# Stellar Astrophysics

## Lecture 5

# Want to calculate Flux we observe at surface

* Opacities based on temperature, density, electron density, etc

* Generally pick an optical depth of 100-1000 which is about 1 percent of the way into the star

* Below this, assume isotropic radiation i.e. not much contribution to what we see

* Must respect Saha, Boltzmann, hydrostatic equilibrium, stellar structure, radiative transfer, etc.

# Grey Atmosphere

* Assume frequency independent opacity, opacity is colorless "grey", optical depth is frequency independent

$$\int u \frac{dI_\nu}{d\tau} d\nu = \int [I_\nu - S_\nu] d\nu = I - S$$

* J = S (using radiative equilibrium eq 3.54)

$$J(\tau) = S(\tau) = B(\tau) = \int_0^\infty B_\nu(\tau) d\nu = \frac{\sigma T^4(\tau)}{\pi}$$

* Integrating over solid angles and dividing by 4 pi gives the first moment

$$\frac{1}{4\pi}\frac{d}{d\tau}\oint uI d\Omega = \frac{1}{4\pi}\oint(I-S)d\Omega = J - S = \frac{dH}{d\tau}$$

which equals zero for J = S meaning the integrated flux is constant in the atmosphere

Recalling the definition of the Eddington flux in terms of the monochromatic flux

$$H = \frac{\sigma T_{eff}^4}{4\pi}$$

but

$$\frac{dK_2}{d\tau} = H$$

So

$$K_2 = H\tau + c$$

and J = 3K$_2$ so

$$J = 3(H\tau + c)$$

rewriting c=yH

$$J(\tau) = 3H(\tau + y)$$

and recalling

$$J(\tau) = \sigma T^4(\tau)/\pi$$

and

$$H = \frac{\sigma T_{eff}^4}{4\pi}$$

Leads to

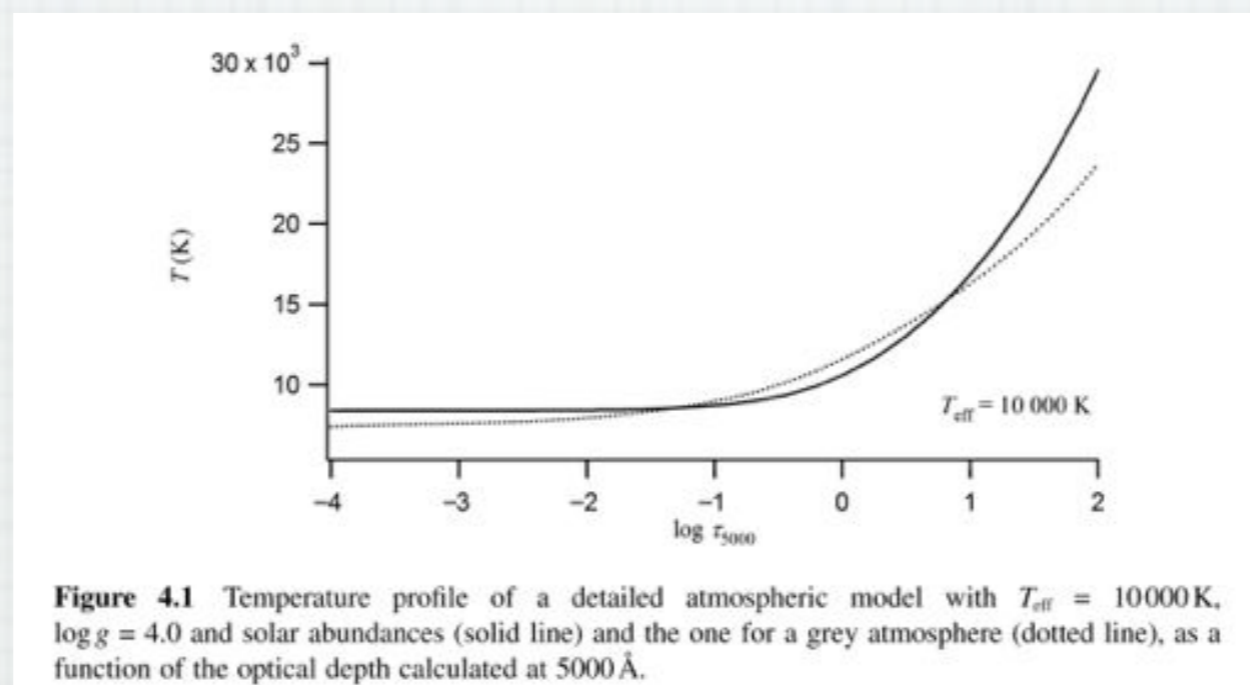$$\frac{\sigma T^4(\tau)}{\pi} = 3\frac{\sigma T_{eff}^4}{4\pi}(\tau + y)$$

or

$$T(\tau) = T_{eff}[\frac{3}{4}(\tau + y)]^{1/4}$$

# Which says

* The temperature inside a star increases with optical depth

* Assuming that the effective temperature and true temperature are equal at a depth where there is a 50 percent probability of photons scattering upwards or inwards leads to $e^{-\tau} = .5$

* or tau ˜ 2/3

* resulting in $y=2/3$ or $T(\tau) = T_{eff}[\frac{3}{4}(\tau + \frac{2}{3})]^{1/4}$

* This is a RESULT, we see radiation from tau = 2/3

# This result is an approximation

* More detailed calculations are required

* Beyond the scope of this course

* Does a pretty good job though



**Figure 4.1** Temperature profile of a detailed atmospheric model with $T_{eff} = 10\,000\,K$, $\log g = 4.0$ and solar abundances (solid line) and the one for a grey atmosphere (dotted line), as a function of the optical depth calculated at $5000\,Å$.

# Limb Darkening

$$J(\tau) = 3H(\tau + \frac{2}{3})$$

$$I_\nu(0,u) = \int_0^\infty S_\nu(t) e^{\frac{-t}{u}} \frac{dt}{u}$$

Integrating over frequency gives and recalling S=J

$$I(0,u) = \int_0^\infty J(t) e^{\frac{-t}{u}} \frac{dt}{u} = \int_0^\infty 3H[t + \frac{2}{3}] e^{\frac{-t}{u}} \frac{dt}{u} = 3H[u + \frac{2}{3}]$$

