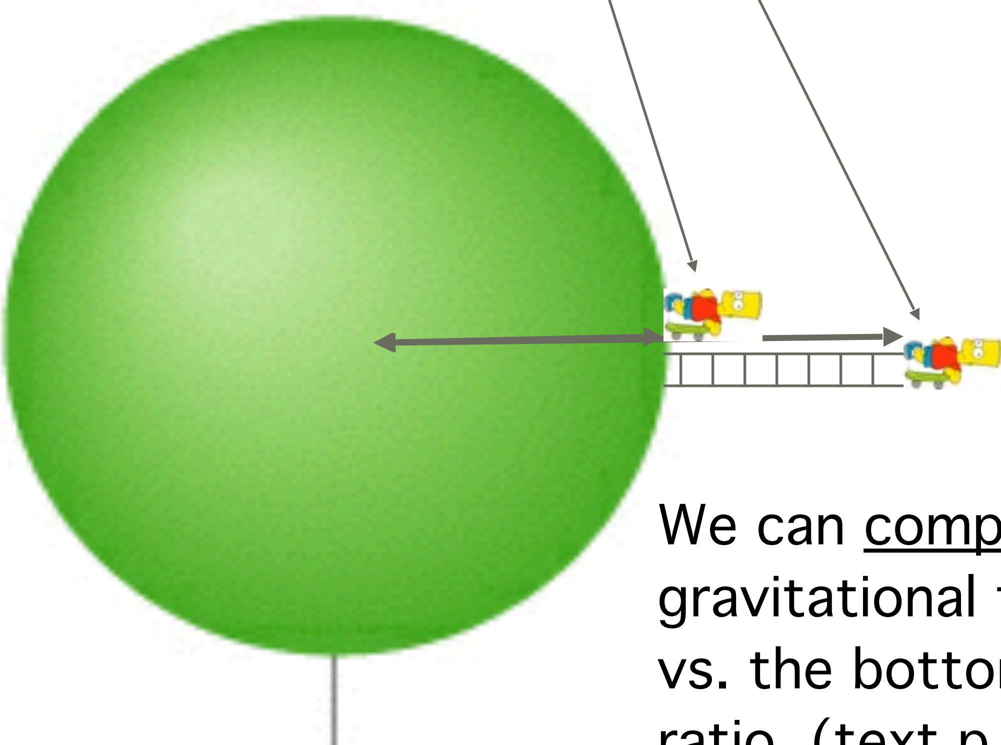
Same equation applies to M_2 standing on M_1



d is the distance to the center of the planet

Now imagine Bart on a ladder whose length equals Earth's radius

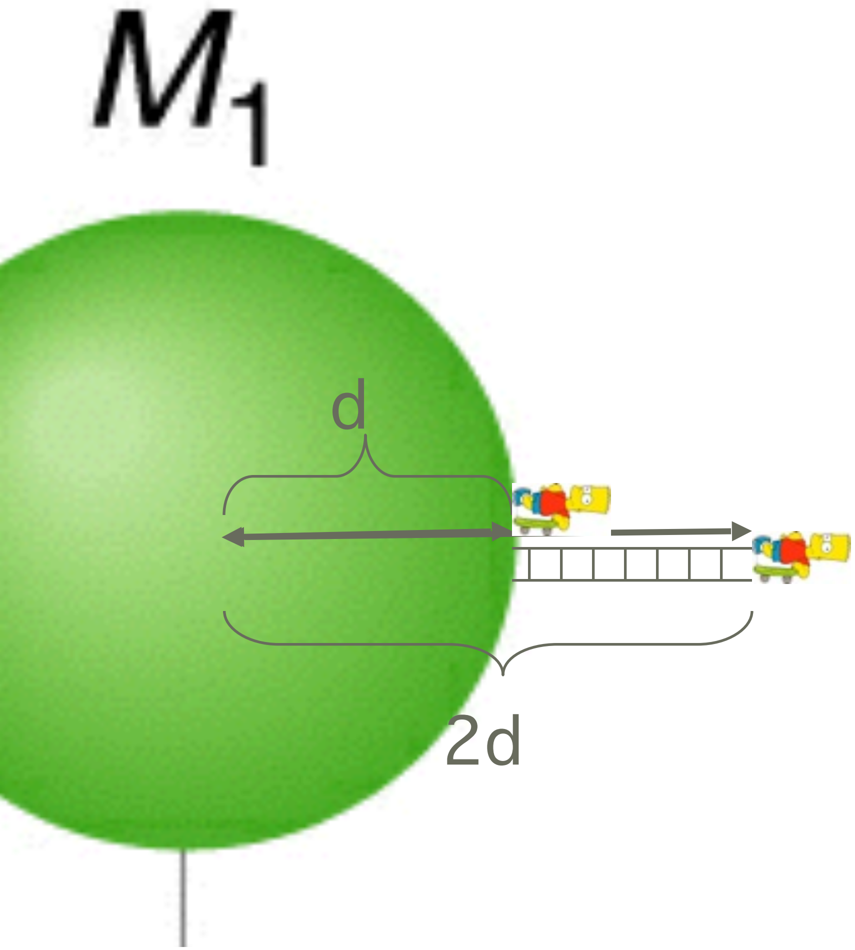
The masses are the same,
but Bart has moved from d
to 2d from Earth's center



We can <u>compare</u> the gravitational force at the top vs. the bottom by taking a <u>ratio</u>. (text p. A-11)

Thought Question

Ratios: The mathematical way



$$F_{top} = \frac{GM_1M_2}{d_{top}^2} = \frac{GM_1M_2}{(2d)^2} = \frac{GM_1M_2}{4d^2}$$

$$F_{bot} = \frac{GM_1M_2}{d_{bot}^2} = \frac{GM_1M_2}{d^2}$$

What's the next step?

A. Divide F_{top} by F_{bot}

B. Rearrange both equations so one side contains only GM_1M_2

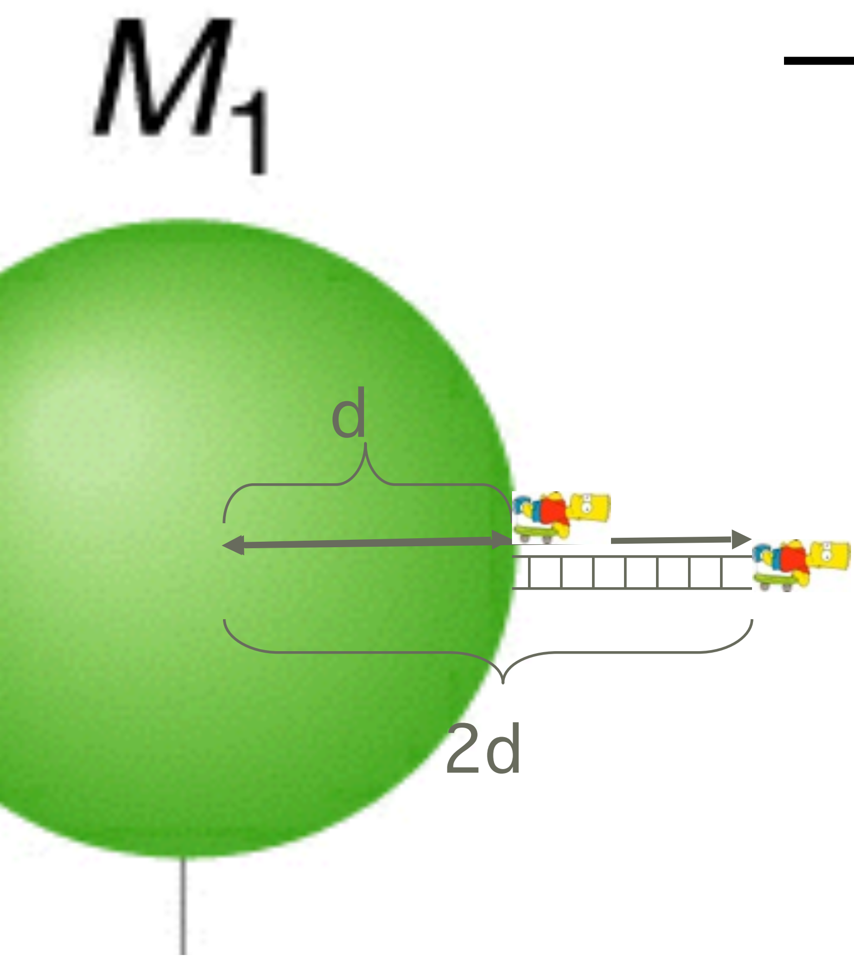
C. Cancel the gravitational constants (G) for both equations

D. Subtract F_{top} from F_{bot}

E. Divide F_{top} by 4

Ratios: The mathematical way

$$\frac{F_{top} = \dfrac{GM_1M_2}{d_{top}^2} = \dfrac{GM_1M_2}{(2d)^2} = \dfrac{GM_1M_2}{4d^2}}{F_{bot} = \dfrac{GM_1M_2}{d_{bot}^2} = \dfrac{GM_1M_2}{d^2}}$$

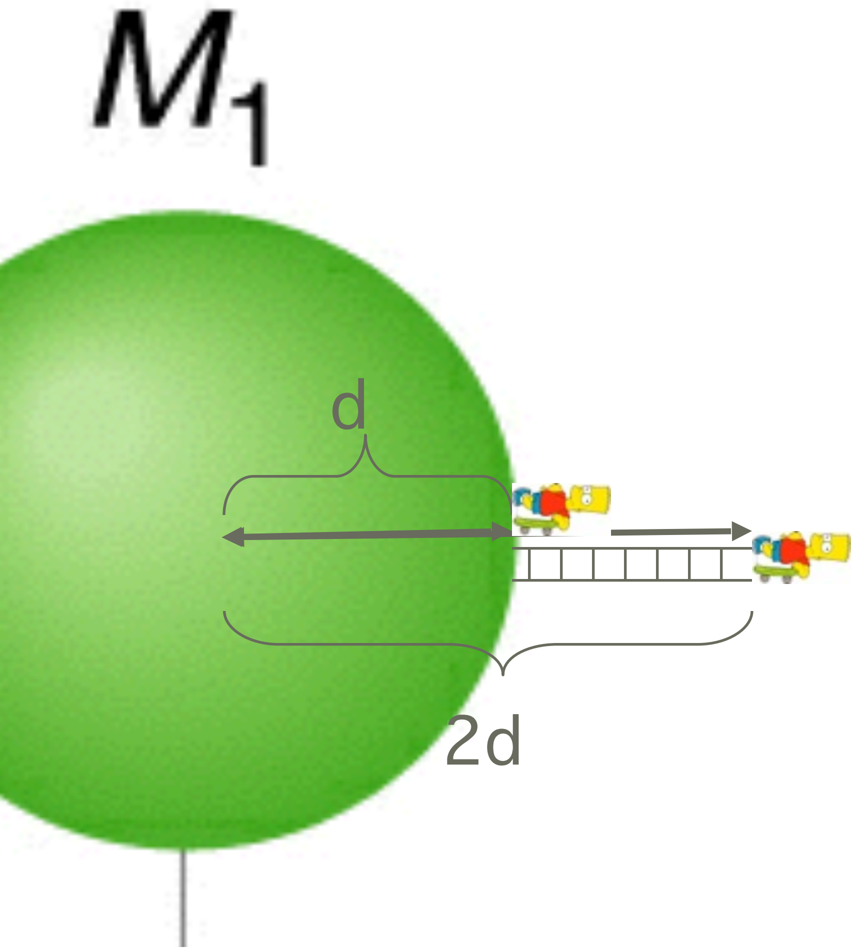


$$\frac{F_{top}}{F_{bot}} = \frac{\dfrac{\cancel{G}\cancel{M_1}\cancel{M_2}}{4\cancel{d^2}}}{\dfrac{\cancel{G}\cancel{M_1}\cancel{M_2}}{\cancel{d^2}}} = \frac{\dfrac{1}{4}}{\dfrac{1}{1}} = \frac{1}{4}$$

$$F_{top} = \frac{1}{4} F_{bot}$$

So the grav. force at the top of the ladder is only 1/4 as strong as at the bottom

Ratios: The logical way



$$F = \frac{GM_1M_2}{d^2}$$

What variable changes when Bart moves from the bottom to the top?

$d \Uparrow 2$

$d^2 \Uparrow 4$

$1/d^2 \Downarrow 4$

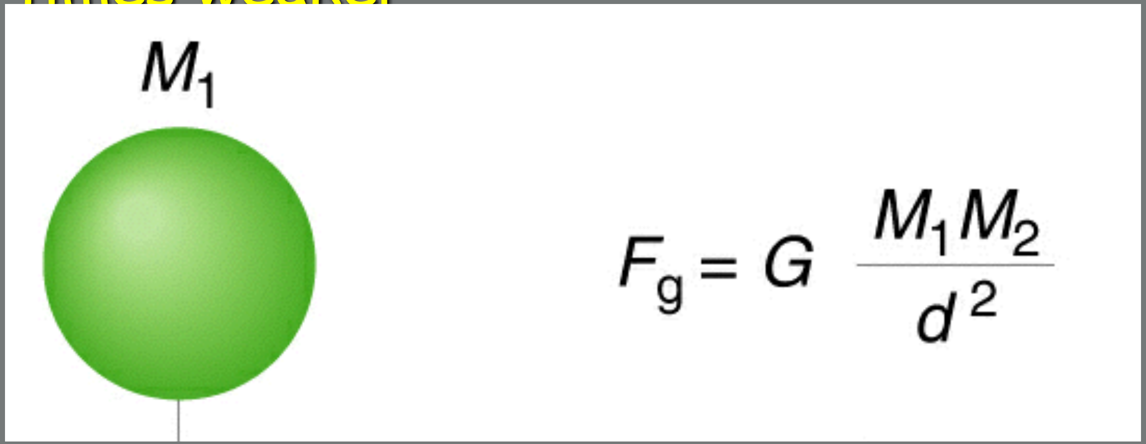
$F \Downarrow 4$

So if d (distance) goes up by 2, F (Force of gravity) goes down by 4

Thought Question

How strong would gravity be at the surface if Earth were replaced by a styrofoam ball of the same mass but 10 times the radius?

