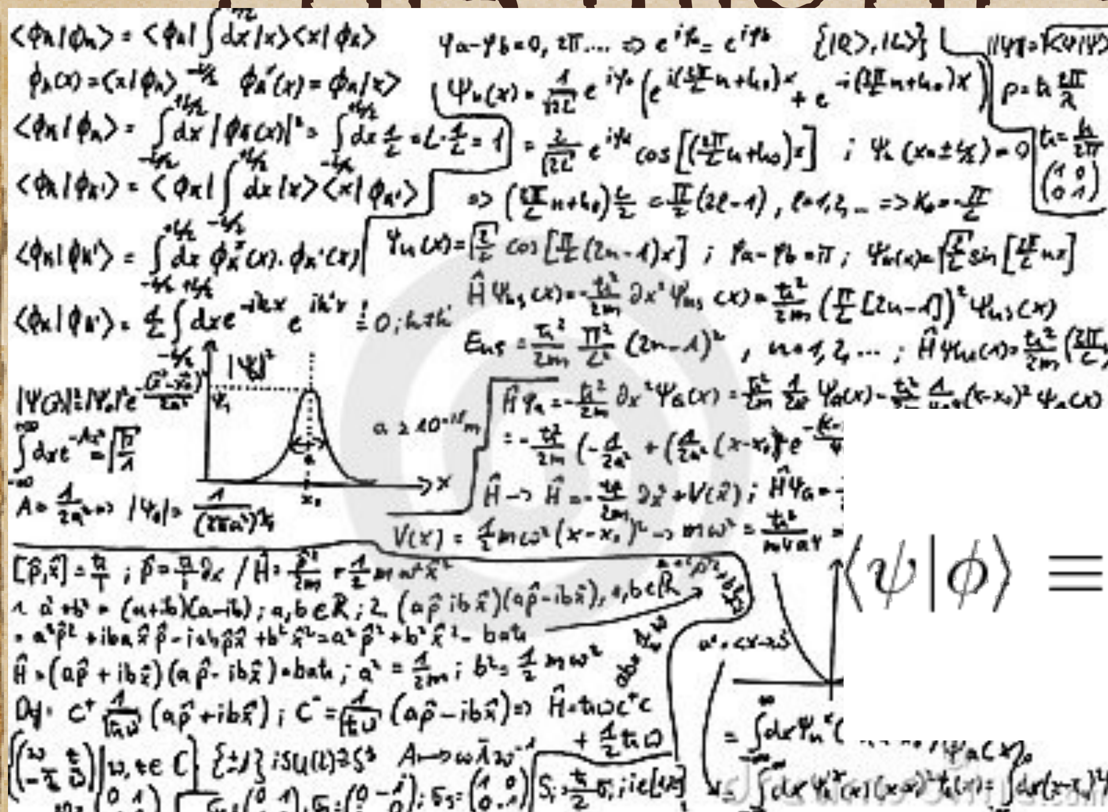
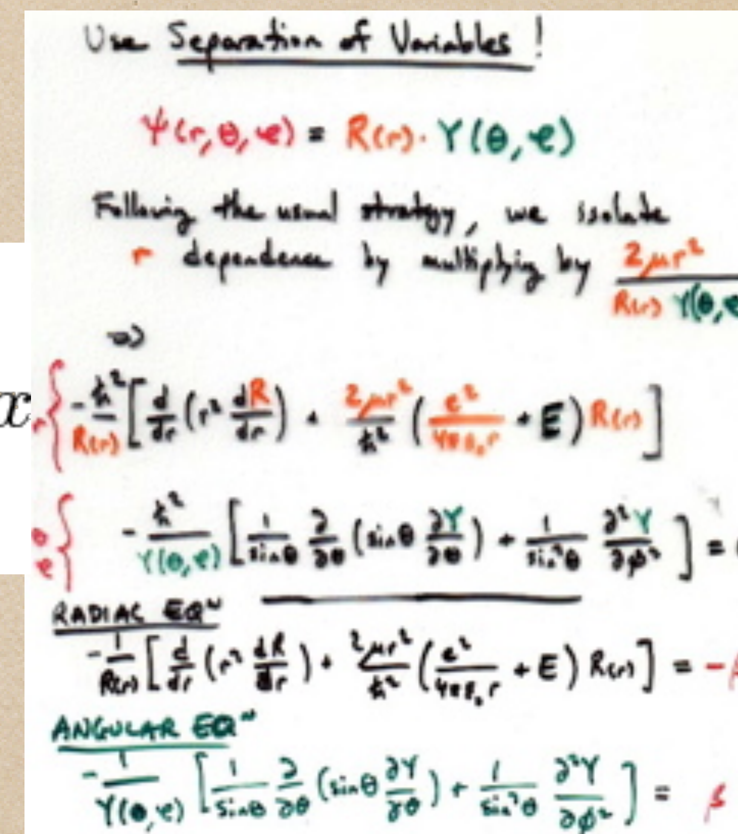
Welcome to Quantum Mechanics