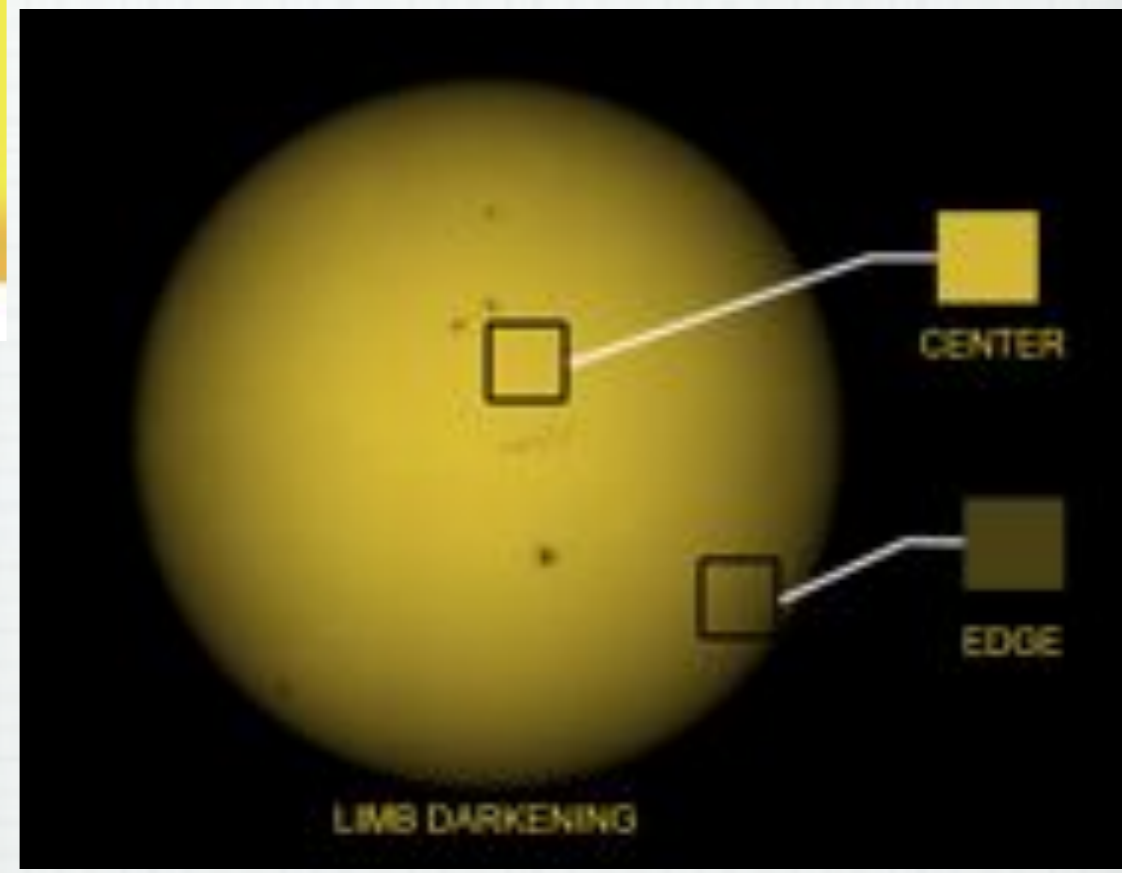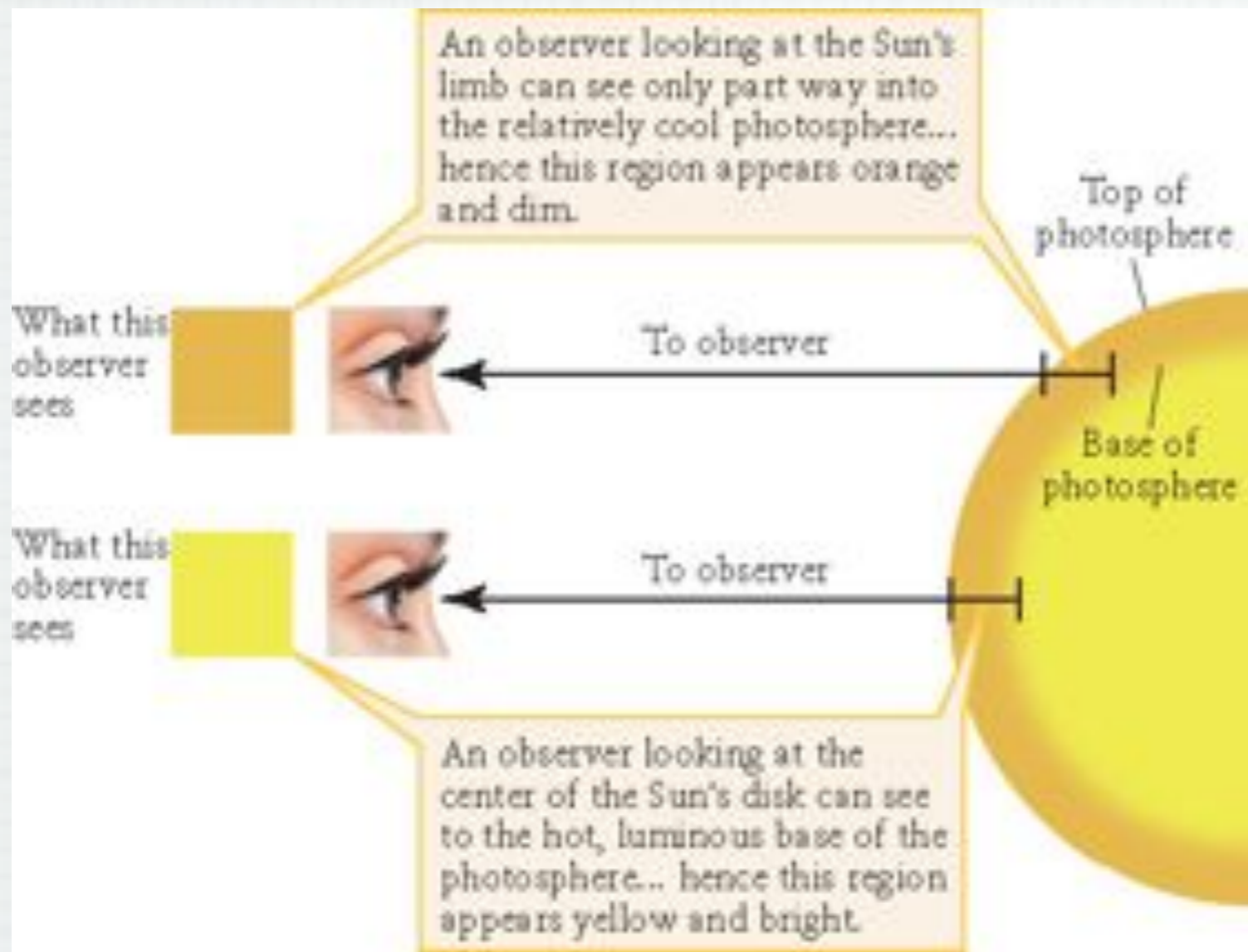Or

$$\frac{I(0,u)}{I(0,1)} = \frac{3}{5}[u + \frac{2}{3}]$$
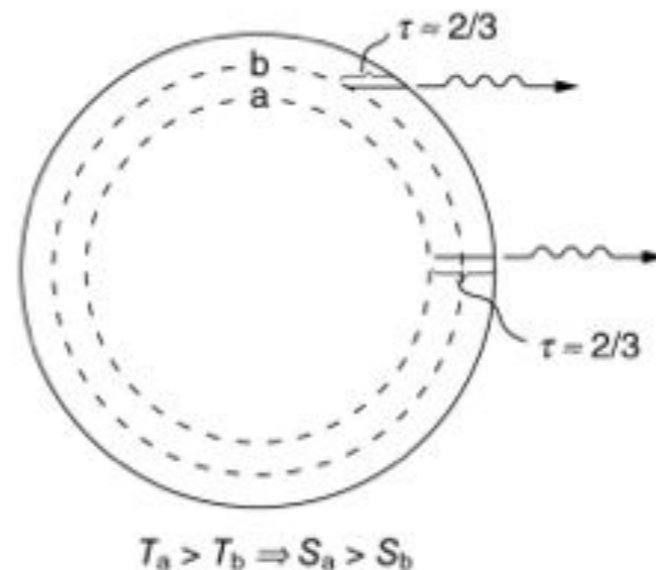
**Figure 4.2** Illustration of the limb-darkening effect. The radiation from the disc's centre (here the observer is on the right side of the star) comes from hotter layers (with a larger value for the source function) than the radiation coming from the portions of the disc near the limb.

Essentially, when you are looking at the disk center you are looking through less atmosphere at a higher temperature source function
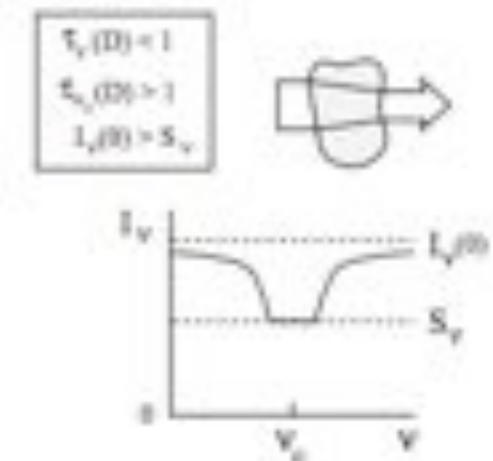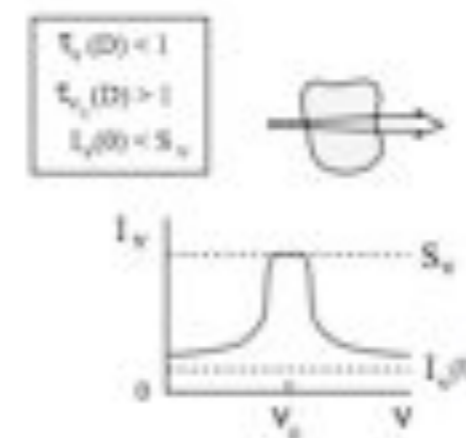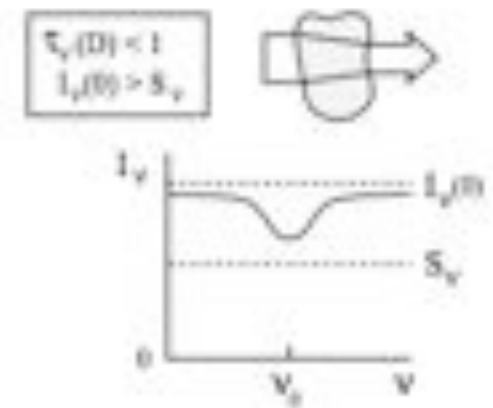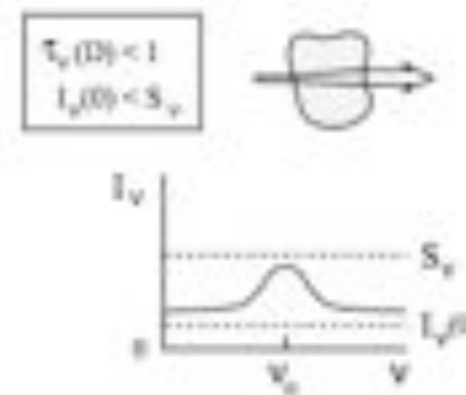When you look at the limb you are looking through more atmosphere to a lower temperature source function

An observer looking at the Sun's limb can see only part way into the relatively cool photosphere... hence this region appears orange and dim.