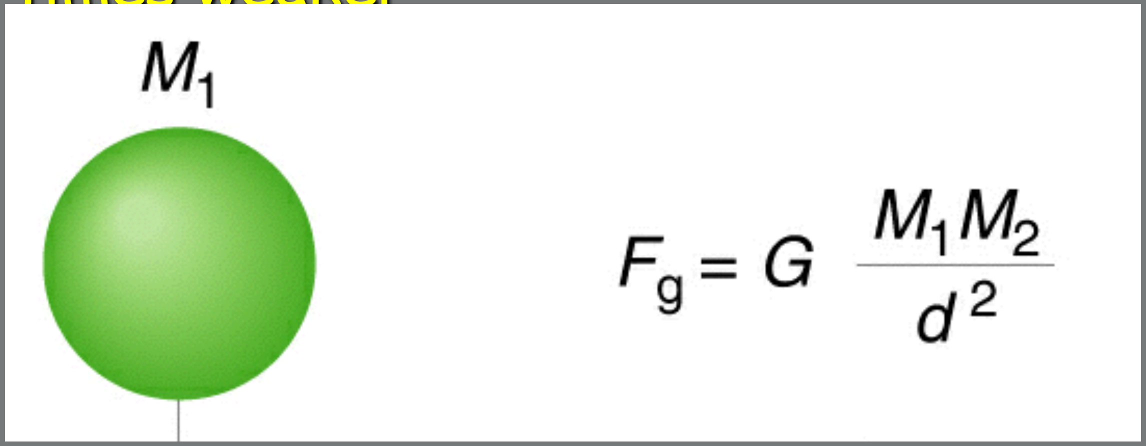
A. 100 times stronger
B. 10 times stronger
C. unchanged
D. 10 times weaker
E. 100 times weaker



Thought Question

How strong would gravity be at the surface if Earth were replaced by a ball of iron with 10 times the mass but the same radius (as regular Earth)?

A. 100 times stronger
B. 10 times stronger
C. unchanged
D. 10 times weaker
E. 100 times weaker



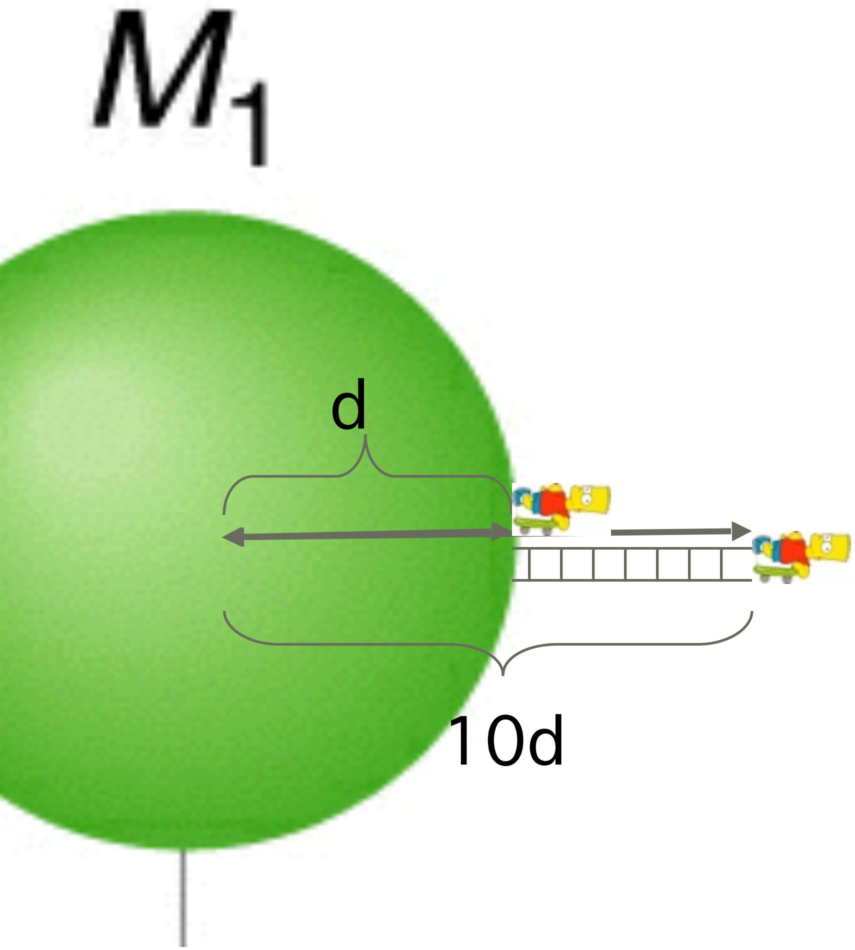
Reading Question

Which of the following is NOT a type of energy?

A. Kinetic energy
B. Quantum energy
C. Thermal energy
D. Gravitational potential energy
E. Radiative energy

Ratios: The logical way

REVIEW



$$F = \frac{GM_1M_2}{d^2}$$

What variable changed?

d⇑10

d^2⇑100

$1/d^2$⇓100

F⇓100

So if d (distance) goes up by 10, F (Force of gravity) goes down by 100

So far we've only been talking about the FORCE of gravity

- Today: How do we find out the acceleration due to gravity?
  - I.e. How fast do things fall towards the Earth?

Acceleration Due to Earth's Gravity

$$\boxed{F = G\frac{M_1 M_2}{d^2}}$$

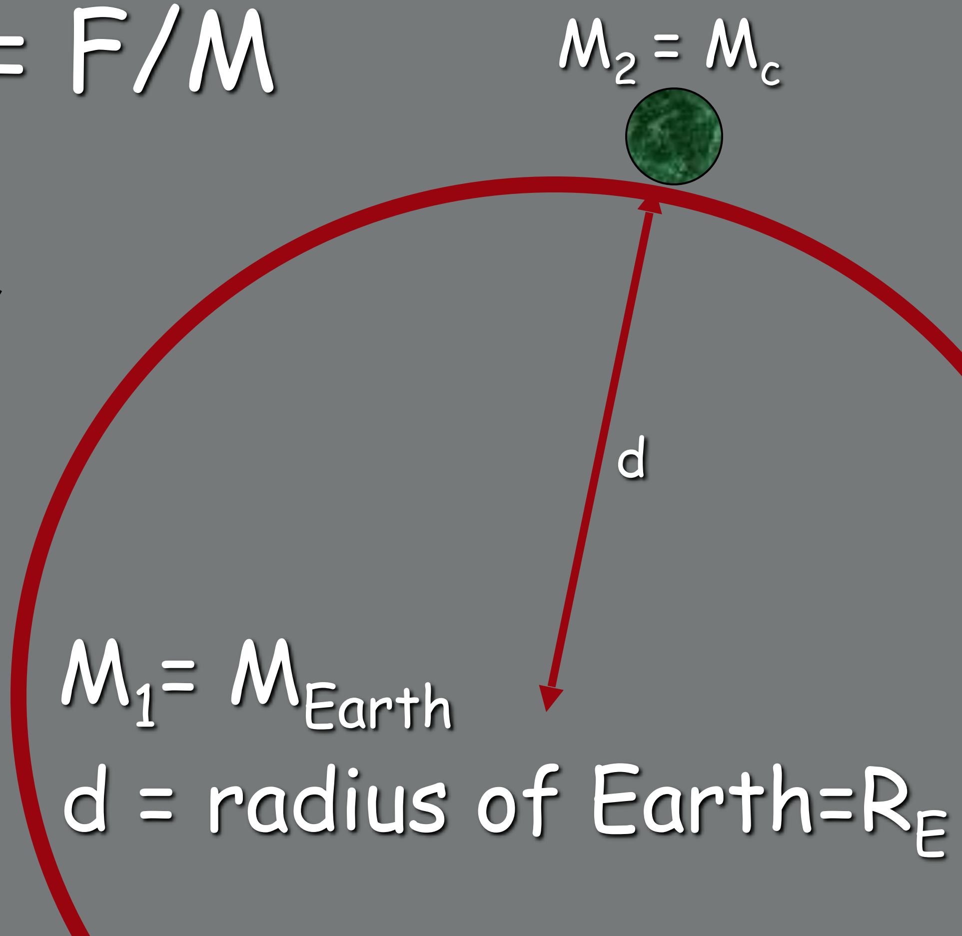
F = MA

A = F/M

$$A_c = [G\, M_E\, \cancel{M_c} / R_E^2] / \cancel{M_c}$$

$$\boxed{= G\, M_{Earth} / R_{Earth}^2}$$

$= 9.8$ meters/sec^2



$M_1 = M_{Earth}$

d = radius of Earth $= R_E$

How much does the Earth accelerate due to the cabbage?

$$F = G\frac{M_1 M_2}{d^2}$$

F = MA

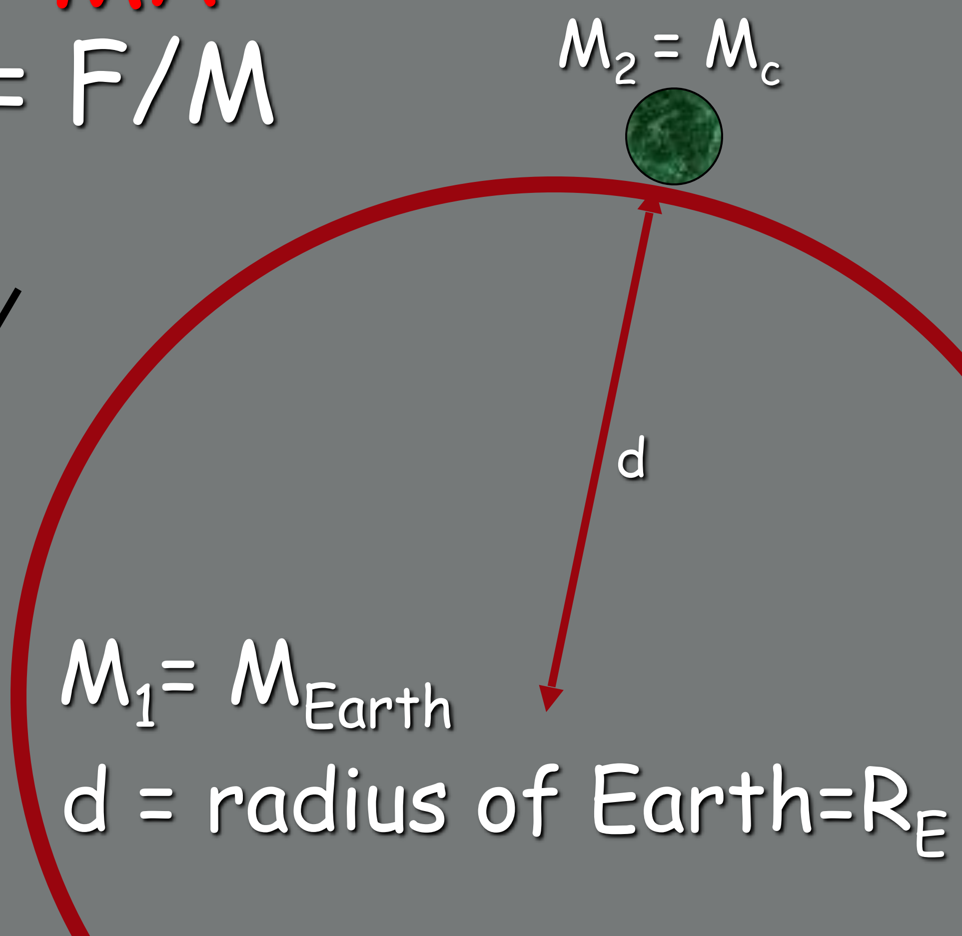
A = F/M

$A_E = [G\ \cancel{M}_E\ M_c\ /\ R_E^2\ ]\ /\ \cancel{\ }$

M_E

$= G\ M_c\ /\ R_E^2$

$= 2 \times 10^{-24}\ m/s^2$



Thought Question

Which falls faster - a cabbage or a medicine ball?

A. The cabbage will fall faster and hit the ground first
B. The medicine ball will fall faster and hit the ground first
C. They both hit at the same time

Thought Question

Why did they hit at the same time?

