Physics 321, Fall 2012 Exam 2

Dr. Jared Workman, Wednesday, November 14th 9:00-10:50

RULES

You may use an equation sheet. This sheet may NOT have solutions to problems but may contain any formulae or integrals you choose to include.

Problems

1: Construct $\Psi_{3,2,1}$. Don't worry about normalizing $R_{n,l}$.

2: The Hamiltonian for a certain system is given by

$$\frac{L^2}{2I} + \alpha L_z \tag{1}$$

Leaving I as I write down the possible energies for a system in the n = 3 state.

3: What is the probability that the electron in a hydrogen atom is found in the nucleus? Let the radius of the nucleus be denoted by b, calculate this for $\Psi_{1,0,0}$. Give an analytic answer as well as an approximation using $a = .5 * 10^{-10}m$ and $b = 10^{-15}m$.

4: A spin vector begins in the state $\chi = A(\begin{matrix} 1-2i \\ 2 \end{matrix})$. Determine A, $< S_x >, < S_y >, < S_z >$.What are the possible out comes of measuring S_x What is the probability of measuring spin up and spin down for S_z?

5: A particle begins in the state $R_{21}[\sqrt{1/3}Y_1^0\chi_+ \sqrt{2/3}Y_1^1\chi_-]$. What are the possible outcomes of measuring $L^2, L_z, S^2, and S_z$? What are the probabilities for each outcome?

6: The Hamiltonian for the interaction energies between two particles with interacting spins is given by

$$(\frac{ge}{2m_n})^2 * \frac{1}{a^3}(S_1 \cdot S_2 - 3S_{1z}S_{2z}) \tag{2}$$

Write down the energy levels when this Hamiltonian is applied to a composite, 2 particle $|S, m_s >$ state where each particle has spin 1/2. Remember, this produces 4 possible states, the triplet and singlet states. Create a chart for the possible $S \& m_s$ states and the corresponding energies.

Formulas

$$P_l(x) = \frac{1}{2^l l!}(\frac{d}{dx})^l(x^2-1)^l \tag{3}$$

$$P_l^m(x) = (1-x^2)^{|m|/2}(\frac{d}{dx})^{|m|}P_l(x) \tag{4}$$

$$Y_l^m(\theta,\phi) = \epsilon \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}} e^{im\phi} P_l^m(\cos\theta) \tag{5}$$

$$R_{n,l} = \frac{\rho^{l+1} e^{-\rho} v(\rho)}{r} \tag{6}$$

where $\rho = r/(an)$ and $v(\rho) = \sum c_j \rho^j$ with

$$c_{j+1} = c_j \frac{2(j+1+l-n}{(j+1)(j+2l+1)}. \tag{7}$$