Top of photosphere

What this observer sees

To observer

Base of photosphere

What this observer sees

To observer

An observer looking at the center of the Sun's disk can see to the hot, luminous base of the photosphere... hence this region appears yellow and bright.
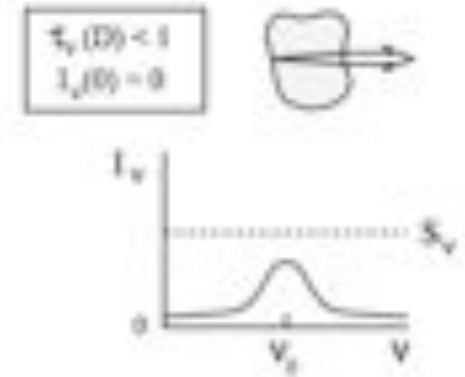
CENTER

EDGE

LIMB DARKENING

# Line Formation



Figure 2.2  Spectral lines from a homogeneous object with $S_\nu = S_\nu^l = S_\nu^c$ everywhere, according to (2.35)–(2.36). No lines emerge when the object is optically thick (top left). When it is optically thin, emission lines emerge when the object is not back-lit ($I_\nu(0) = 0$, top right), or when it is illuminated with $I_\nu(0) < S_\nu$. Absorption lines emerge only when the object is optically thin and $I_\nu(0) > S_\nu$. The emergent lines saturate to $I_\nu = S_\nu$ when the object is optically thick at line center.
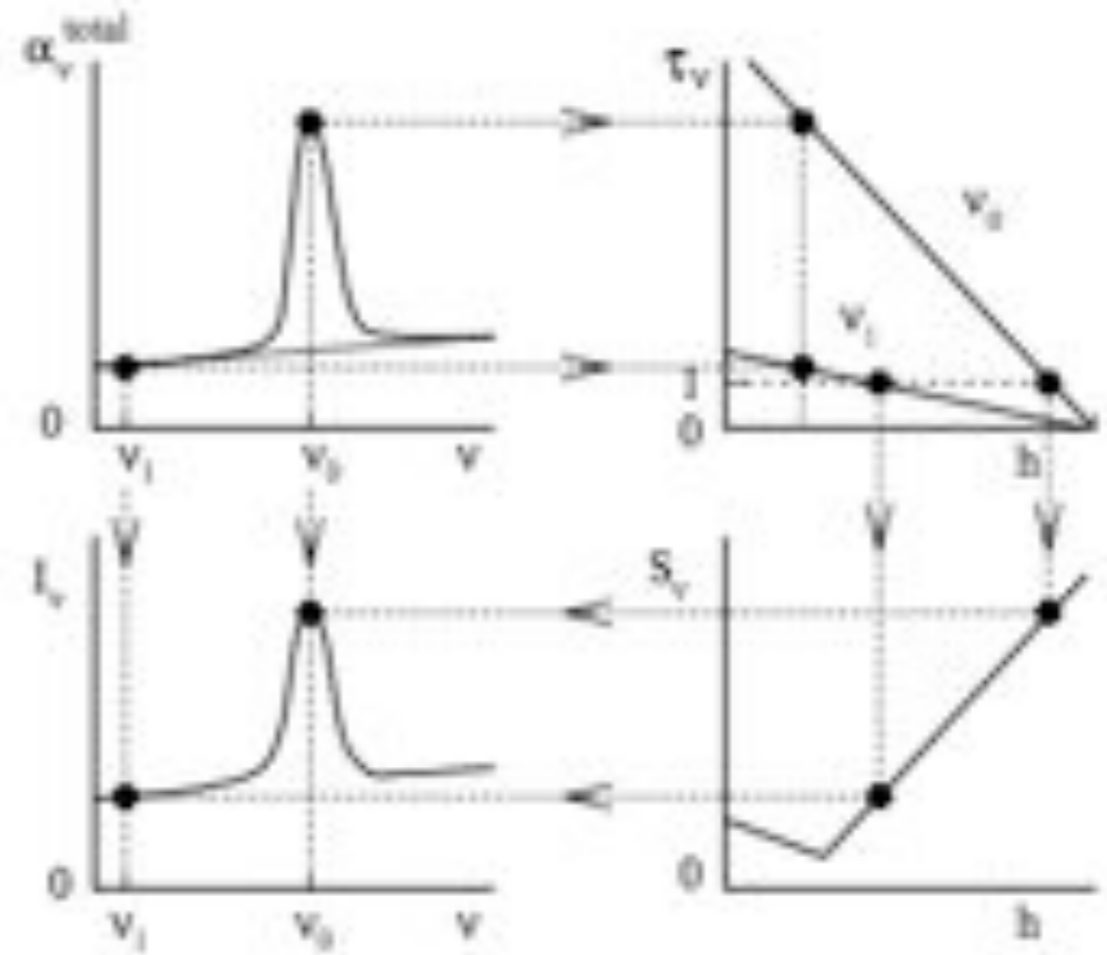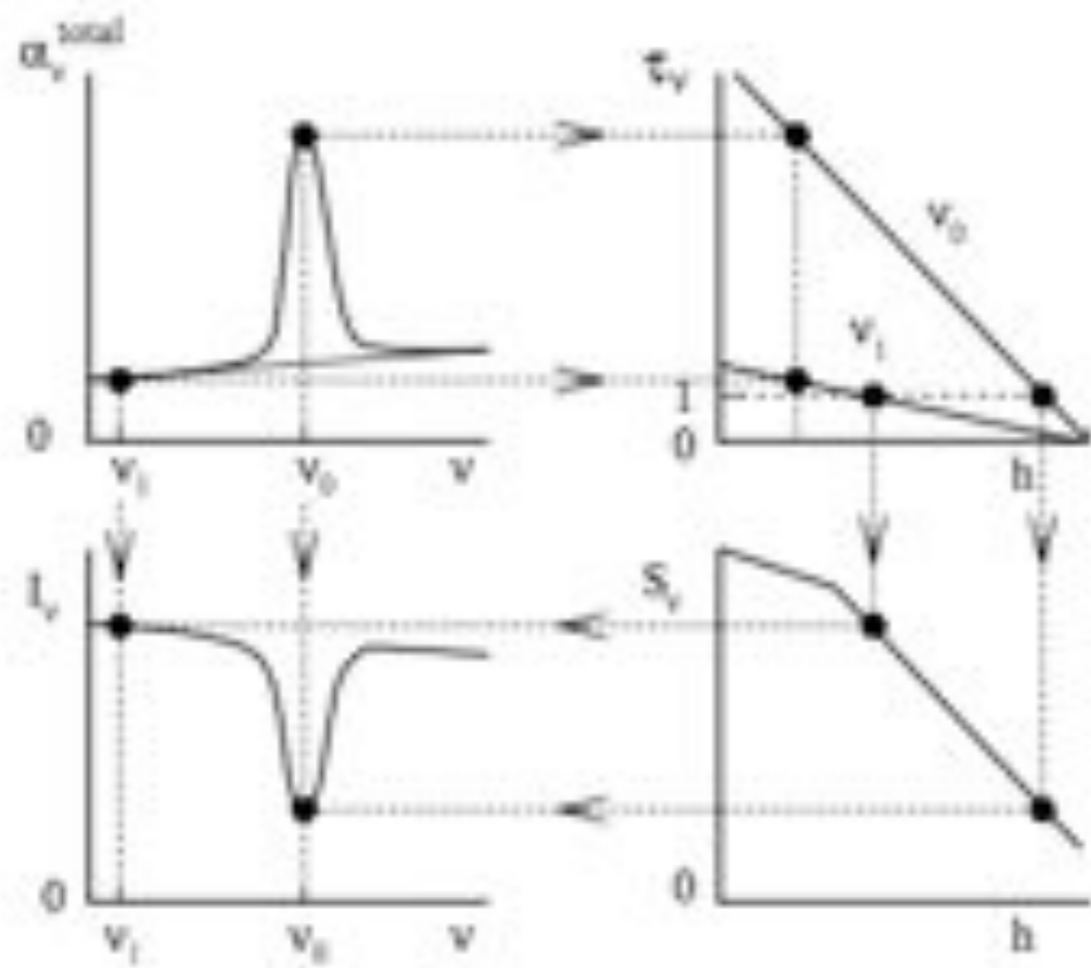
# Line Opacities and broadening

* We must differentiate between continuum (total) opacity and line opacity

* How much the overall spectrum is reduced across the board vs how much single lines are reduced

* Think the reduction in the blackbody spectrum vs a single line like the Hydrogen Balmer line

# Line Opacity

$$k_\nu \rho = \frac{\pi e^2}{m_e c} f_{ij} n_i \psi_\nu = \alpha(\nu) n_i$$

* $f_{ij}$ is the oscillator strength and ranges between 1 and 0, describes the probability of a transition from lower level i to upper level j