A. The masses of the cabbage and the medicine ball were tiny compared to the mass of the Earth.
B. The mass of the falling object doesn't matter
C. They actually fell at slightly different speeds but over that small distance it wasn't noticeable
D. No idea

Acceleration Due to Earth's Gravity

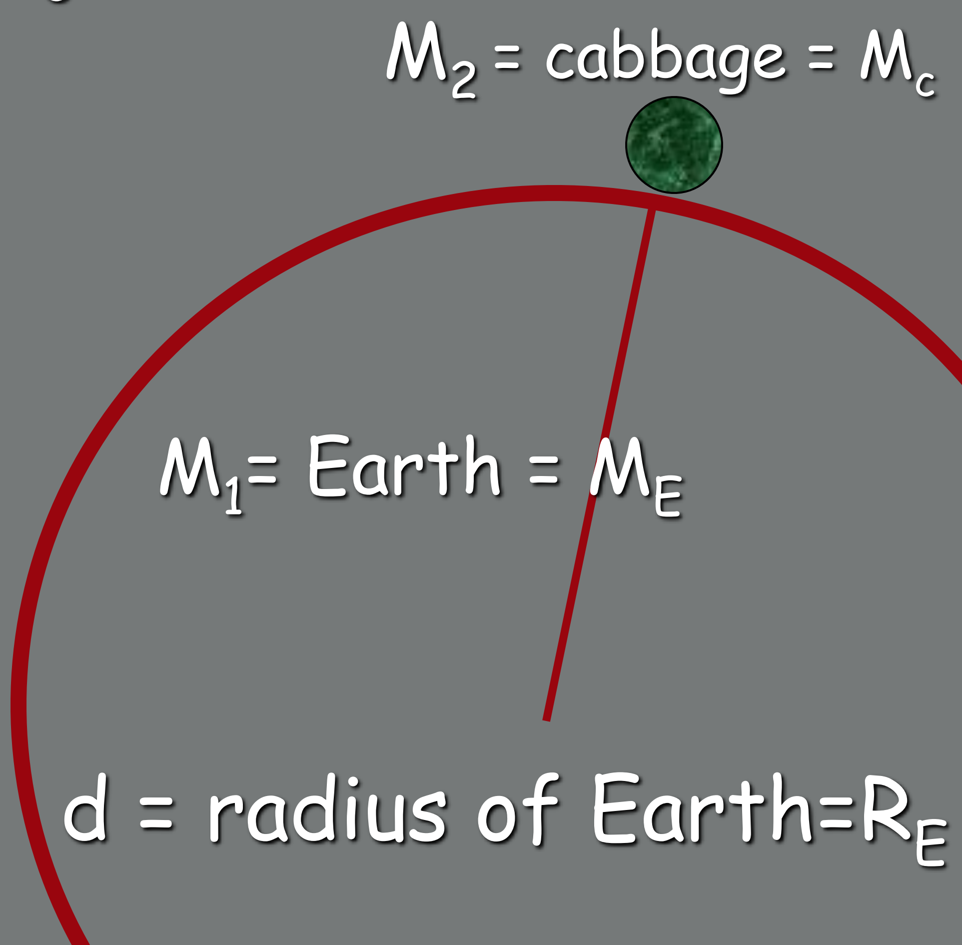
$$A_c = [G\, M_E\, M_c / d^2\,] / M_c$$

$$= G\, M_E / d^2$$

$$g = G\, M_{Earth} / R_E^{\,2}$$

$$= 9.8 \text{ meters/sec}^2$$

Does not depend on the mass of the falling object!!!



Acceleration on the Moon

$$g_{planet} = G\, M_{planet} \;/\; R_{planet}^{\;2}$$

Mass Moon ~ $M_{earth}/100$

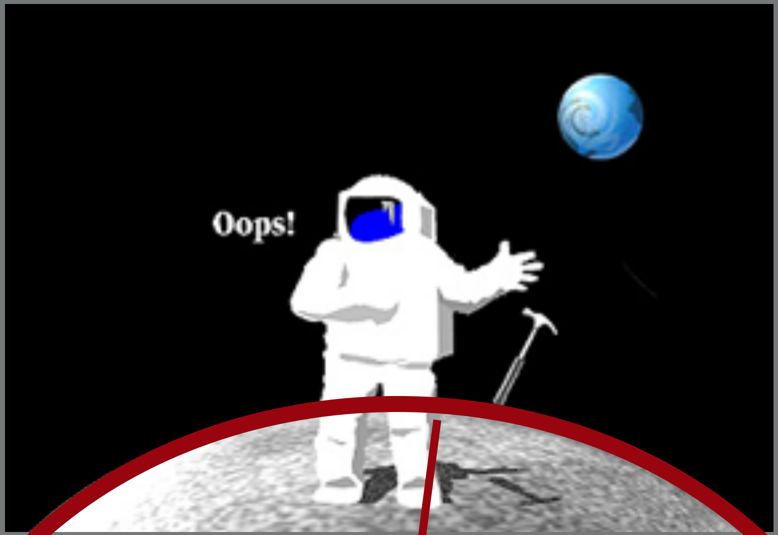
Radius Moon ~ $r_{Earth}/4$

What is g_{moon}

How about Jupiter?

Mass ~ $300M_{earth}$

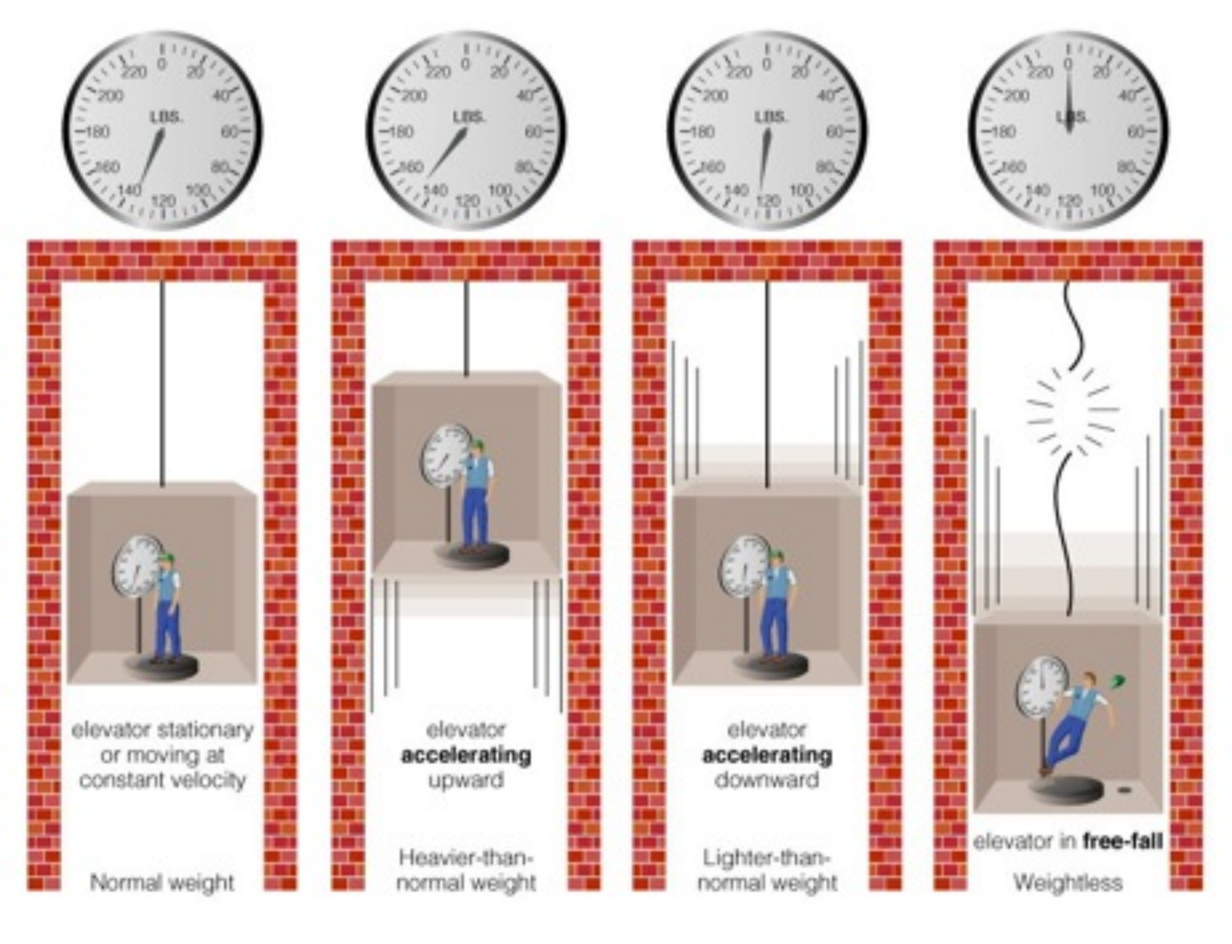
Radius ~ $11r_{Earth}$



d = radius of Moon

How is **mass** different from **weight**?

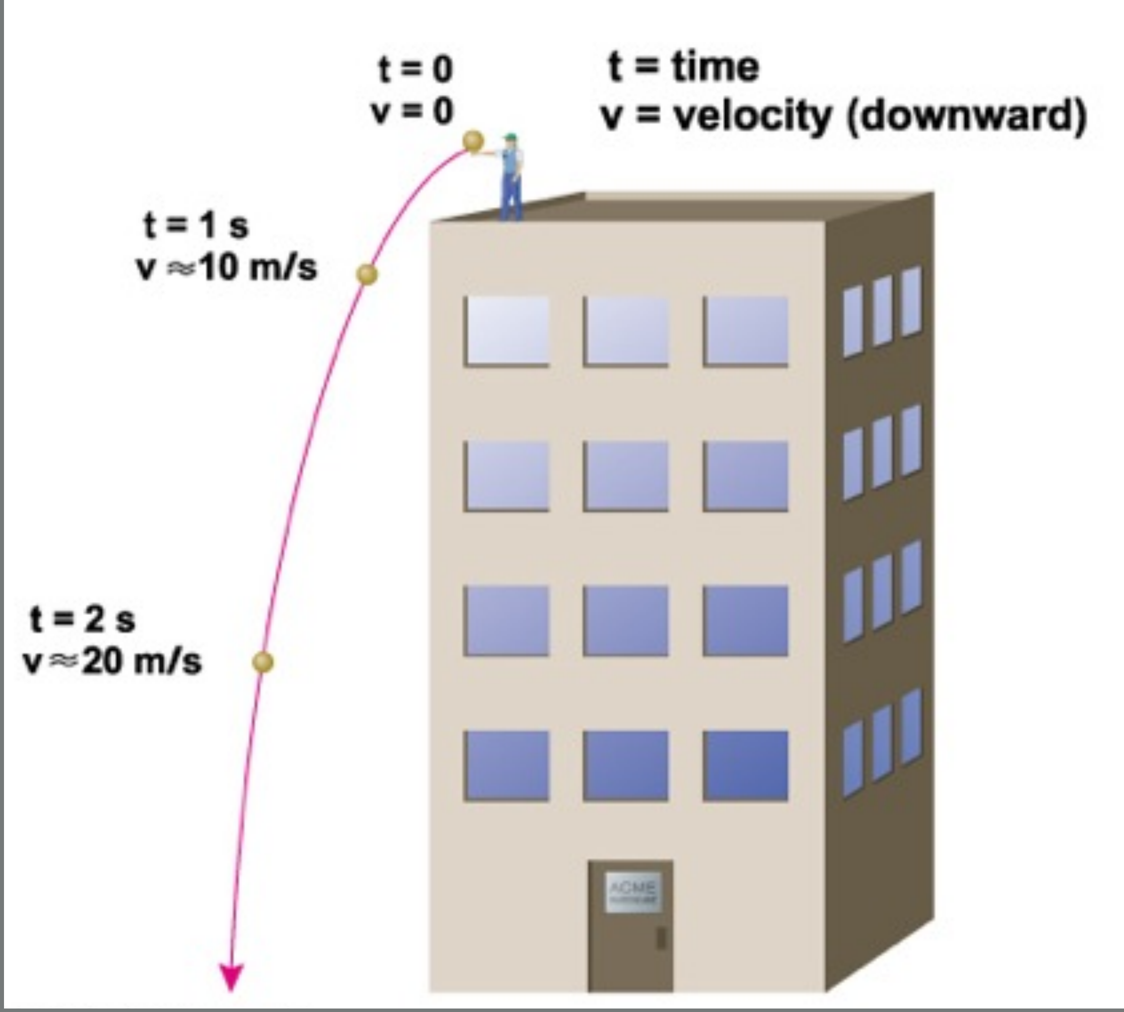
- **Mass** – the amount of matter in an object
- **Weight** – the force that acts upon an object (based on acceleration and mass)



Your weight can change a lot depending on where you are but you mass doesn't.

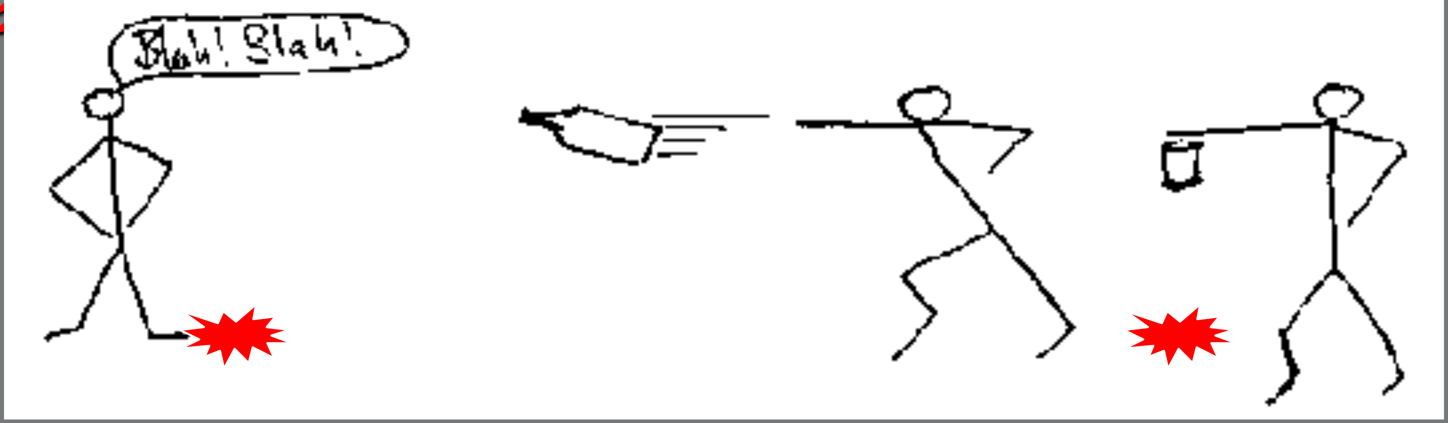
The Acceleration of Gravity

- All falling objects accelerate at the same rate (not counting friction of air resistance).
- On Earth, $g \approx 10$ m/s^2
  - Speed increases 10 m/s with each second of falling.
    - 10 m/s per s



Thought Question

At exactly the same time that you drop a glass, your roommate throws a bottle perfectly horizontally at your professor. The bottle lands at the feet of your professor. Which smashes first, the glass or the bottle?