This might sting a little bit



$$\langle\psi|\phi\rangle \equiv \int_{-\infty}^{\infty} \psi^*(x)\phi(x)dx$$



$$\mathbf{1} = \int \mathrm{d}\rho\rho^2 \,|\rho\rangle\,\langle\rho|$$

$$\langle\rho|\rho'\rangle = \frac{\delta(\rho-\rho')}{\rho^2}$$

$$\langle\rho|R\rangle = R(\rho)$$

$$\langle\rho|\hat{H}|R\rangle = \frac{1}{\rho^2}(-\tfrac{1}{2}\rho^2R')' + (V + \tfrac{1}{2}\frac{l(l+1)}{\rho^2})R$$

$$\hat{H}\,|R\rangle = E\,|R\rangle$$

$$\langle p\rangle_\psi = \langle\psi|p|\psi\rangle = \int_{-\infty}^{\infty} \psi^*(x)p^{(op)}\psi(x)dx$$

Policies

- Homework 40%
    - This is where you learn QM
    - Weekly sets
    - 10% penalty per day late
- Exams 60%
    - Three, two each 15%, 30% last one

Let Us Start With The Obvious

- What is quantum mechanics?
- Why do we need quantum mechanics?
- How is it different from classical mechanics?
- Brief segue for the postulates.
- This stuff seems so abstract, what real world value does it have?

What is QM?

- The framework by which we understand and predict the behaviour of the very small, from molecular excitation to atomic and sub-atomic behaviour.
- QM tells us all we can (at present) know about the world of the tiny.
- Remember this -Schroedinger's Equation solves for the wave function(Ψ) of a molecular, atomic, or subatomic particle from which we can determine various "physical" quantities.
    - This is analogous to solving F=ma and determining the position, location, temporal evolution, energy, etc. for a macroscopic particle.

Why Do We Need QM?

- As it turns out, classical mechanics is completely incorrect on small scales.
- We get everything wrong sticking with classical mechanics when it comes to light and the behaviour of the tiny.

How Is QM Different?

- It is non-deterministic, particles do not exist in a specific state, rather they have probabilities of being found in various states and are not given definite form until measured.
- Systems are comprised of different states with various amplitudes describing their contribution to the wave function and these amplitudes correspond to how likely the object is to be in a given state when a measurement is made. Basically, a wave function is made up of different things in different amounts.

Think about this

- Classical mechanics views objects as having distinct states.
- Even non-linear systems are viewed as deterministic if sufficient precision is available concerning their initial conditions.
- In QM a particle is not in any given distinct state until it is measured and then its location is still only probabilistically determined.

The Famous Cat

$$\Psi(cat) = \frac{1}{\sqrt{2}}(|dead > +|alive >)$$

- Cat is placed in box with radioactive material
- If material decays it triggers a cyanide capsule to open
- Material decays or doesn't only once an observation has been made
- Cat is a linear superposition of alive and dead until someone observes it
- As stated in the book, this is patently absurd

And Yet

- This is how QM works
- Measure systems in the exact same initial configuration
- Get different outcomes
- Can only describe the most probable state for an ensemble of identical systems
- And this “expectation Value” may be one that may never be measured

Example

- What are your ages?
- What's the average value for ages in this room?
- Is anyone this age?

From CM To QM

$$\vec{r}(\vec{x},t) \rightarrow |\Psi(\vec{x},t)|^2 dx$$

$$where \int_{-\infty}^{+\infty} |\Psi(\vec{x},t)|^2 dx = 1$$

$$m\frac{d^2\vec{r}(\vec{x},t)}{dt^2} = \vec{F} \rightarrow \hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

Observables (location, momentum, energy, etc.
are replaced by hermetian operators, determinate
states of a wave function are eigenfunctions of the
operator. H in the above equation, is the Hamiltonian of the system or the
sum of its kinetic and potential energy

$$\hat{Q}\Psi = q\Psi$$

$$\hat{Q}(\vec{x},\vec{p}) = \hat{Q}(\vec{x},\frac{\hbar}{i}\vec{\nabla}_x) - PositionSpace$$

$$\hat{Q}(\vec{x},\vec{p}) = \hat{Q}(-\frac{\hbar}{i}\vec{\nabla}_p,\vec{p}) - MomentumSpace$$

The Postulates of QM

- Griffiths doesn't touch this
- I think its important
- You don't have the math to see them in full form so I'll put them in my own words

Postulate #1

- For a given system at a time t_0 there exists a wave function $\Psi(t_0)$ which specifies the state of the system. This wave function belongs to Hilbert space, the space of square integrable functions $\int_{-\infty}^{+\infty} f(x)^2 < \infty$

Postulate #2

- Every measurable physical quantity of this system is defined by an operator acting on this wave function $\hat{Q}\Psi = q\Psi$

Postulate #3

- The only possible result of an operator acting on this wave function is one of the eigenvalues of this operator

Postulate #4

- The probability of obtaining the eigenvector q associated with operator Q depends on the square of how much of the wave function is made up of eigenvectors associated with q.
- I'll show you how to do this latter.