  * determined experimentally or from quantum mechanics

    * Take Quantum 2 if you want to see how this is done

* $\psi_\nu$ is the line profile function, it may be thought of as the likelihood of an atom to absorb a photon of frequency nu

  * Essentially $n_i \psi_\nu$ is the number density of atoms in state i that are able to absorb a photon of frequency nu and, as a consequence, able to re-emit the photon later

# Line Profile
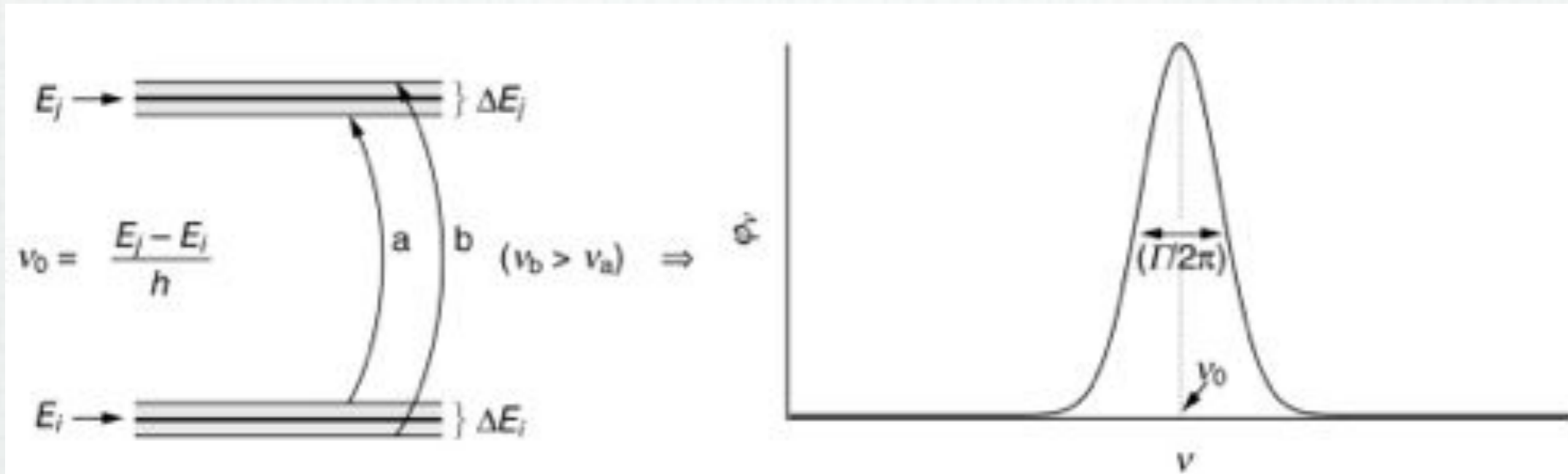


**Figure 4.4** Illustration of the effect of the width of atomic energy levels due to the uncertainty principle on the profile of an atomic transition between levels $i$ and $j$. Here, the energy levels $E_i$ and $E_j$ are those obtained by Schrödinger's equation, while $\Delta E_i$ and $\Delta E_j$ are the corresponding uncertainties predicted by Heisenberg's uncertainty principle. The value $\Gamma/2\pi$ represents the full width of the profile at half-intensity.

# Line Profiles

* Several physical effects can change the line profile

  * Natural broadening

  * Doppler broadening

  * Pressure broadening

  * Rotational broadening

  * There are more, B fields, turbulence, etc

  * We'll concern ourselves with a few

  * Since this is essentially a probability distribution function

$$\int_0^\infty \psi_\nu d\nu = 1$$

# To understand spectra

* Need to include all effects and calculate line profiles then fit them

# Natural Broadening

* Heisenberg uncertainty principle leads to a broadening of photon frequencies able to interact with an atom in a given excitation state

$$\Delta E \Delta T \geq \frac{h}{4\pi}$$

* Leads to the Lorentz profile $\psi_\nu = \dfrac{\frac{\Gamma}{4\pi^2}}{(\nu - \nu_0)^2 + (\frac{\Gamma}{4\pi})^2}$

$$\Gamma = \sum_{n'}^{n} A_{nn'}$$

* Where $\Gamma$ is the radiative damping constant and $\dfrac{\Gamma}{2\pi}$ is the full width at half maximum of the line profile.

* The maximum of the line profile reached is $\psi(\nu_0) = \dfrac{4}{\Gamma}$ and at the half maximum frequency is $\psi(\nu_{hm} = \nu_0/2) = \dfrac{2}{\Gamma}$

* **Leading to a frequency dependent cross section**

$$\alpha(\nu) = \frac{\pi e^2}{m_e c} f_{ij} \frac{\frac{\Gamma}{4\pi^2}}{(\nu - \nu_0)^2 + (\frac{\Gamma}{4\pi})^2} \rightarrow \kappa_\nu \rho = \alpha(\nu) n_i$$

* **For general line the lifetime is $10^{-8}$ seconds** $\Delta\nu \sim 10^8 hz \rightarrow \Delta\lambda \sim 10^{-4} angstroms$

* **Meaning natural broadening only plays a small role, other mechanisms contribute much more**

# Doppler Broadening

* The thermal motions of particles in an ideal gas or in LTE follow a Maxwellian distribution

* Essentially, particles that are characterized by some temperature T suffer collisions with each other that tend to impart momentum to some at the expense of the momentum of others

* This distribution also depends on the mass of the particles, heavier particles transfer momentum less effectively as their inertias are greater

# Maxwellian Distribution

* The Maxwellian distribution $F(v_x,v_y,v_z)$ represents the probability of finding a particle with a given velocity.  It is normalized to one

* For N particles the number with a given velocity is $N\ F(v_x,v_y,v_z)$

* This may be derived from statistical mechanics

* The probability of finding a particle with velocity vector $\vec{V}$ is

$$F(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} e^{\frac{-m(v_x^2 + v_y^2 + v_z^2)}{2k_b T}}$$

* More generically we are interested in finding the probability of finding a particle with a speed in between V and V+dV in a given volume. This is derived by recasting the above equation in terms of V and multiplying by a spherical shell of width dV in velocity space. This gives

$$F(V) = 4\pi V^2 \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} e^{\frac{-mV^2}{2k_b T}}$$

* Where $\quad V^2 = v_x^2 + v_y^2 + v_z^2$

# Different likely speeds

* Most probable speed comes from maximizing above function $V_p = \sqrt{\dfrac{2k_bT}{m}}$

* The mean or average velocity comes from finding the expectation value of velocity

$$< V >= \int_0^\infty VF(V)dV = \sqrt{\frac{8k_bT}{\pi m}} = \frac{2}{\sqrt{\pi}}V_p$$