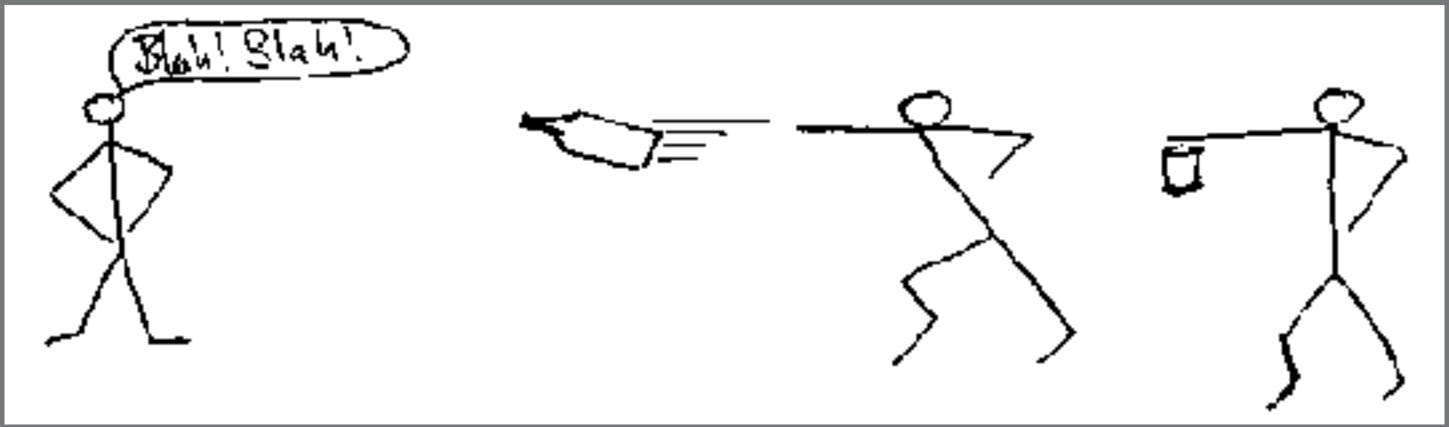


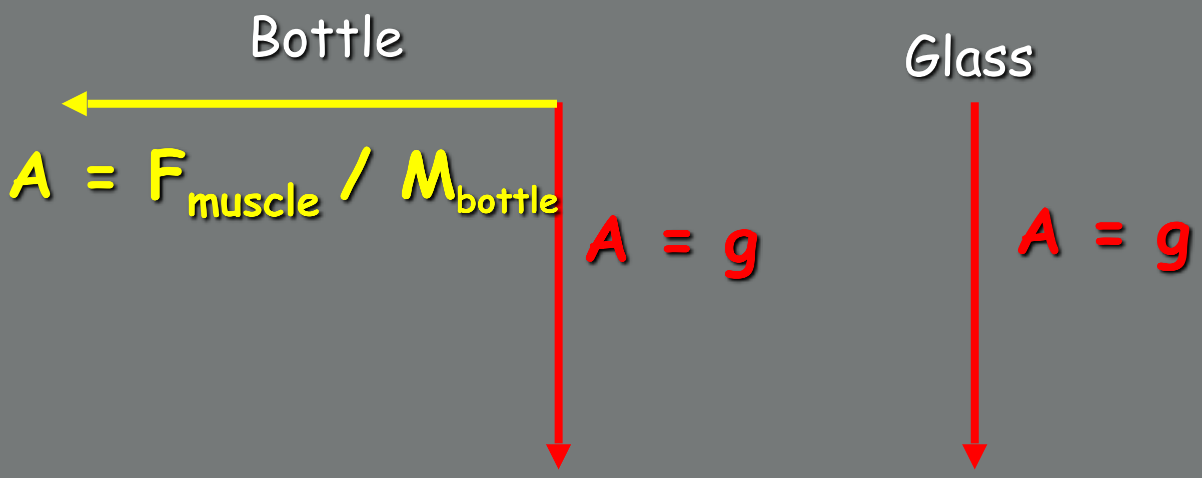
A. The glass

B. The bottle

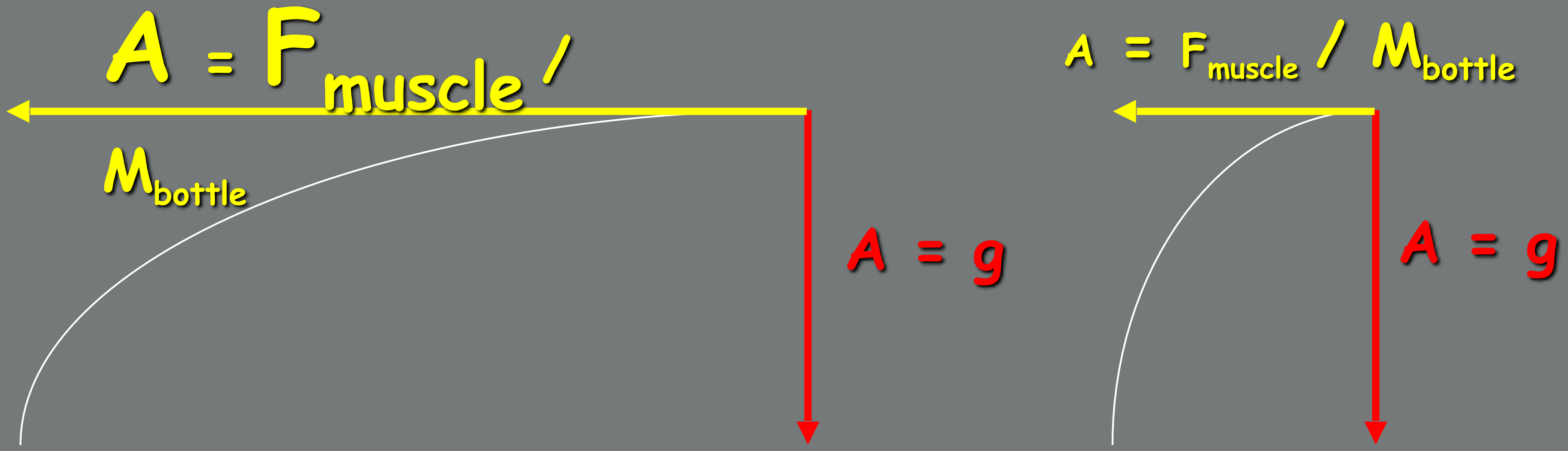
C. Depends on how hard he threw the bottle

D. Both at same time





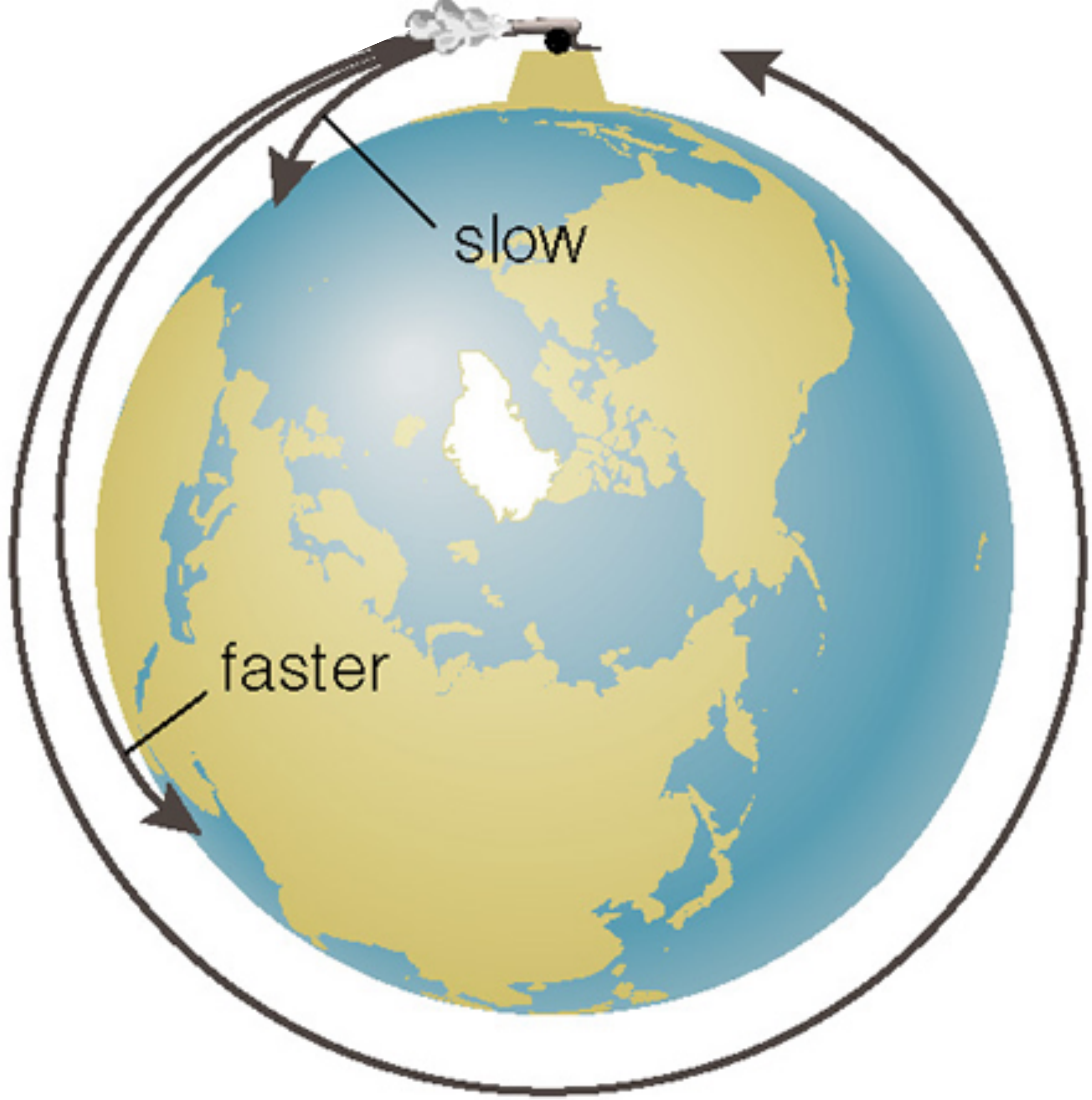
Separating horizontal and vertical motions



The harder you throw, the farther it goes … horizontally.

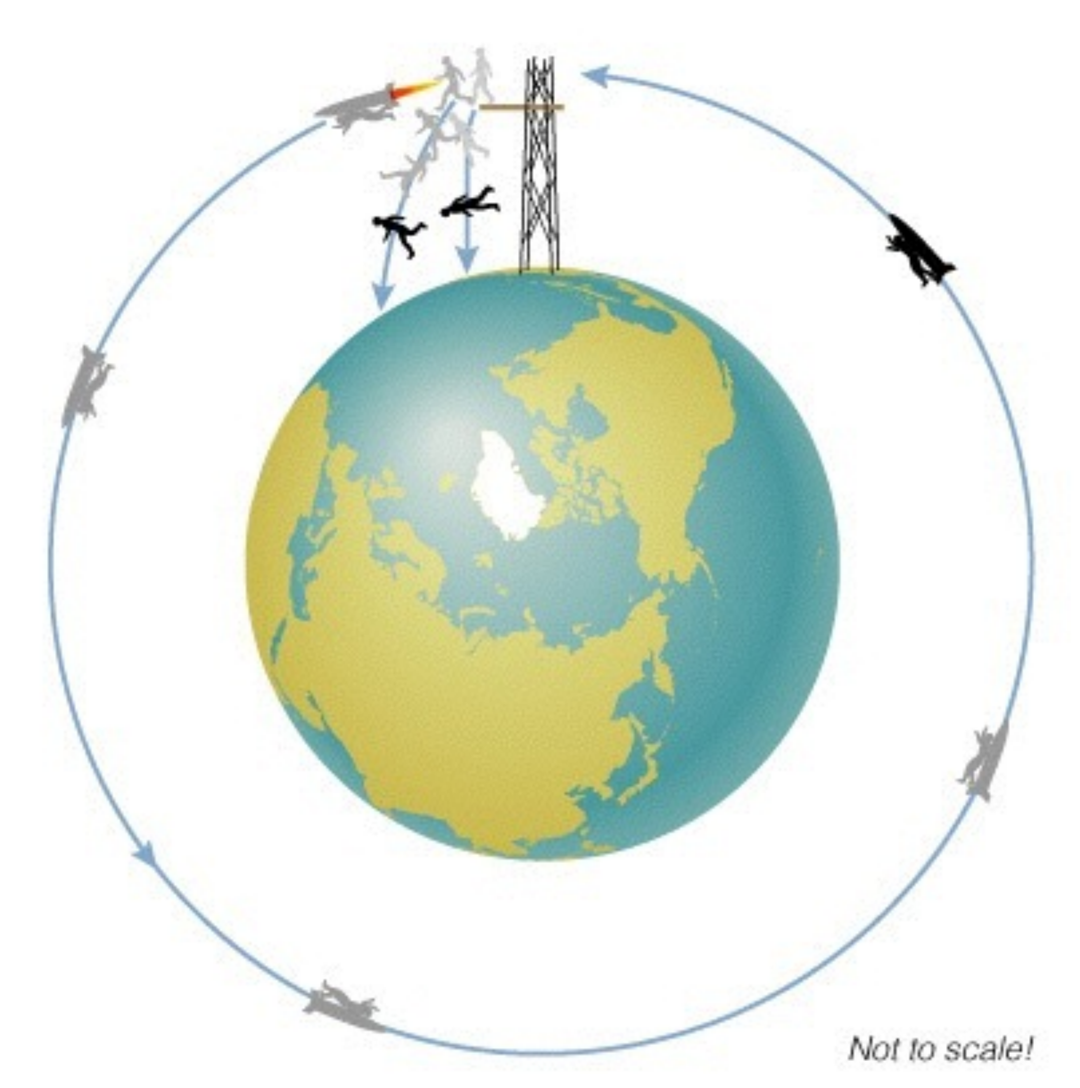
But the downward force of gravity remains the same