Postulate #5

- I have no good way to put this one non-mathematically, I'll show you how to do it in chapter 3.
- Once you make a measurement you collapse the wave function and pick out the piece corresponding to what was measured. This means projecting the original function onto the subspace spanned by the eigenvectors of the eigenvalue measured then normalizing it. Think of it as $\frac{\vec{r} \cdot e_x}{\sqrt{\vec{r} \cdot e_x)^2}}$ where e_x is the measured eigenvalue.
- The full mechanics of the fifth postulate is within the scope of the course but the abstract notation for it is not. Basically, once a measurement is made the new wave function is the normalized projection onto the measured components basis vectors. This is the collapsed wave function you have heard of.

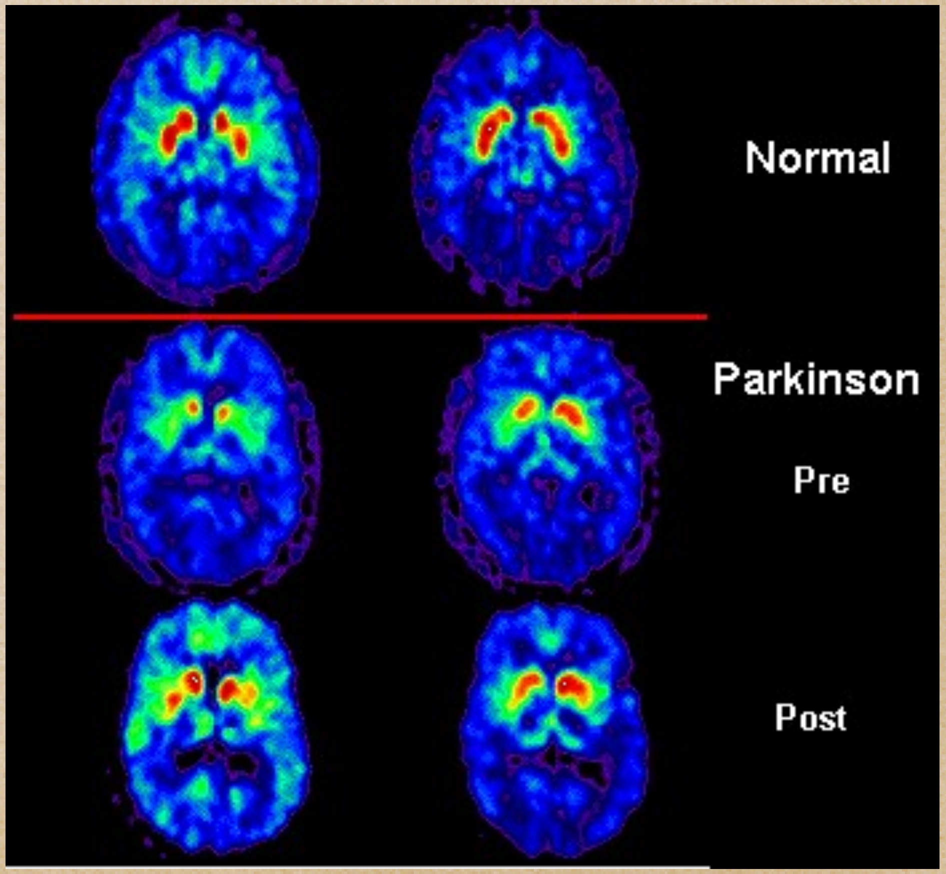
Postulate #5

- The time evolution of the wave function is given by $i\hbar \frac{d\Psi}{dt} = \hat{H}\Psi$