* The root mean square speed

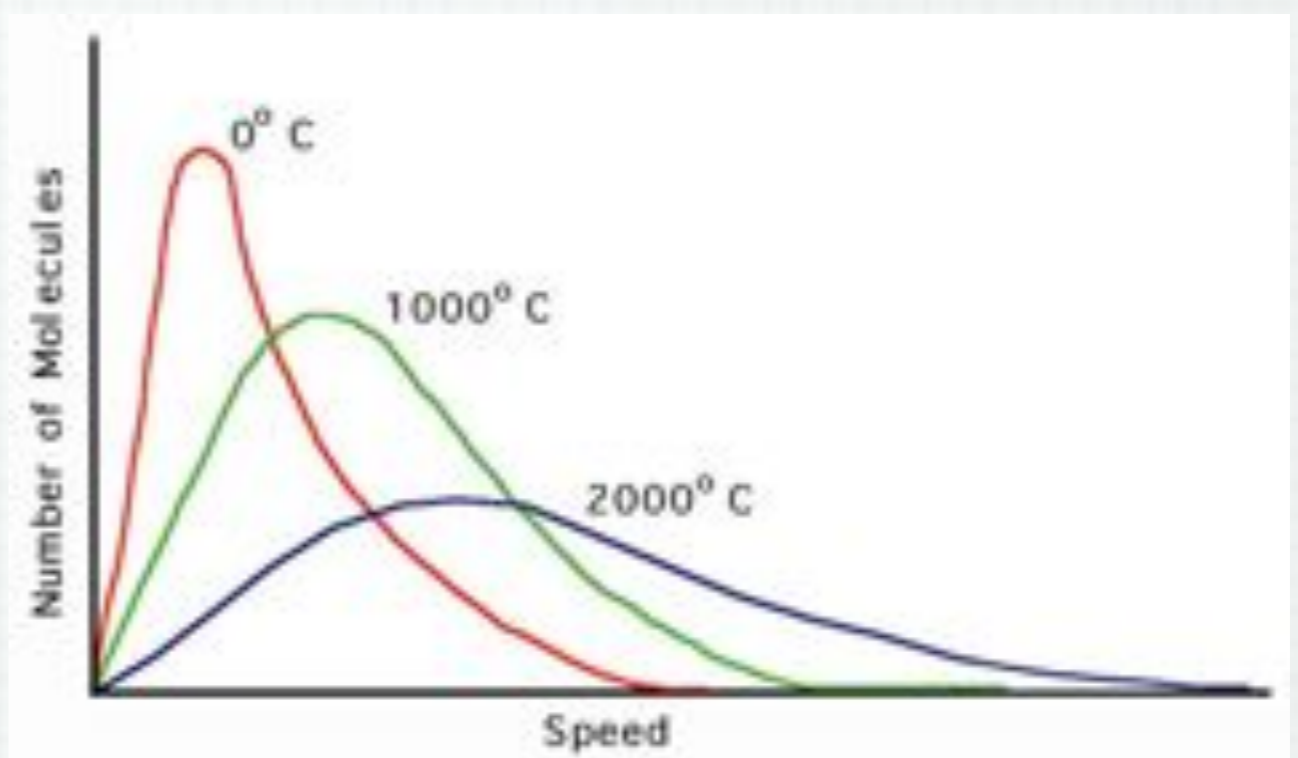$$V_{rms} = [\int_0^\infty V^2F(V)dV]^{1/2} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3}{2}}V_p$$
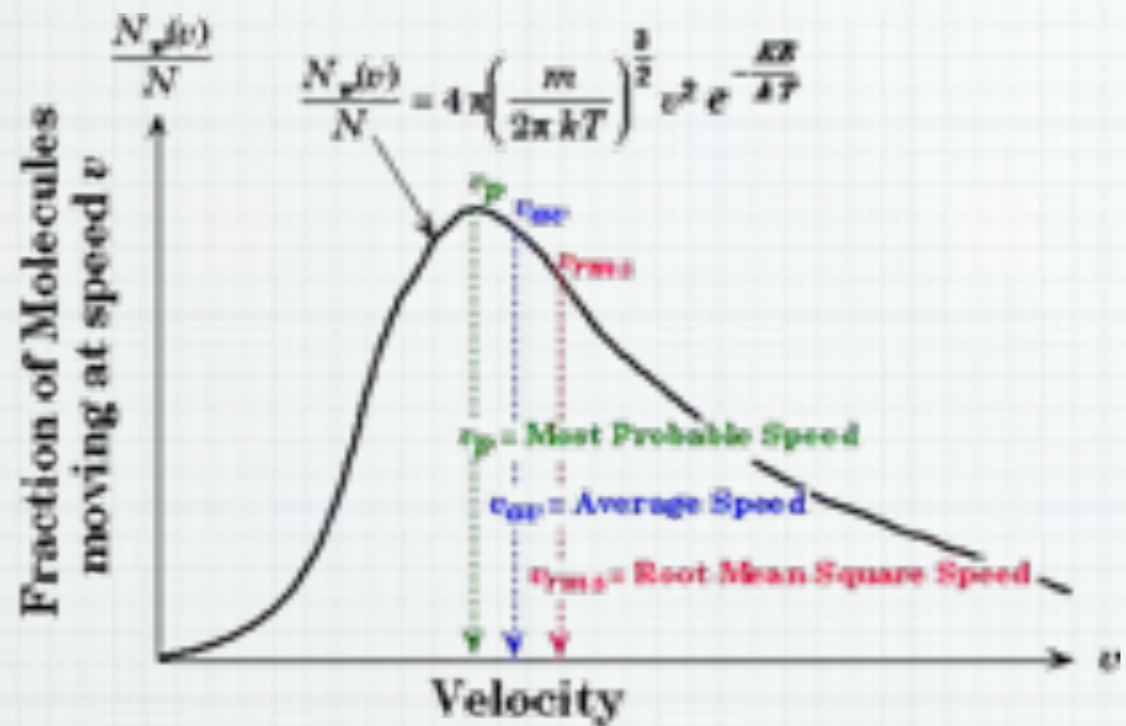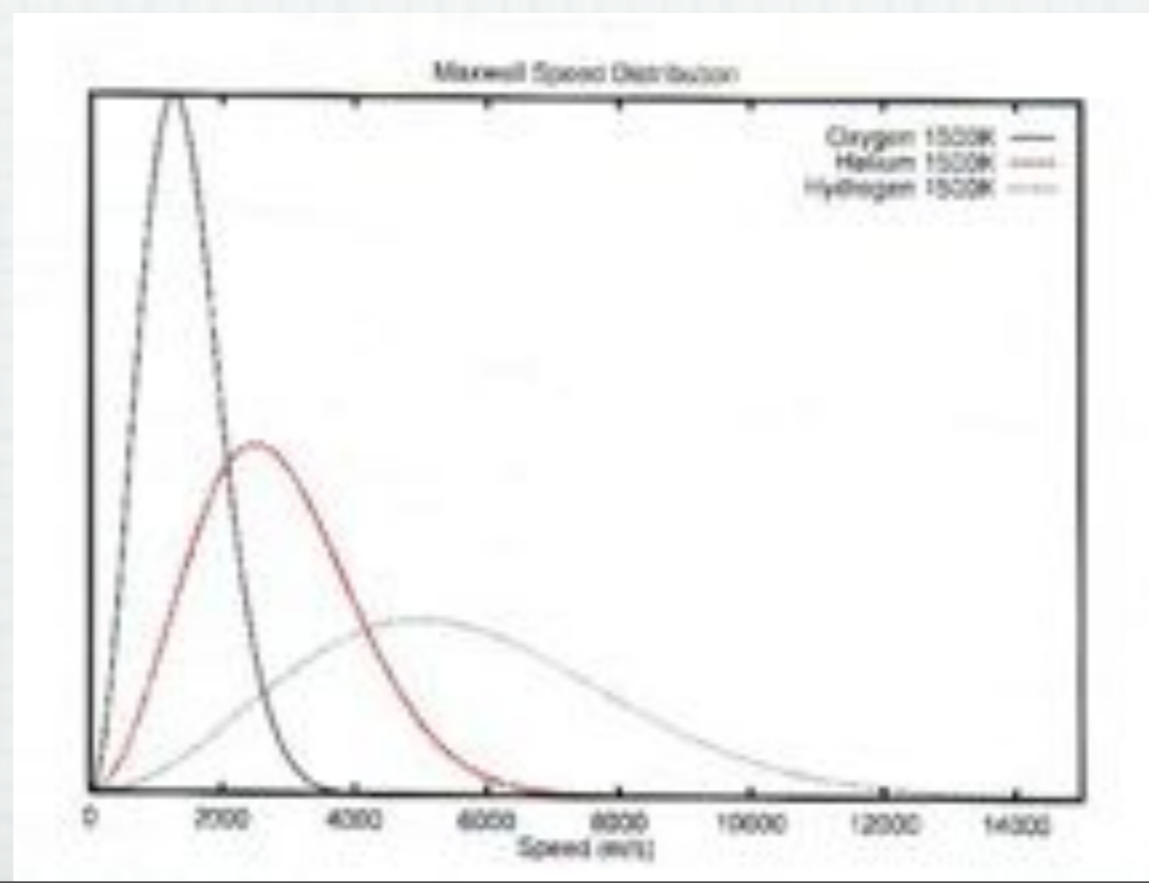
# Why this is important

* The frequency of light that a moving particle sees is doppler shifted due to its velocity and the particle then sees a frequency other than the rest frame frequency of the photon given by

$$\nu_{observed} = (\nu - \frac{\nu V}{c})$$

* So now our frequency dependent cross section is really dependent on the observed frequency of light