Into Orbit!



orbital velocity

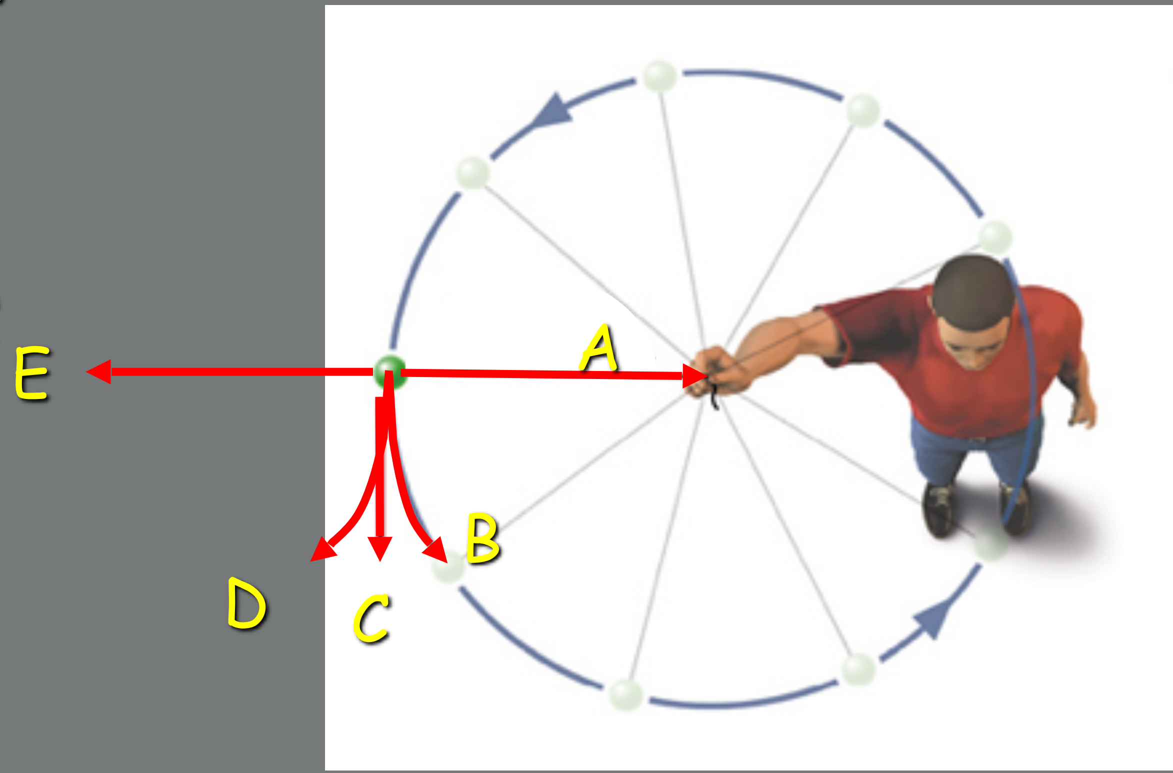
What is orbit?



Objects in orbit are simply falling constantly around the Earth

Thought Question

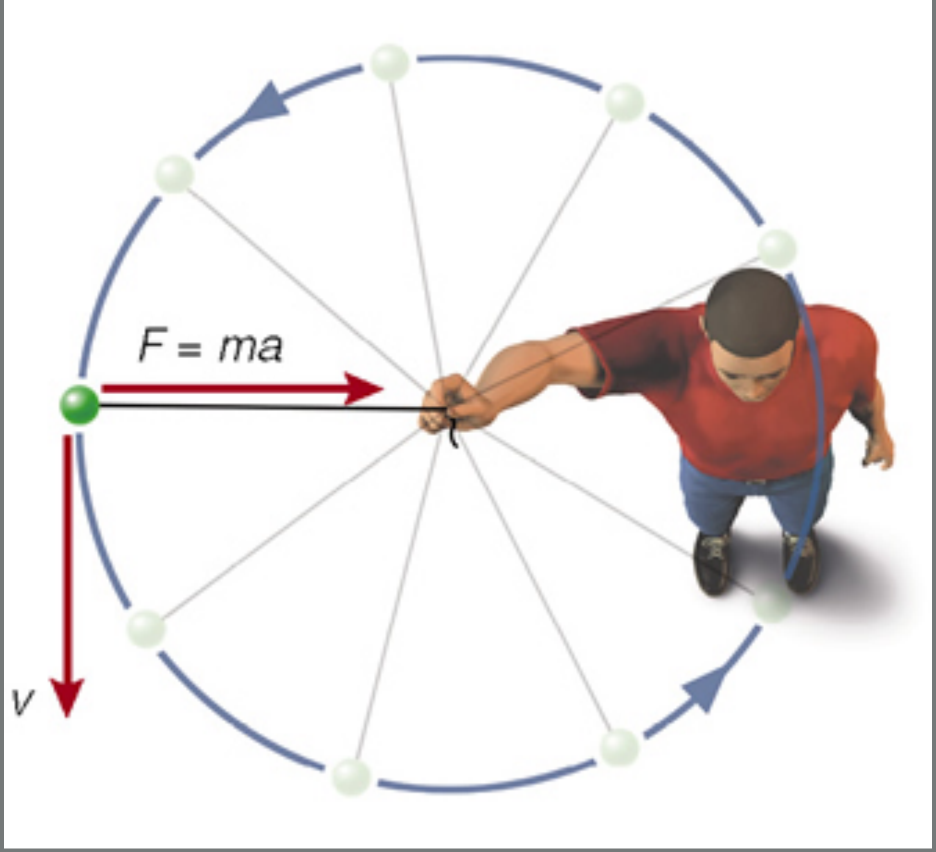
What happens if he lets go of the string?



Does the object move along path A, B, C, D, or E?

What keeps an orbiting object in orbit?

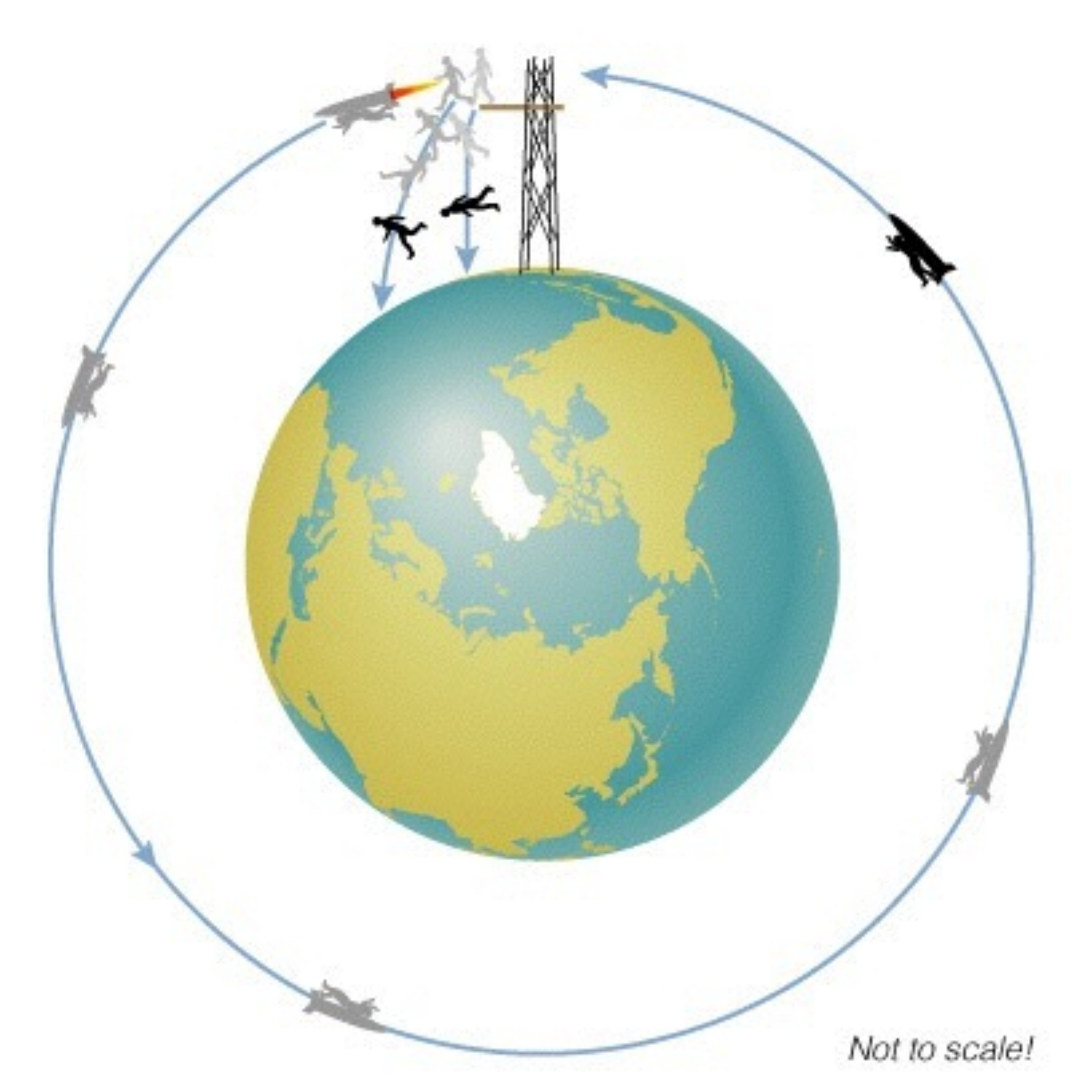
- Gravity!
  - Since the direction is changing, there must be an acceleration
    - Newton's 1st law
- Gravity is a constant force pulling inward



More on Orbits

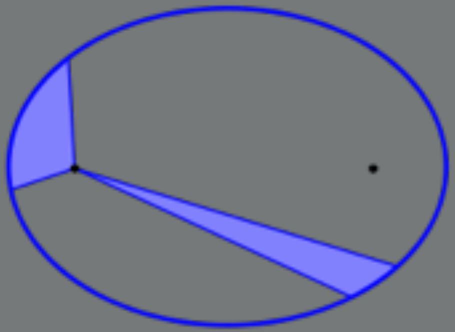
- Two ways to look at it, internal (energy) balance or external (force) balance.
- $PE+KE=C$ (<0)
- $F=ma=mv^2/r=mMG/r^2$
- If $C>0$?

Why are astronauts weightless in space?



- There *is* gravity in space
- Weightlessness is due to a constant state of free-fall

How Fast Do Things Orbit?