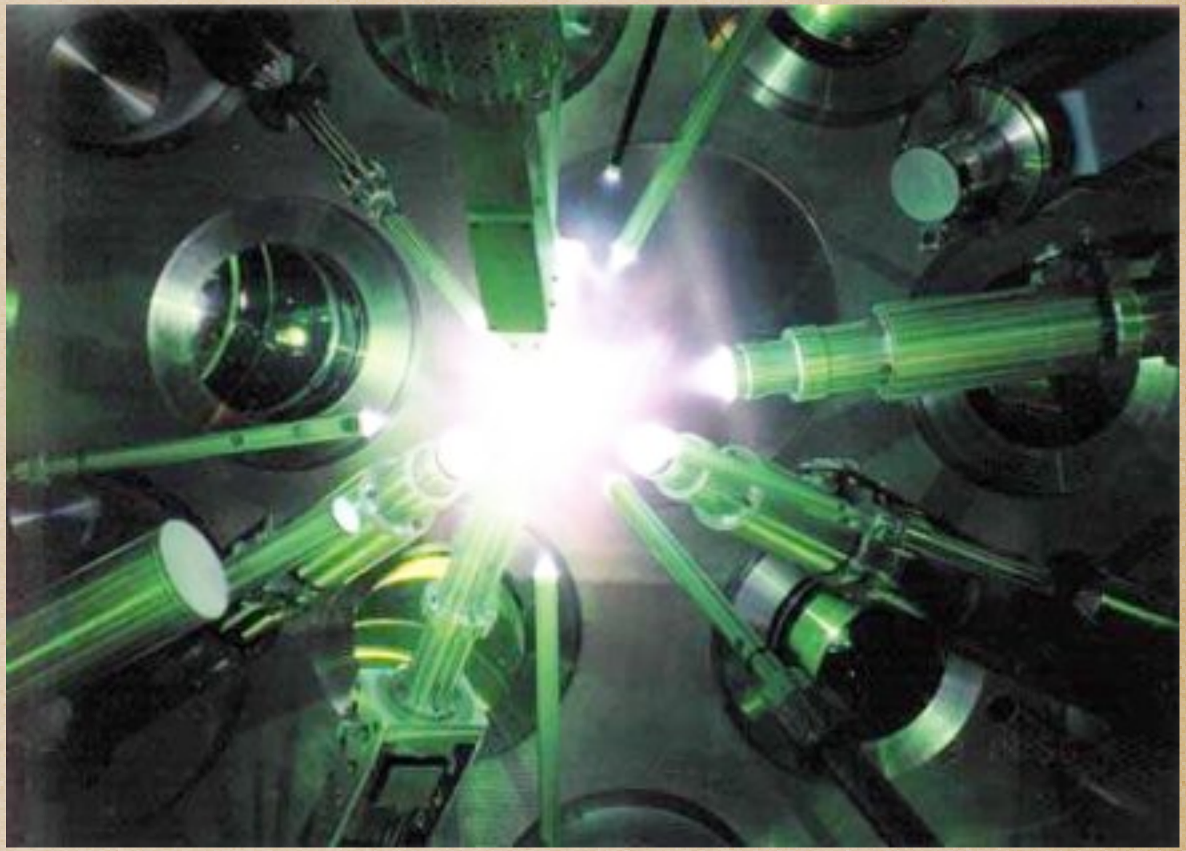
We'll come back to that stuff, why is it important?