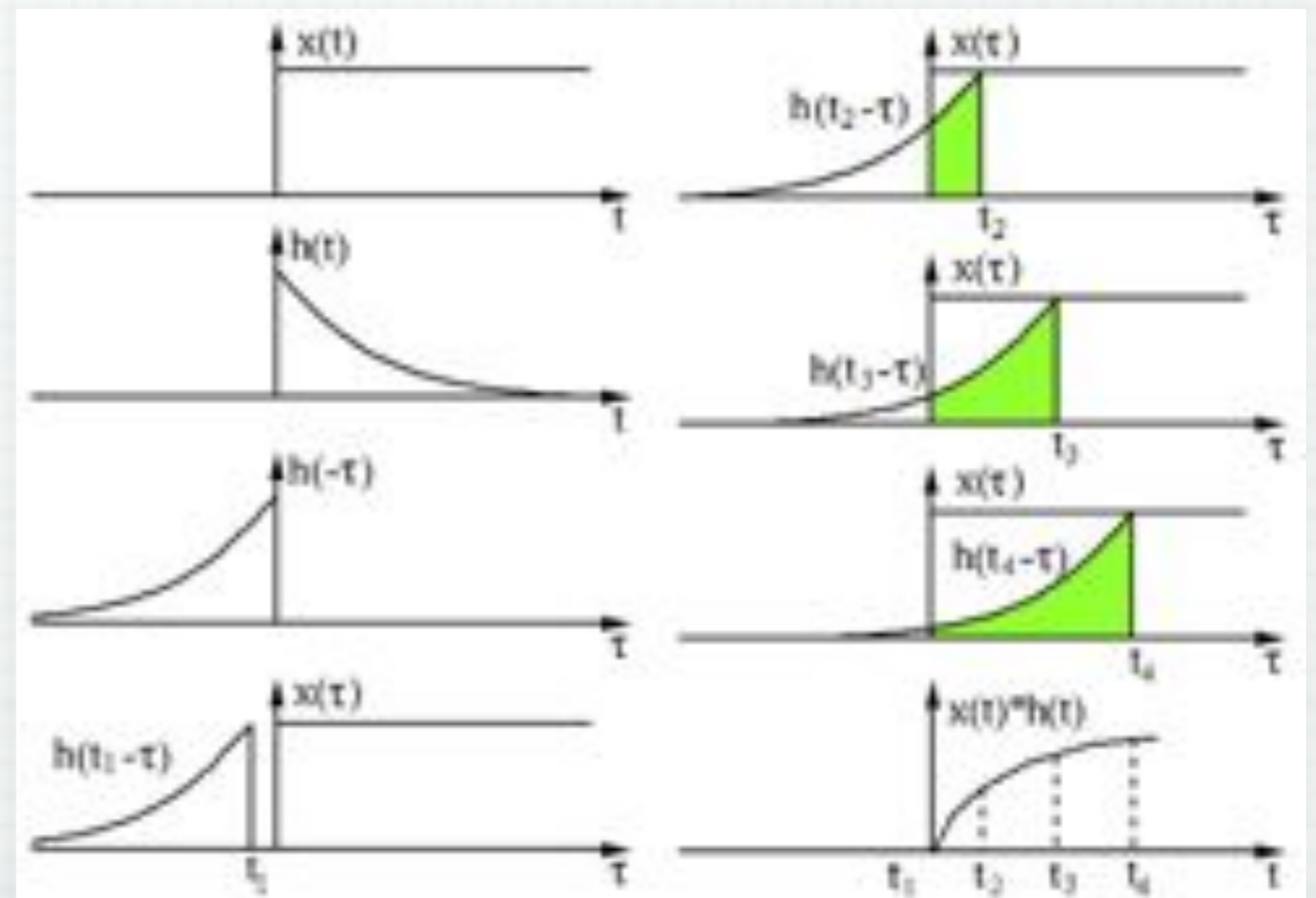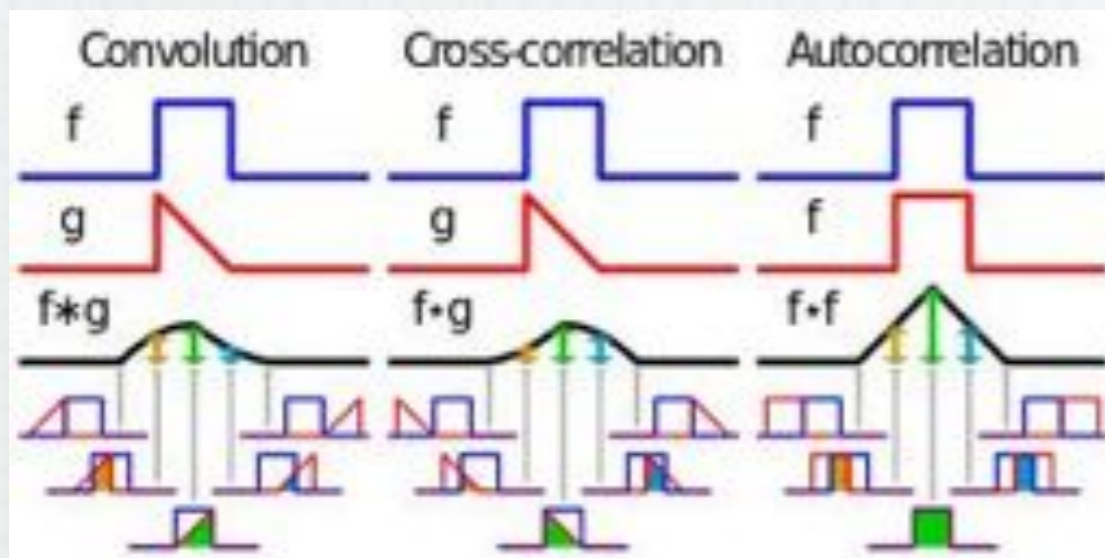* The true frequency dependent cross section is now a convolution of the natural cross section with the velocity distribution

$$\alpha(\nu) = \int_{-\infty}^{\infty} \alpha(\nu - \frac{\nu V}{c}) F(V) dV$$

# You should be scratching your head

* Convolution?

* Take two functions

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(x)g(t-x)dt$$

* The convolution is the measure of overlap as the y mirrored g function is translated across f

# Now the line profile is (with the following definitions)

$$\alpha(\nu) = \frac{\sqrt{\pi}e^2}{m_e c} f_{ij} \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ \frac{(\frac{\Gamma}{4\pi})e^{\frac{-V^2}{V_p^2}}}{(\nu - \frac{\nu V}{c} - \nu_{observed})^2 + (\frac{\Gamma}{4\pi})^2} \right] \frac{dV}{V_p}$$

$$\Delta\nu = \frac{\nu V}{c} \quad \Delta\nu_D = \frac{\nu_{observed} V_p}{c} \quad \nu = \frac{\nu - \nu_{observed}}{\Delta\nu_D}$$

$$y = \frac{\Delta\nu}{\Delta\nu_d} \sim \frac{V}{V_p} \quad a = \frac{\Gamma}{4\pi\Delta\nu_D}$$

* Leads to $\alpha(\nu) = \frac{\sqrt{\pi}e^2}{m_e c} f_{ij} \frac{H(a,\nu)}{\Delta\nu_d}$

* Where H is the Voigt function defined as

$$H(a,\nu) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(\nu - y)^2 + a^2} dy$$

* Normalizing the Voigt profile leads to

$$U(a, \nu) = \frac{H(a, \nu)}{\sqrt{\pi}}$$

* and a normalized line profile

$$\psi(\nu) = \frac{U(a, \nu)}{\Delta \nu_D}$$

* In the core doppler wins, in the wings, natural broadening wins
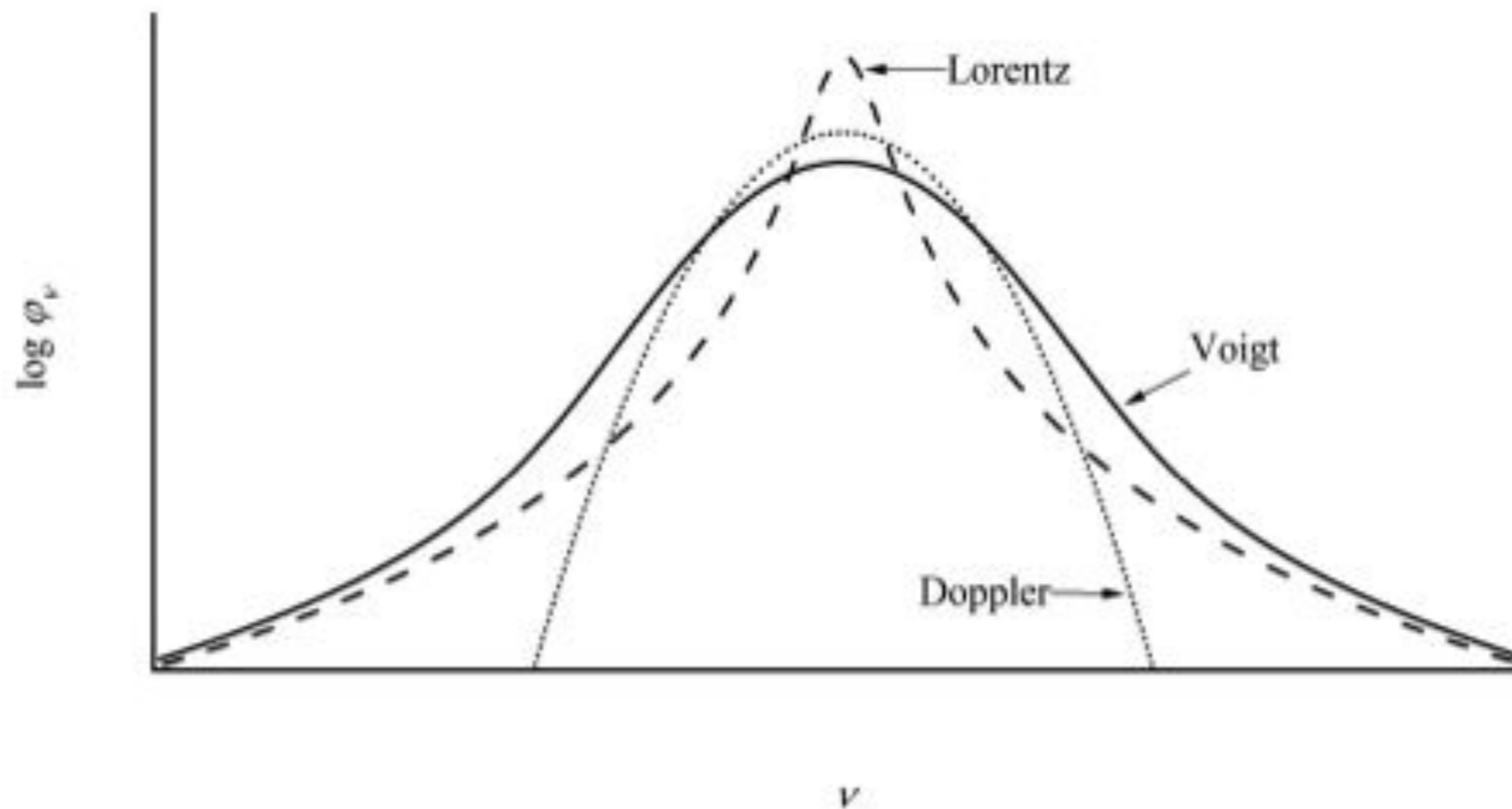