- Conservation Laws
- Angular momentum- Conservation of spinning
- My own work on accretion deals with this

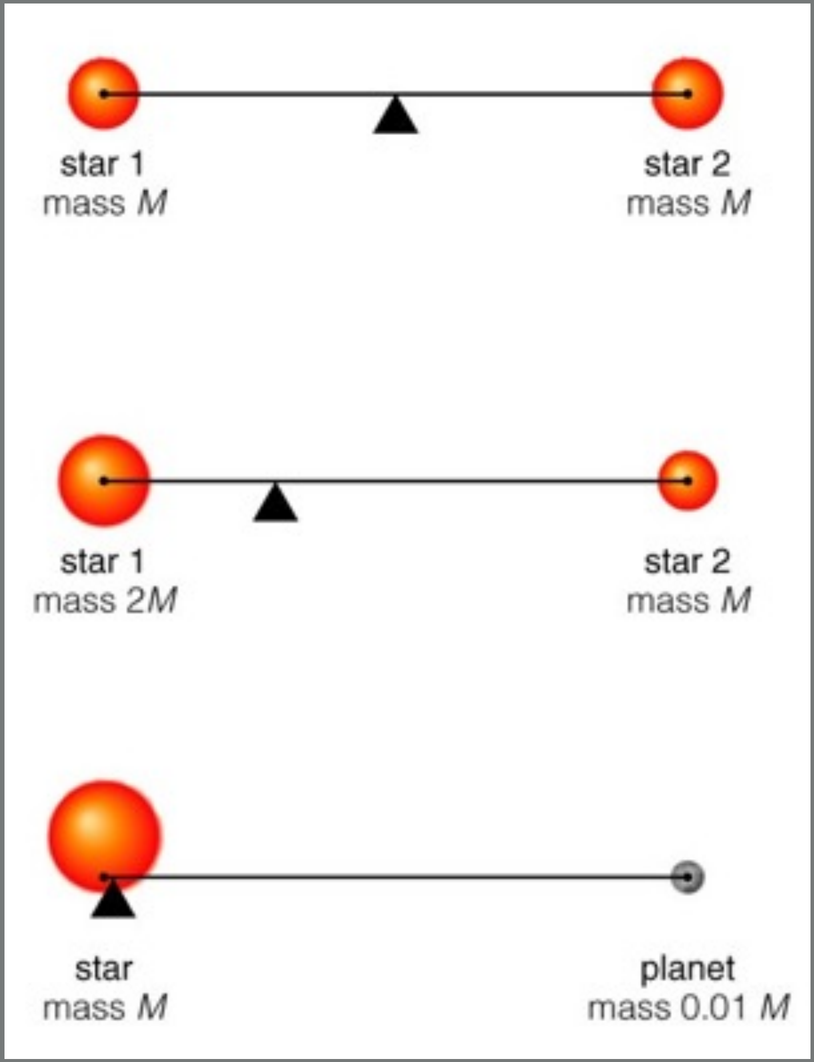
$$\ddot{r} - r\dot{\theta}^2 = -GMr^{-2};$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.$$

$$\frac{d\ell}{dt} = \frac{d(r^2\dot{\theta})}{dt} = r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} = r(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0$$

$$\int_{t_1}^{t_2} \frac{1}{2} \cdot \text{base} \cdot d(\text{height}) = \int_{t_1}^{t_2} \frac{1}{2} \cdot r \cdot r\dot{\theta} dt = \frac{1}{2} \cdot \ell \cdot (t_2 - t_1)$$

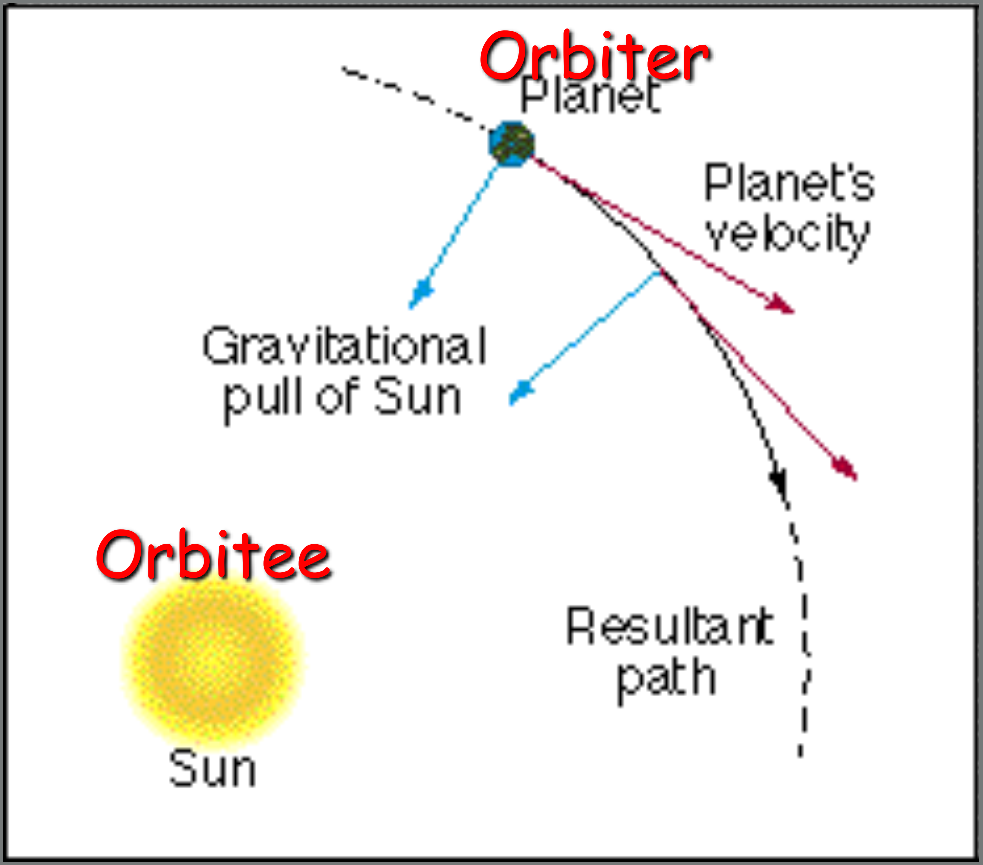
Objects Orbit The "Center of Mass"



- Because of angular momentum conservation, objects orbit around the center of mass of the system

Orbital Speed

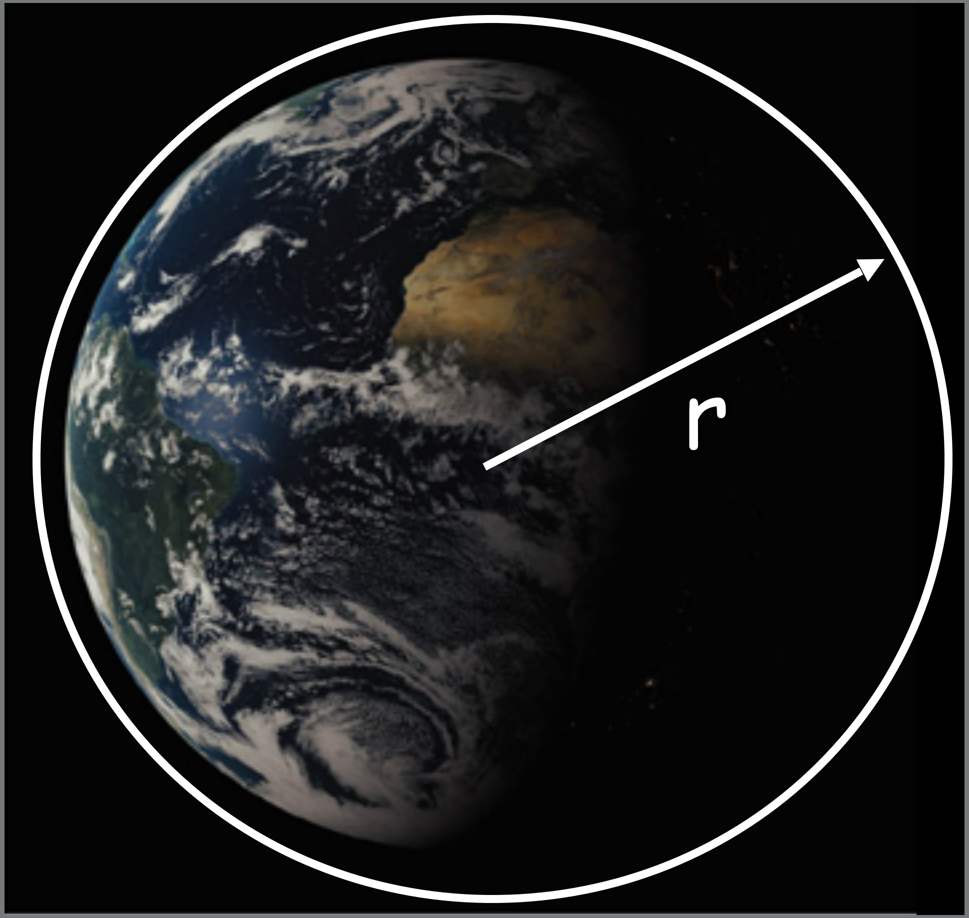
- Planets (orbiters) are constantly trying to get away
  - Gravity (from the orbitee) is constantly pulling them back
- We can use Newton's law of gravity to calculate how fast they move



$$V_{circular} = \sqrt{\frac{GM_{orbitee}}{r}}$$

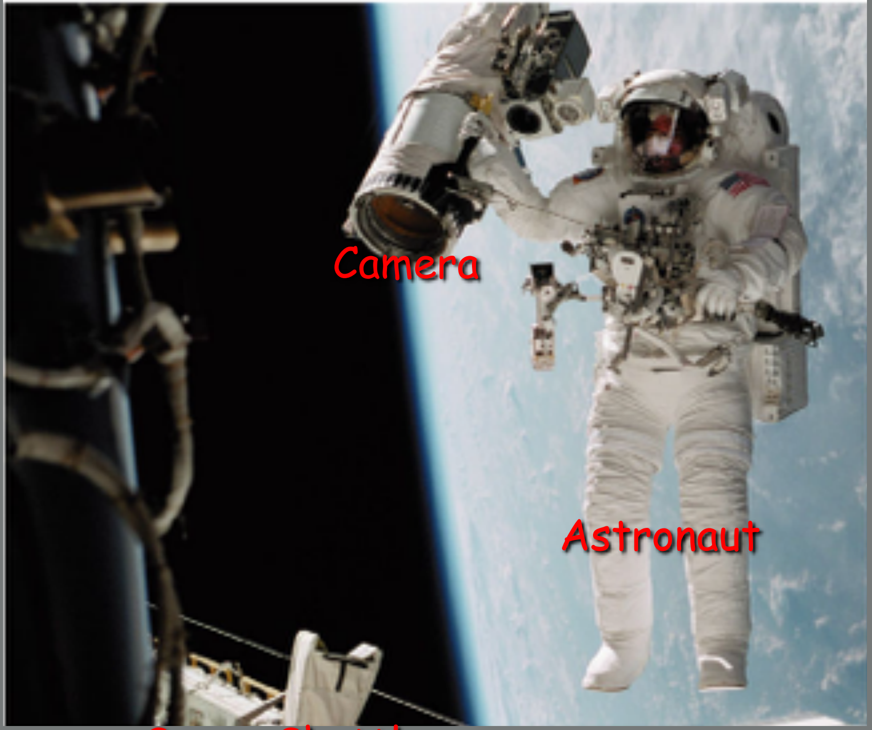
Orbital Speeds for Satellites Around Earth

$$V_{circular} = \sqrt{\frac{GM_{orbitee}}{r}}$$



Thought Question

An astronaut and camera are floating outside the Space Shuttle, which moves fastest?



A. Shuttle has greater speed due to greater mass

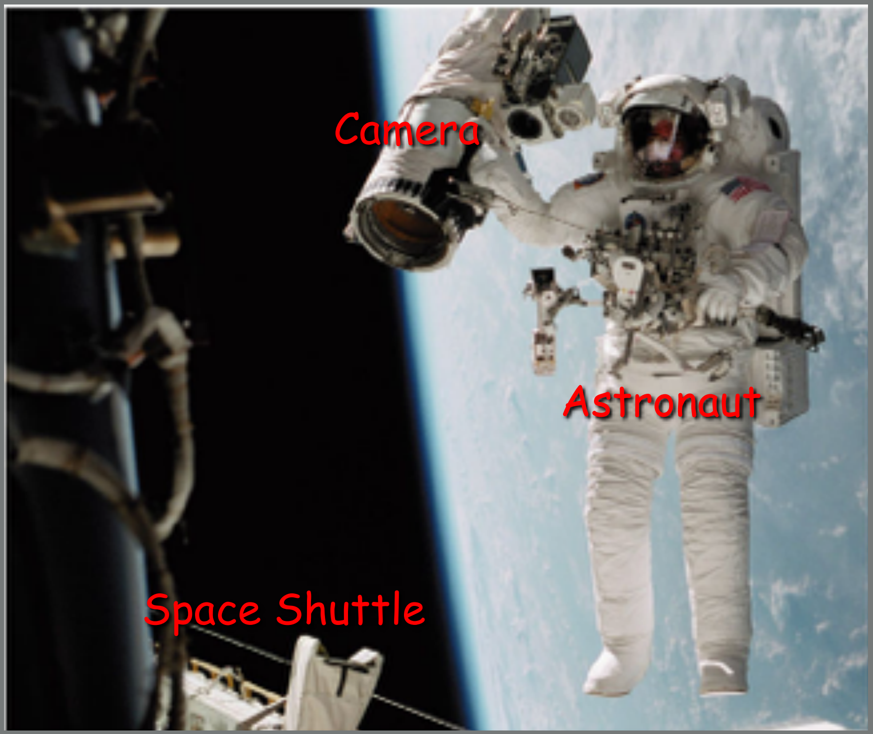
B. Astronaut experiences greater acceleration (and therefore greater speed) than Shuttle due to lower mass