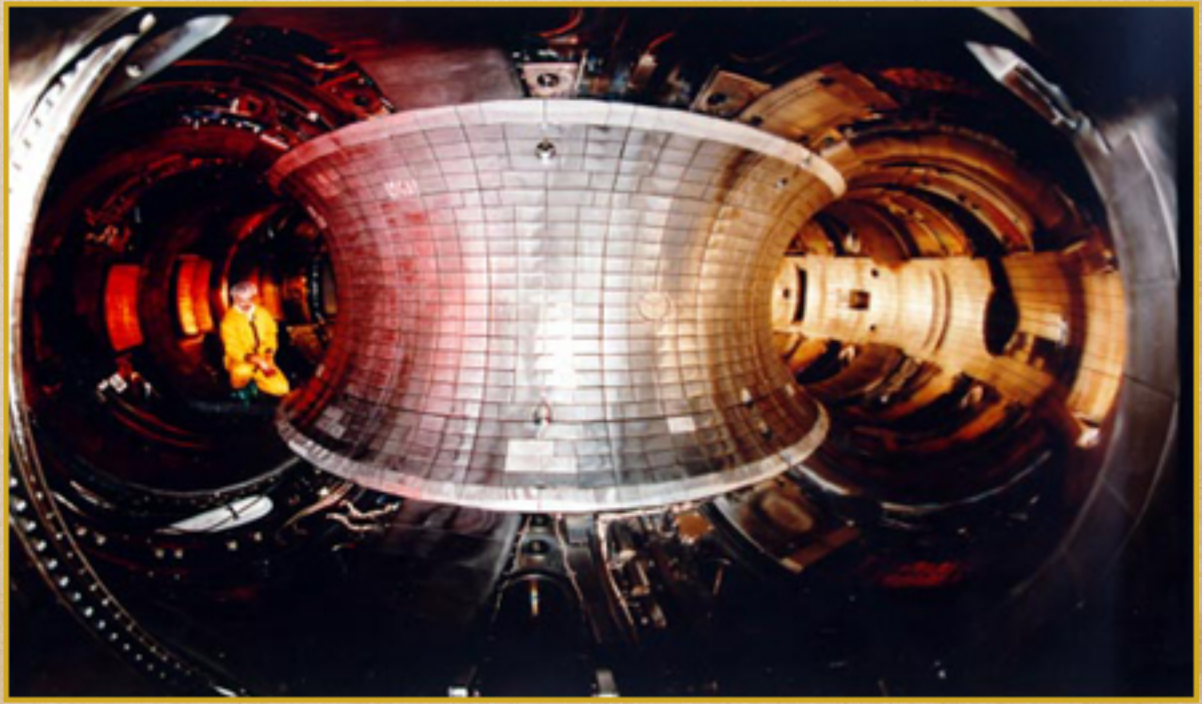
- Many Applications

Medicine

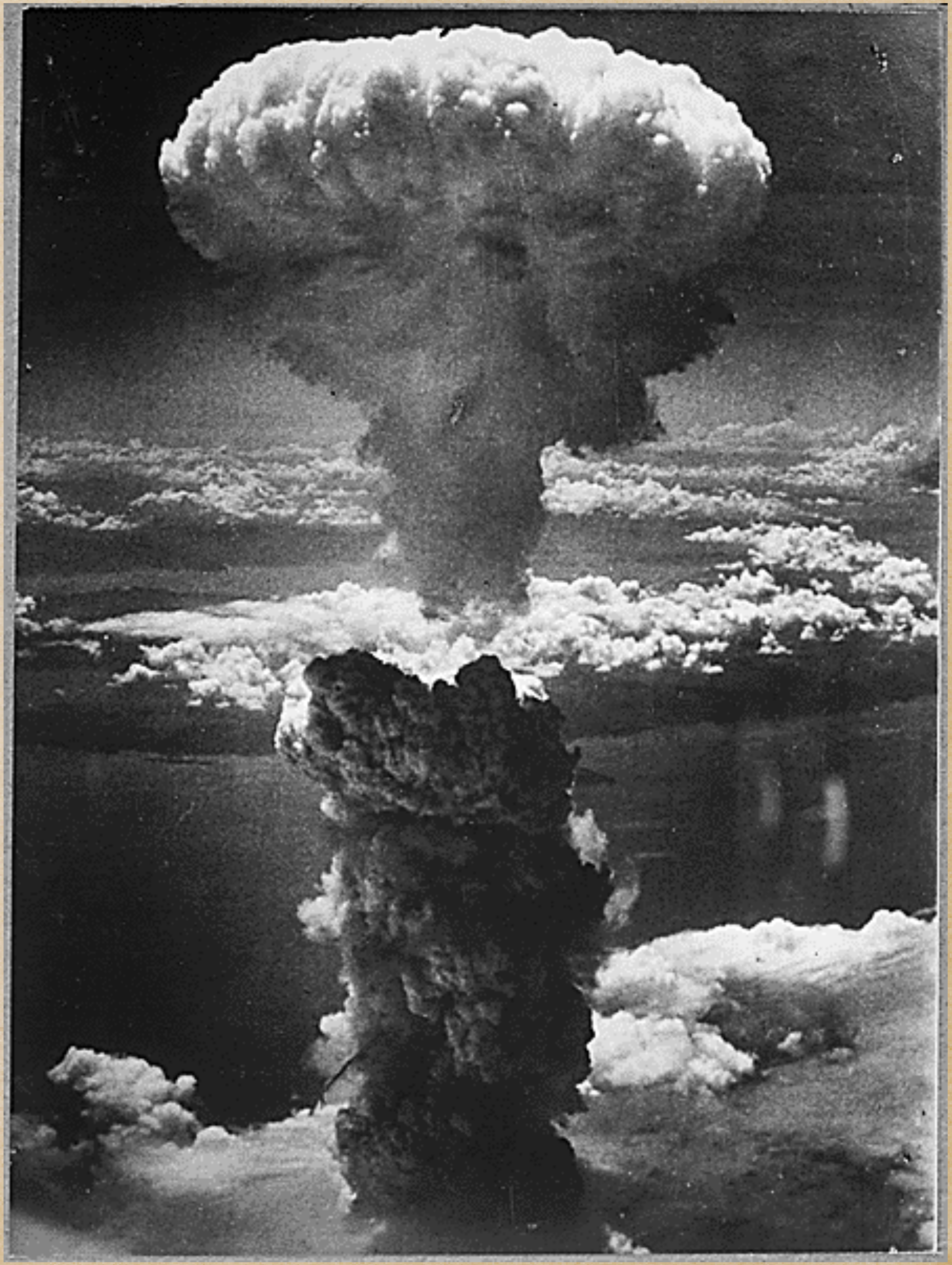