**Figure 4.6**   Lorentz, Voigt and Doppler profiles of a hypothetical atomic line.

# Turbulent Broadening

* If turbulent velocities (say due to convection) play a role in the motions of the particles we add this in quadrature to the definition of $\Delta\nu_D$ given above and it becomes $\Delta\nu_D = \frac{\nu_{observed}}{c}(V_p^2 + v_{turb}^2)^{1/2}$

* The line profile function still proceeds as calculated above

# The absorption profile depends on density
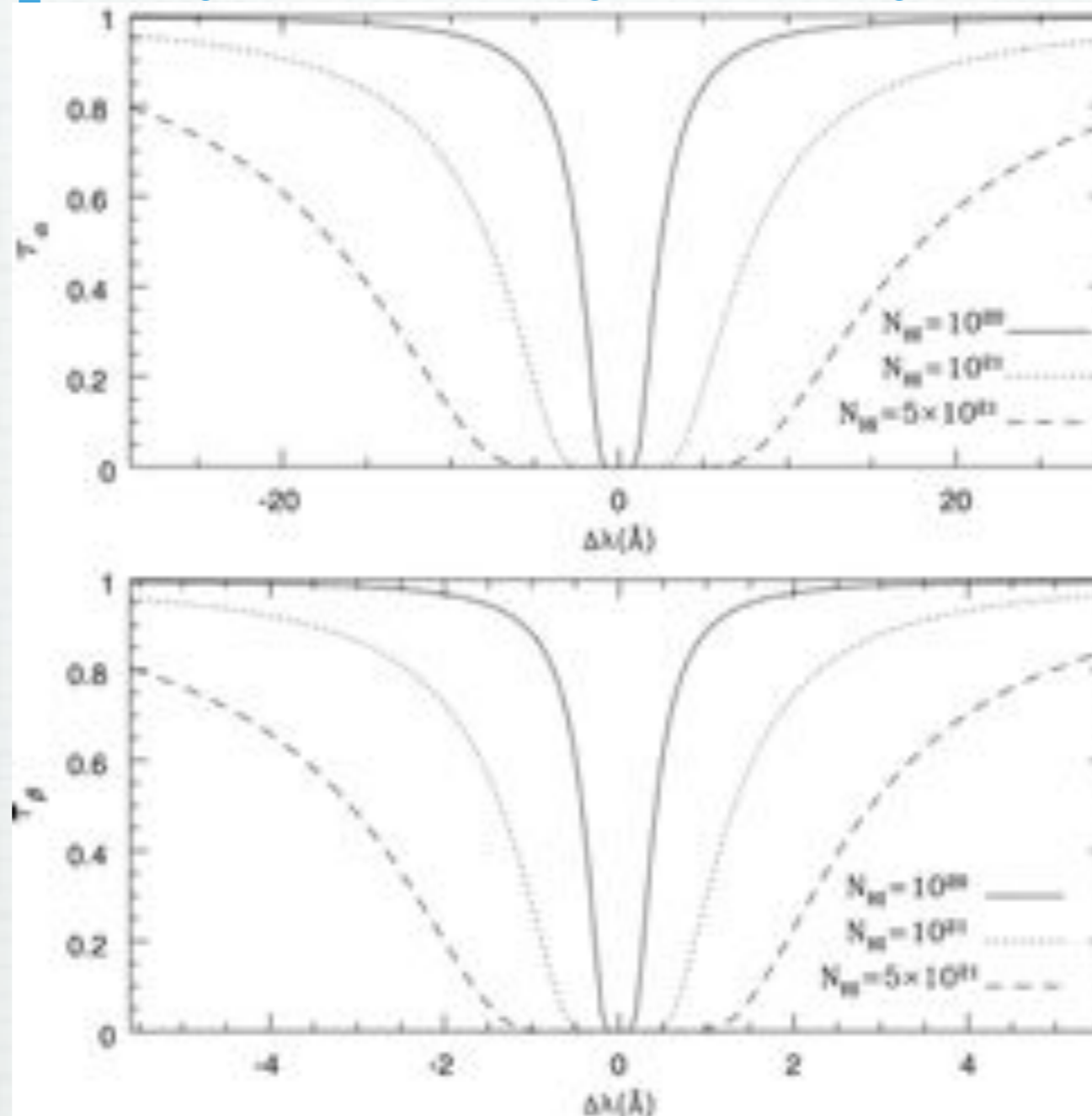
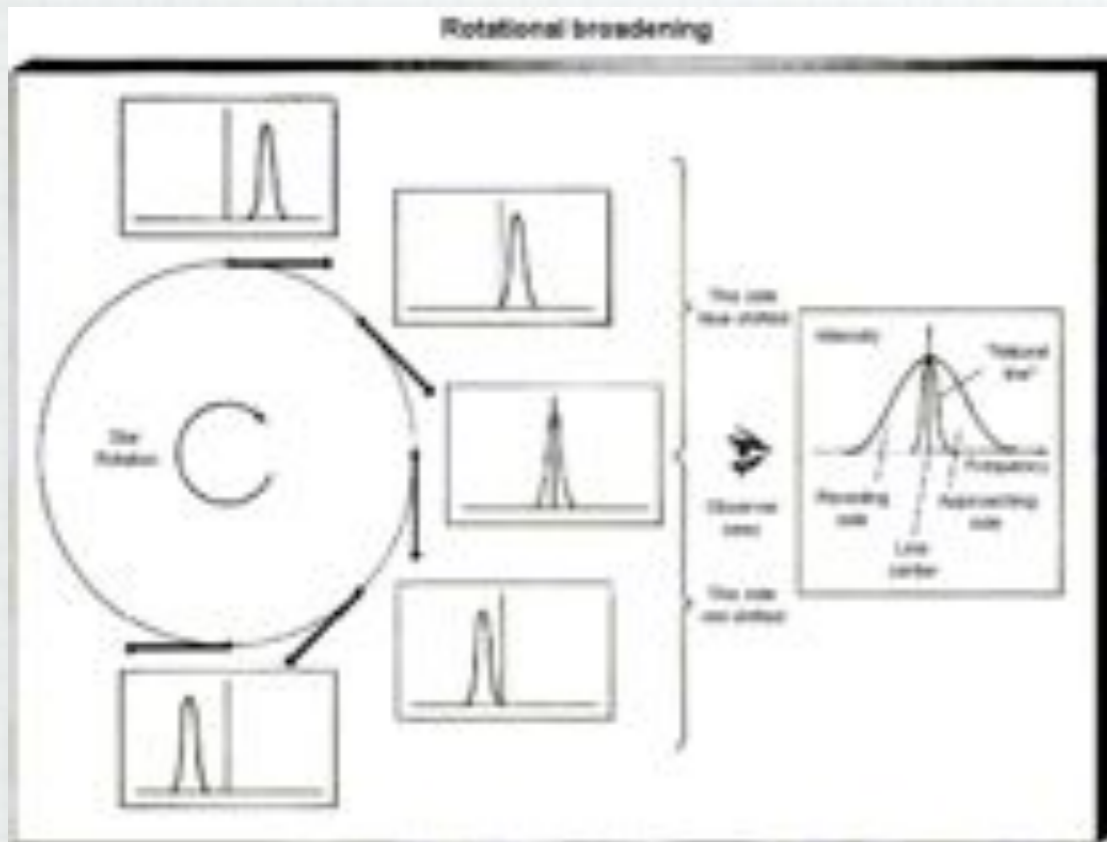# In Practice