C. Camera's speed is fastest due to lowest mass

D. Astronaut, shuttle and camera all have same orbital speed

$$V_{circular} = \sqrt{\frac{GM_{orbitee}}{r}}$$

- Depends only on:
  - mass of orbitEE
  - orbital distance, r



All orbitERS have same orbital speed

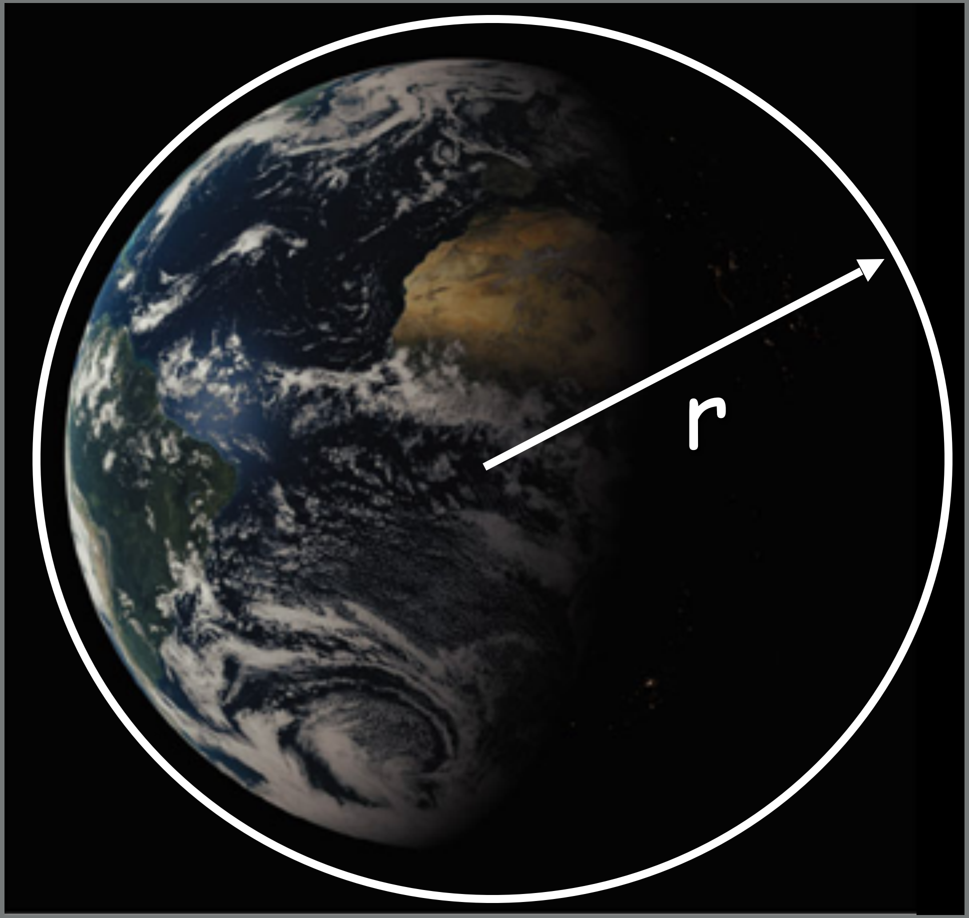
Reading Question

In Newton's version of Kepler's 3rd Law ($p^2=4\pi^2a^3/G(M_1+M_2)$), why is M_1+M_2 usually written as just M?

A. Because Newton's version only works for one mass.
B. Newton's version was designed for planets.
C. You only need the mass of the object you want the period of.
D. Because usually one of the masses is much smaller than the other.
E. The gravitational constant, G, only contains kg, not kg^2.

Orbital Times for Satellites Around Earth

$$V_{circular} \approx 8000 m/s \approx 18{,}000 mph$$



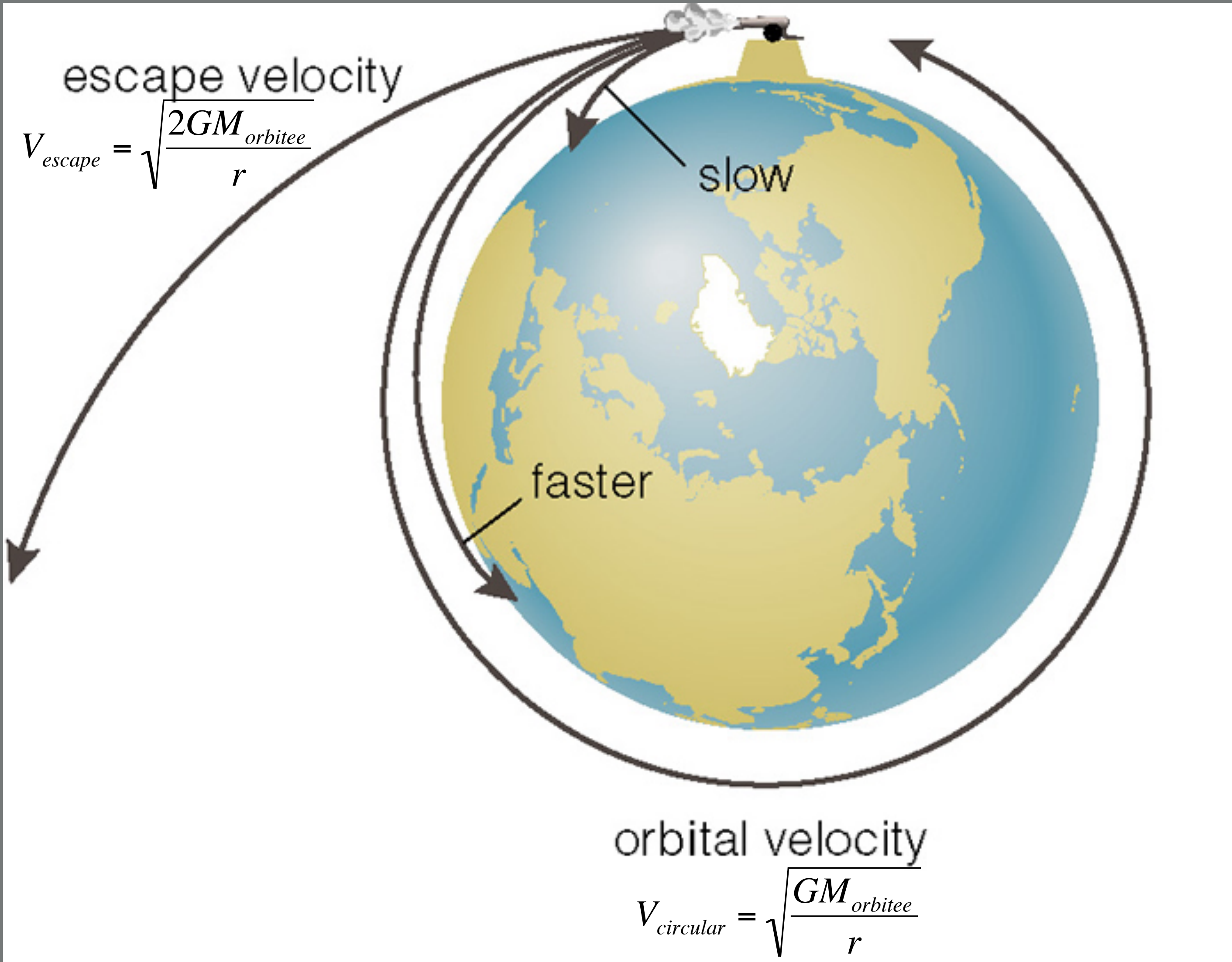
Escape Velocity

The velocity needed to escape the gravity of the orbitEE

Earth

V_{circ}= 8 km/s

V_{esc}= 11 km/s



Question *Suppose Earth were moved to one-fourth of its current distance from the Sun. What would happen to the gravitational force between Earth and Sun?*

Before starting any calculations, make a <u>prediction!</u>

- Mathematical way

$$\frac{F_{new}}{F_{orig}} = \frac{\dfrac{GM_1M_2}{\left(\frac{1}{4}d\right)^2}}{\dfrac{GM_1M_2}{(d)^2}} = \frac{\dfrac{1}{\frac{1}{16}}}{1} = 16$$

$$\boxed{F_{new} = 16 \cdot F_{orig}}$$

- Logic way

$$F = \frac{GM_1M_2}{d^2}$$

d ⇓4 times

d^2 ⇓ 16 times

$1/d^2$ ⇑ 16 times

F ⇑ 16 times

Newton's Version of Kepler's Third Law

Orbital Time = Total Distance / Speed

circular Orbit

$$P = 2\pi a / V_{circ}$$

$$P^2 = (2\pi a)^2 / (V_{circ})^2$$

$$P^2 = 4\pi^2 a^2 / (GM/a)$$

$$\boxed{P^2 = \{4\pi^2/GM\}\, a^3}$$

r = a for circular orbits

What is M again?

M = Mass of the OrbitEE

Newton's Version of Kepler's Third Law

$$p^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

p = orbital period

a = average orbital distance (between centers)

= semi-major axis

$(M_1 + M_2)$ = sum of object masses

- In most cases, $M_1+M_2 \approx M_1$ (Mass of the bigger object)

Kepler's 3rd Law

$$P^2 = a^3$$

Years A.U.

ONLY works for orbiting the SUN

Newton's Version of Kepler's 3rd Law

$$P^2 = \{4\pi^2/GM\}\, a^3$$

sec kg m

$G = 6.67 \times 10^{-11}\ m^3 / (kg\ s^2)$

works for ANYTHING orbiting ANYTHING

Using Newton's Version of Kepler's 3rd Law (NVK3L)

$$M = \{4\pi^2/G\}\, a^3/P^2 \ \text{kg}$$

<u>Three variables</u>
If we know two,
we can solve
for the third!

Measure P

Timing: Count seconds to make 1 orbit

Measure a

Direct Observation, Parallax

Calculate M = mass of orbitEE

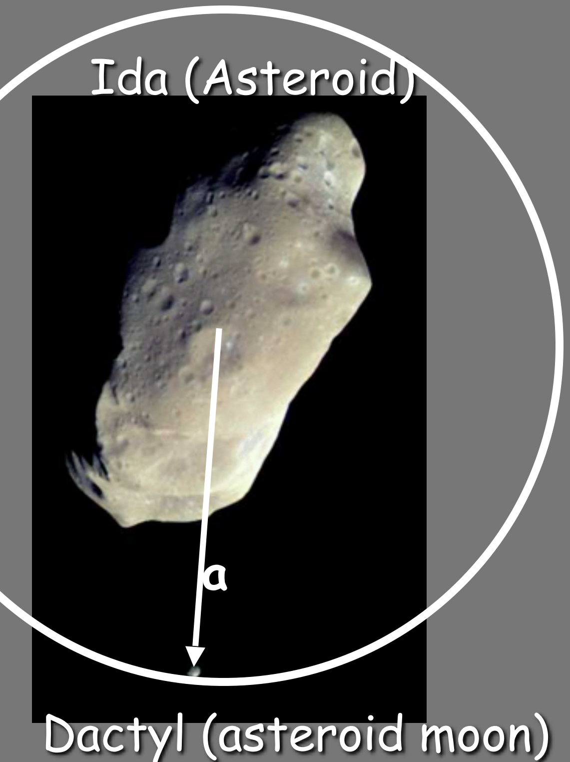
NVK3L can be used on anything that orbits!

Mass of Ida

$= \{4\pi^2/G\}a^3/P^2$

$P \sim 1.5$ days

NVK3L can be used on anything that orbits!



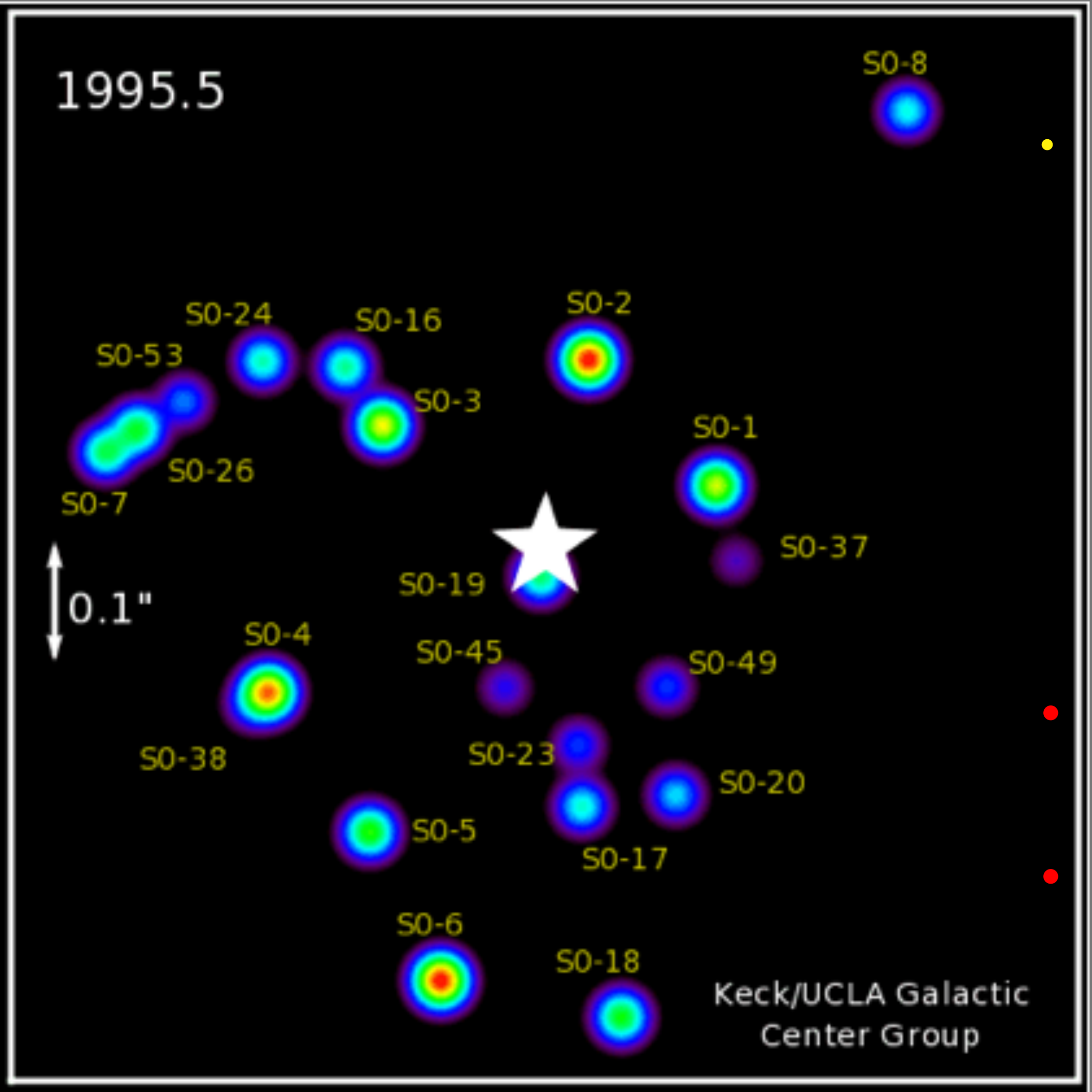
$$M = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

$$M = \frac{4\pi^2}{6.67\times10^{-11}\dfrac{m^3}{kg\cdot s^2}}\frac{\left(100km\cdot\dfrac{1000m}{1km}\right)^3}{\left(1.5day\cdot\dfrac{86400s}{1day}\right)^2}$$

$$\boxed{M = 3.5\times10^{16}kg}$$

Even if you can't see the object in the center!