Energy



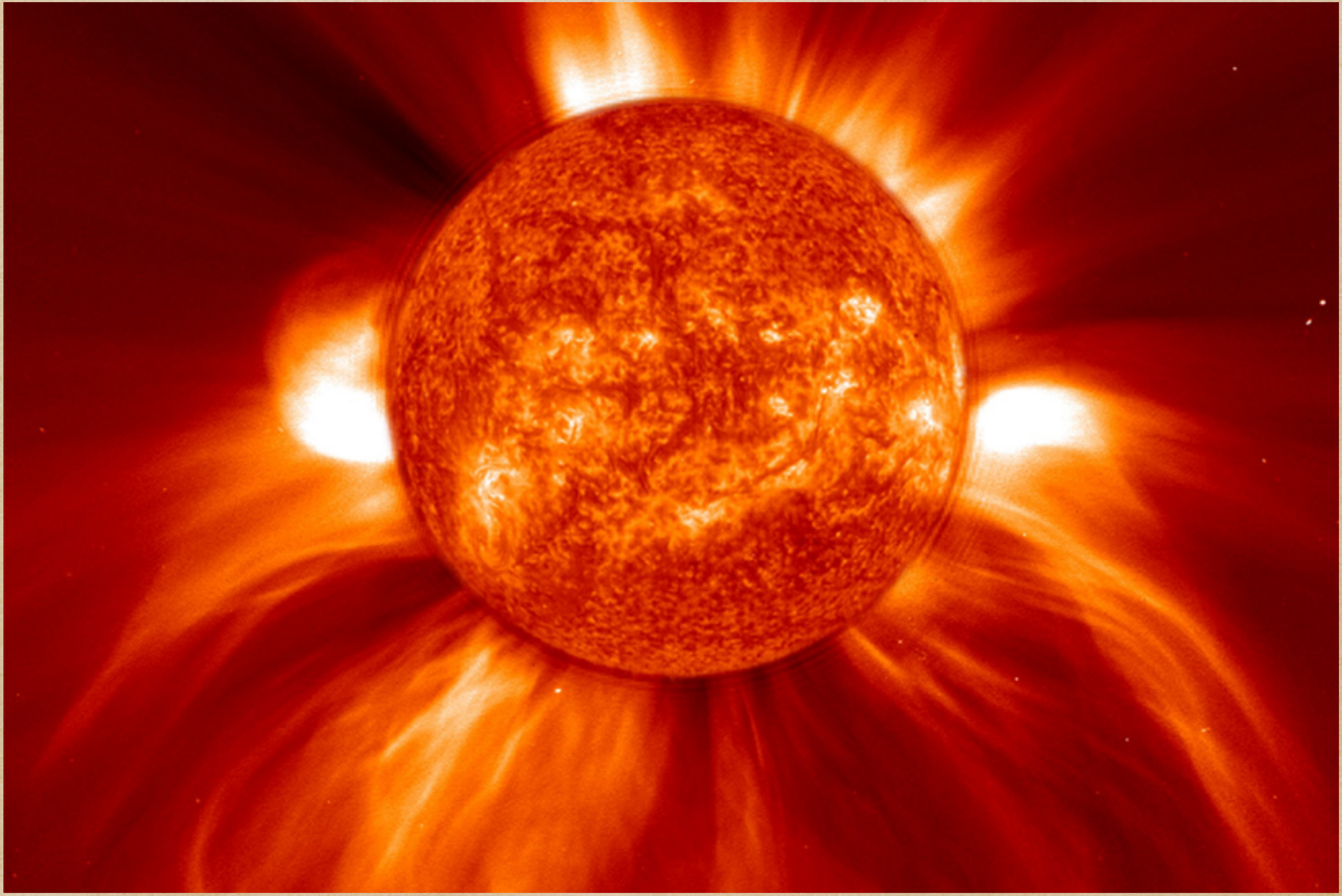


War and likely preventing war