How to weigh a black hole…



- Watch stars orbit an invisible center
  - Mass of supermassive black hole = $\{4\pi^2/G\}a^3/P^2$
- P ~ 15 years
- a ~ 1000 AU

Even on complex problems like NVK3L, you can use the logical method to compare with known quantities

Extra Credit

How long would one orbit (for Earth) take if the Sun was 4 times as massive as it currently is?

A. 1/16th of a year (~3 weeks)
B. 1/4th of a year (3 months)
C. 1/2 of a year (6 months)
D. 1 year (12 months, 52 weeks, 365 days, $\pi \times 10^7$ seconds)
E. 2 years

How long would one orbit (for Earth) take if the Sun was 4 times as massive as it currently is?

$$p^2 = \frac{4\pi^2}{GM} a^3$$

- $M \Uparrow 4$
- $1/M \Downarrow 4$
- $p^2 \Downarrow 4$
- $p \Downarrow \sqrt{4} = 2$

What if the orbitER and orbitEE are of similar masses?

Pluto & Charon

$$P^2 = \frac{4\pi^2 a^3}{GM}$$

$$P^2 = \frac{4\pi^2 a^3}{G(M_1+M_2)}$$

What if the orbitER and orbitEE are of similar masses?

Pluto & Charon

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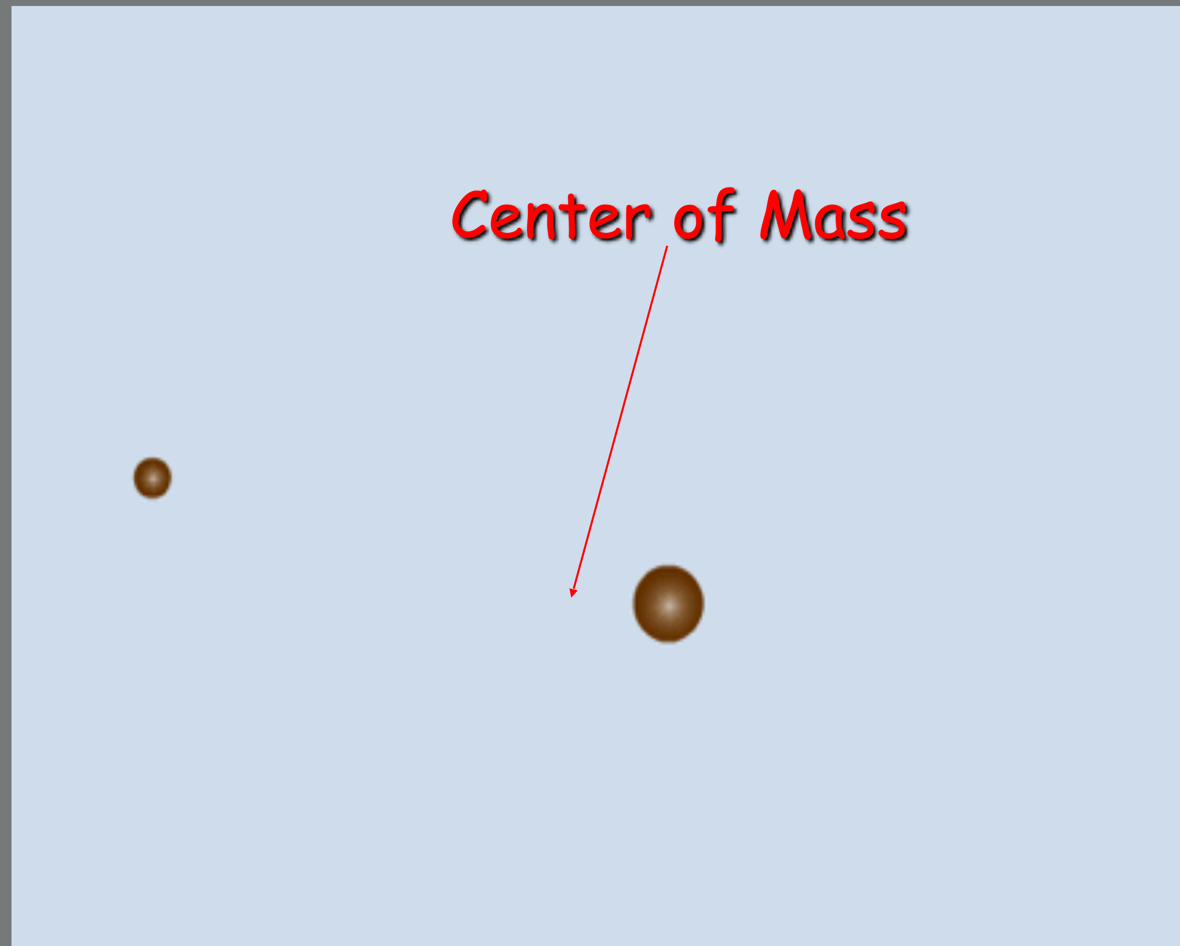

For movie http://pluto.jhuapl.edu/science/everything_pluto/10_binary_planet.html

Tides

- Due to the difference in the strength of gravity across the earth due to the sun and moon
- Tidal friction slows the earths orbit by 1 second/50,000 years
- Earth day may have been 5-6 hours when moon was 10 times closer
- Angular Momentum is conserved, Earth's rotation slows Moons orbital distance grows
- Sun also affects us but by less than half the amount the moon does, 1 million times more massive but 500 times farther

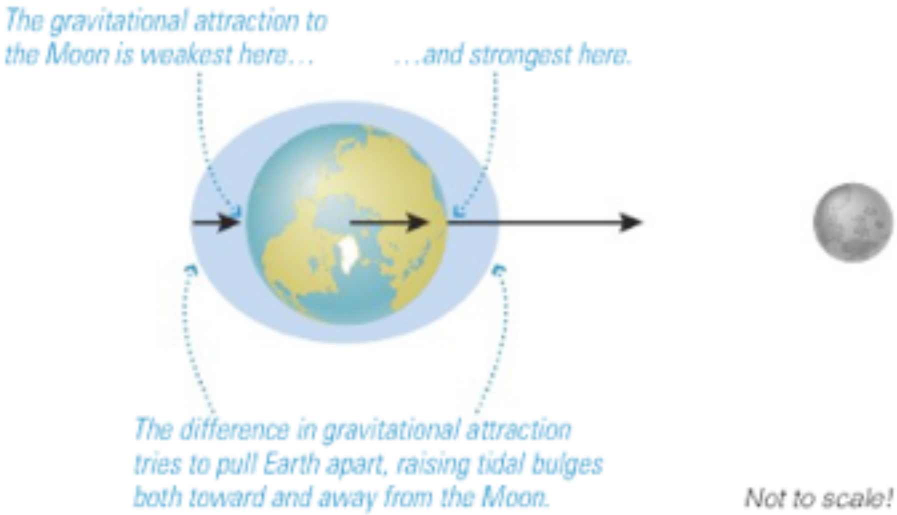

Figure 4.23 Tides are created by the difference in the force of attraction between different parts of Earth and the Moon. There are two daily high tides as any location on Earth rotates through the two tidal bulges. (The diagram greatly exaggerates the tidal bulges, which raise the oceans only about 2 meters and the land only about a centimeter.)

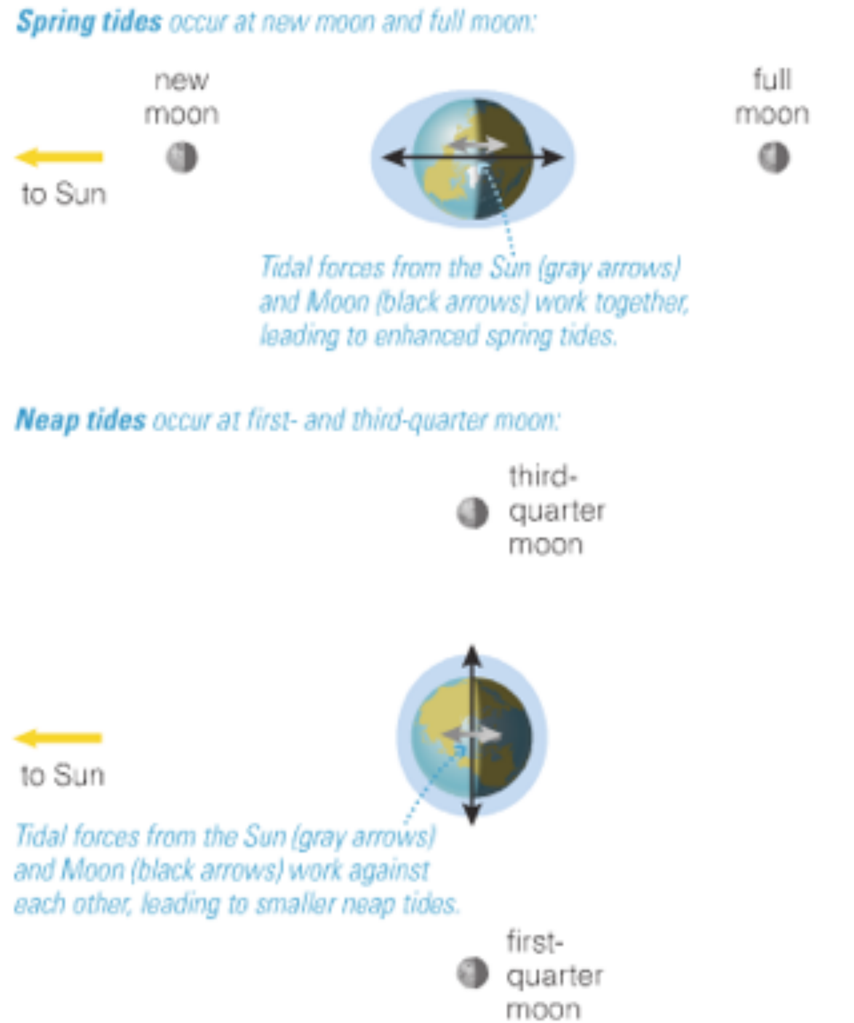

Figure 4.25 Interactive Figure The Sun exerts a tidal force on Earth less than half as strong as that from the Moon. When the tidal forces from the Sun and Moon work together at new moon and full moon, we get enhanced *spring tides*. When they work against each other, at first- and third-quarter moons, we get smaller *neap tides*.

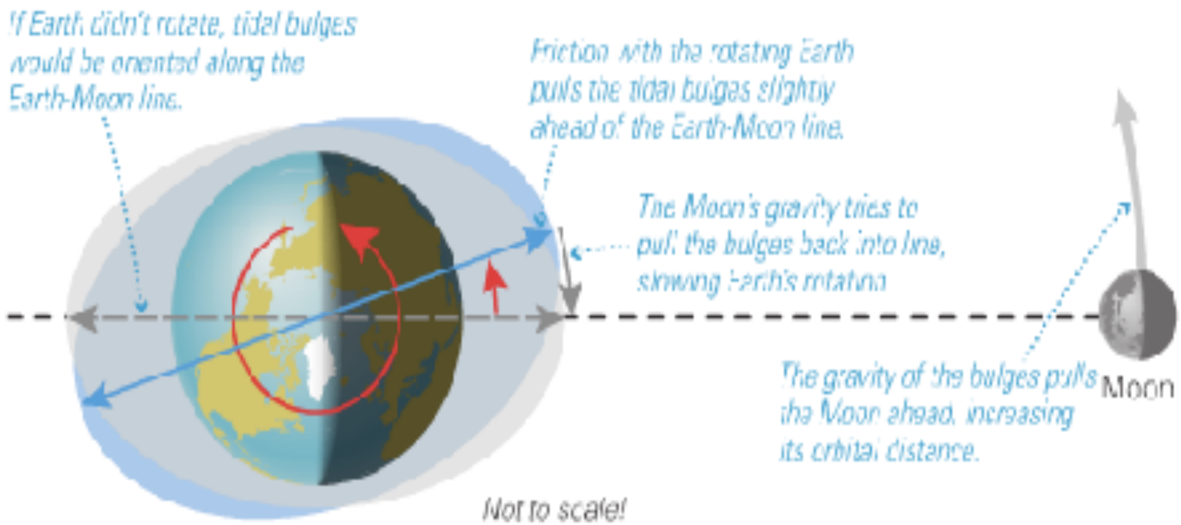

Figure 4.26 Earth's rotation pulls its tidal bulges slightly ahead of the Earth-Moon line, leading to gravitational effects that gradually slow Earth's rotation and increase the Moon's orbital energy and distance.

Tidal friction Elsewhere

- Life on moons? Synchronous Rotation?
- Io