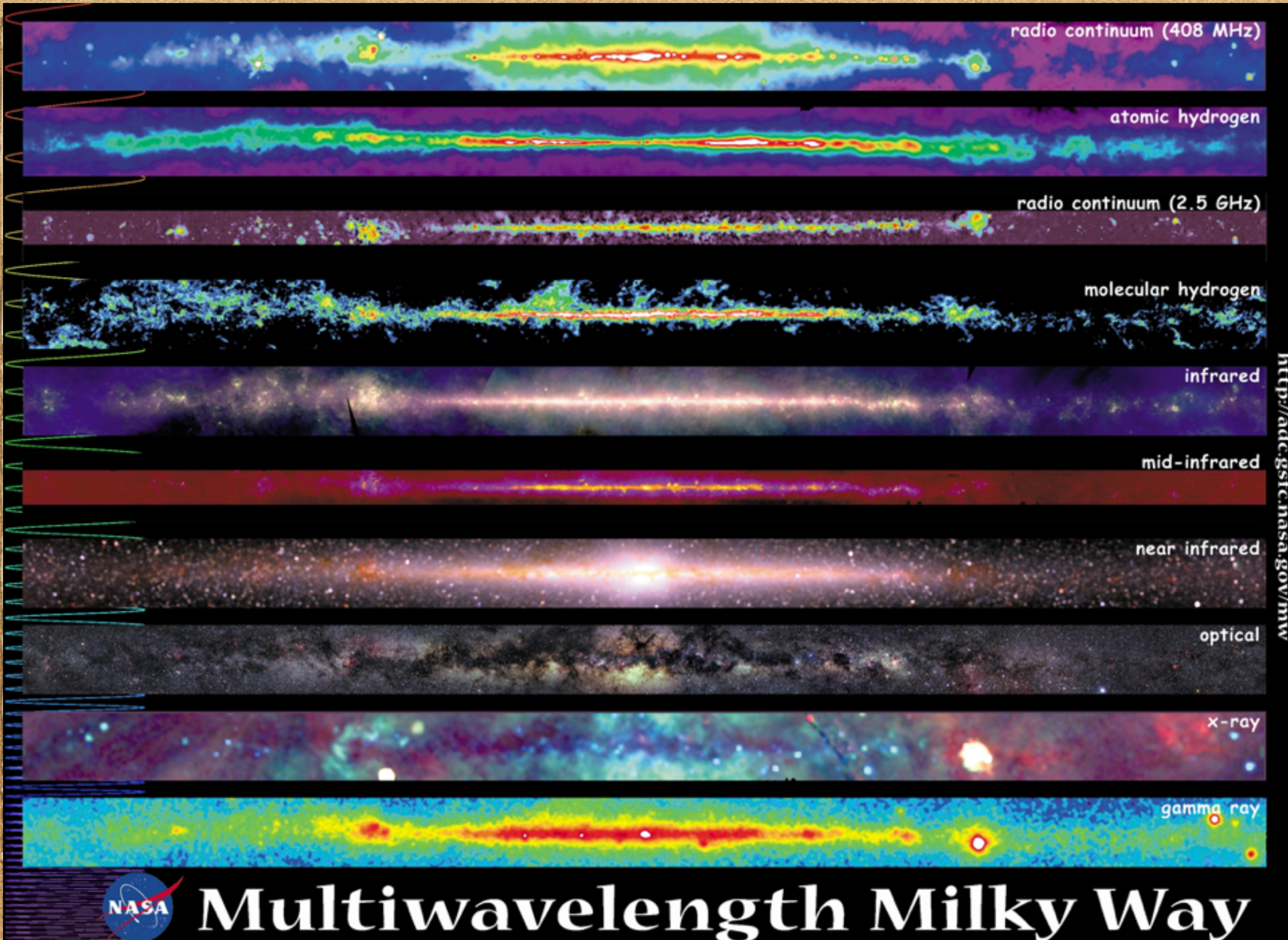
Modern Society, record is 6.8 billion transistors on one, 10 core cpu

All life on Earth

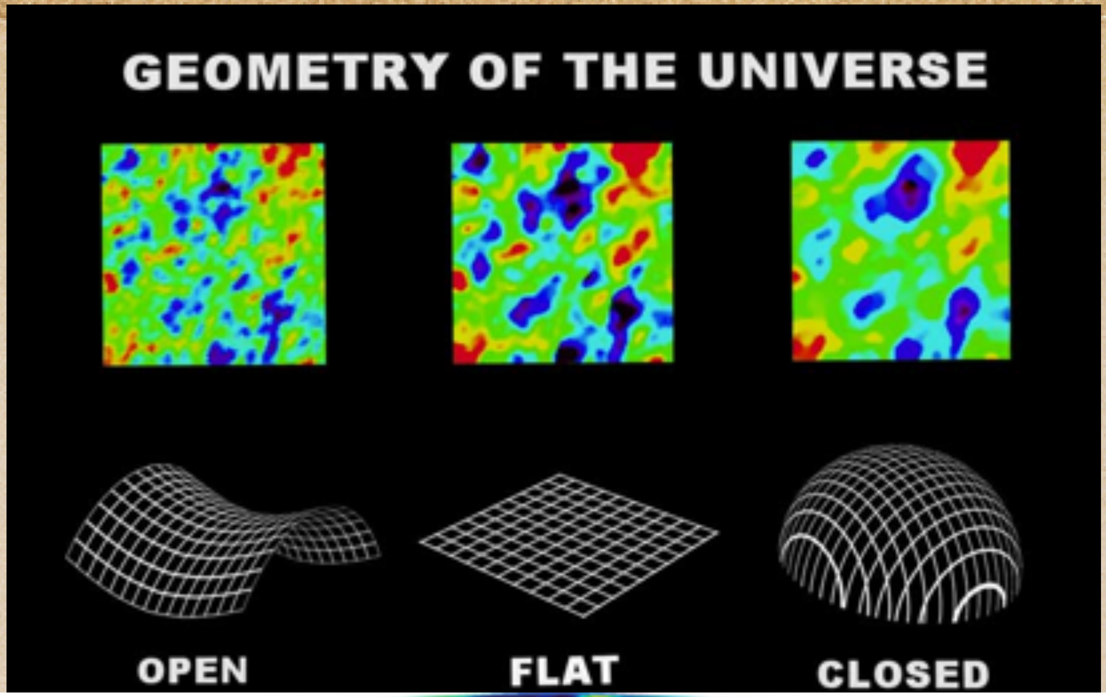


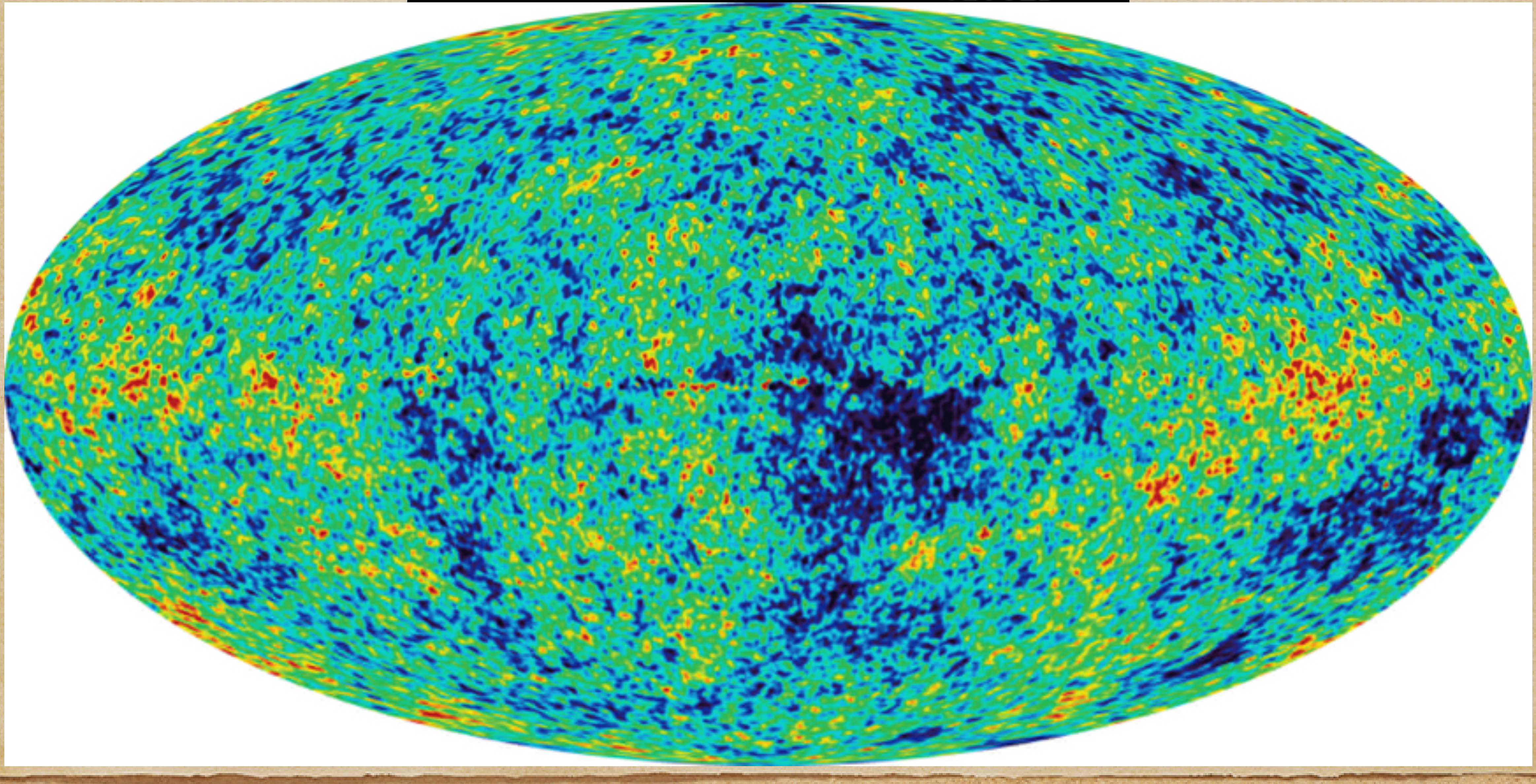
And my favorites!!!

- Astrophysics



radio continuum (408 MHz)
atomic hydrogen
radio continuum (2.5 GHz)
molecular hydrogen
infrared
mid-infrared
near infrared
optical
x-ray
gamma ray
http://adc.gsfc.nasa.gov/mw
NASA
Multiwavelength Milky Way


GEOMETRY OF THE UNIVERSE
OPEN
FLAT
CLOSED



We'll Start Next Class

- Read Chapter 1 for Thursday
- There's going to be a lot of math in this class, not much vector calculus but I'll be introducing
    - Probability Theory
    - dirac orthogonality
    - series solutions to ODEs
    - Linear Algebra
    - Hilbert Spaces
    - Bra Ket Notation
- Come to my office whenever you need help