* **There will be overlapping lines**

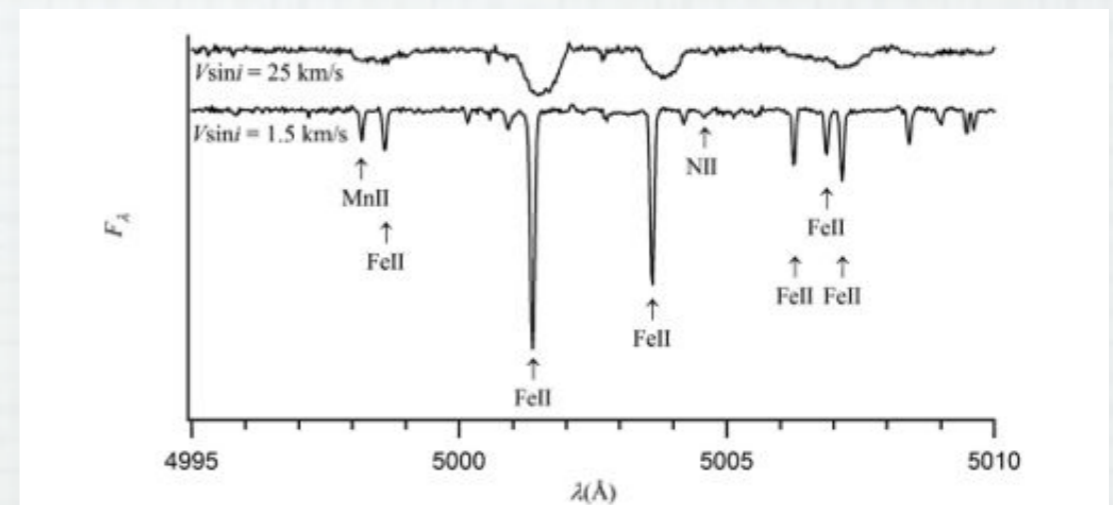* **Need to fit spectra with multiple Voigt profiles for different lines**

# Rotational Broadening



Rotational broadening
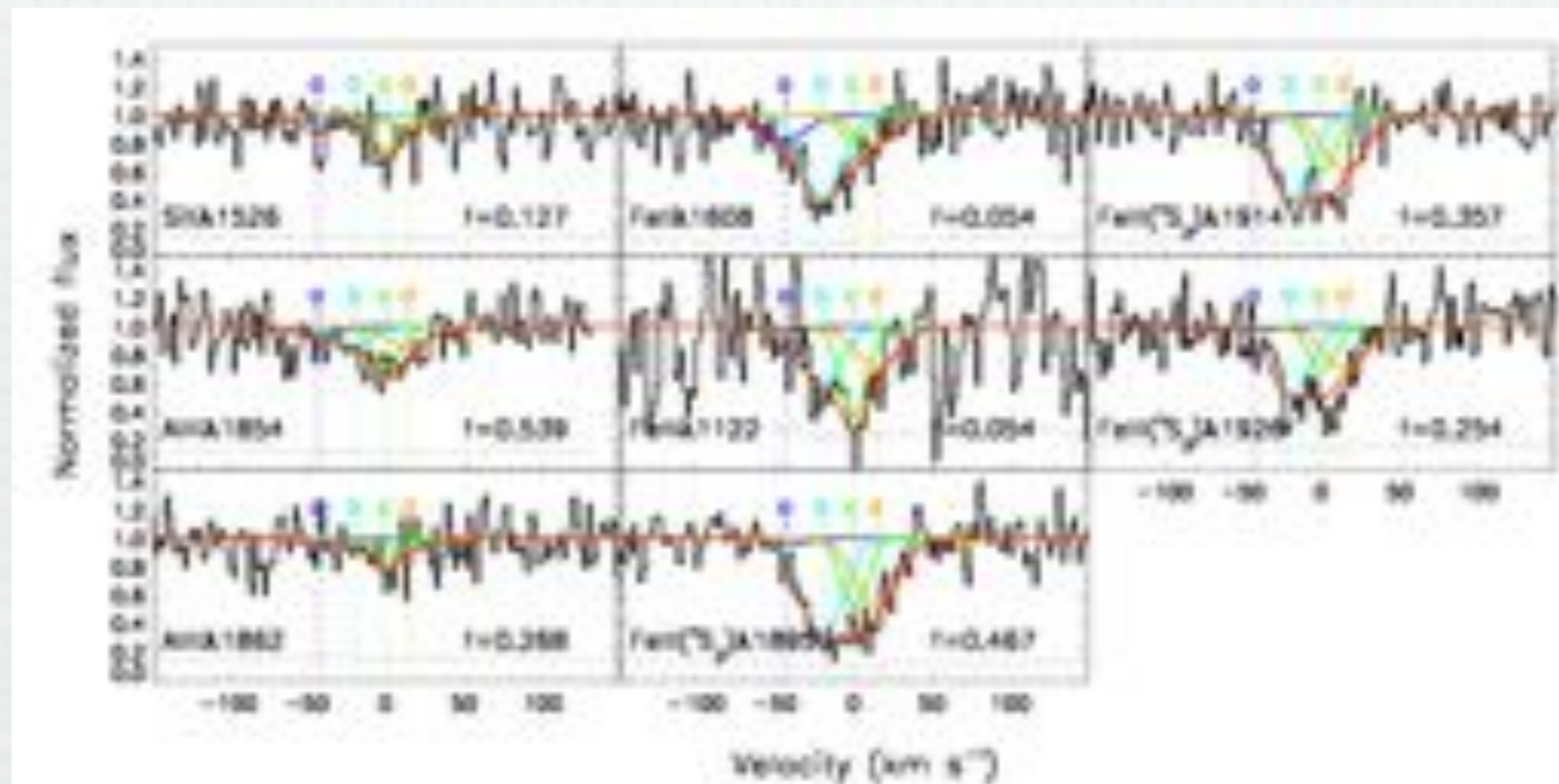


no rotation                          large rotation

**Figure 4.8** Illustration of the blending of two atomic lines. On the left, two atomic lines are shown in a spectrum of a star that does not rotate, while on the right, these two lines are blended together for a star with a large rotational velocity. Note that the blended line is no longer symmetric.



axis of rotation

equator

line-of-sight

$V \sin i$
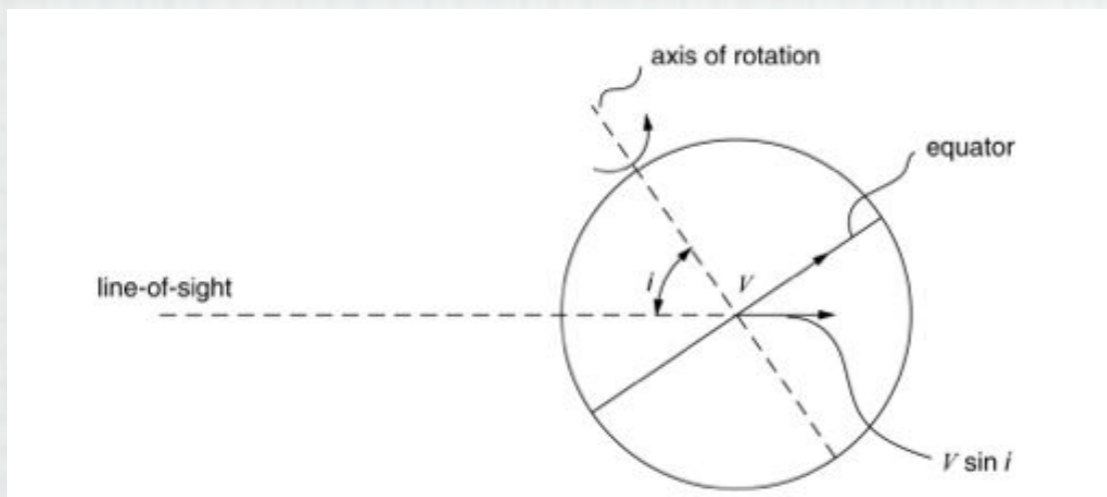
**Figure 4.9** Illustration showing that the component of the rotation velocity at the equator ($V$) along the line-of-sight is $V \sin i$ where $i$ is the angle between the axis of rotation of the star and the line-of-sight.



$V \sin i = 25$ km/s

$V \sin i = 1.5$ km/s

NII

MnII

FeII

FeII

FeII FeII

FeII

FeII

4995        5000        5005        5010

$\lambda(\text{Å})$

**Figure 4.10** Observed flux for two stars of similar effective temperatures ($T_{eff} \approx$ 12000 K) but with different rotational velocities. The upper curve represents a star that has a relatively large rotational velocity ($V \sin i = 25$ km/s) while the star represented by the lower curve has a small rotational velocity ($V \sin i = 1.5$ km/s). The two spectra are horizontally shifted to distinguish the two curves. The lines from several ions are identified in the figure. These spectra were extracted from the European Space Organization (ESO) Archive, and were processed and kindly obtained from Mouhamadou Thiam.
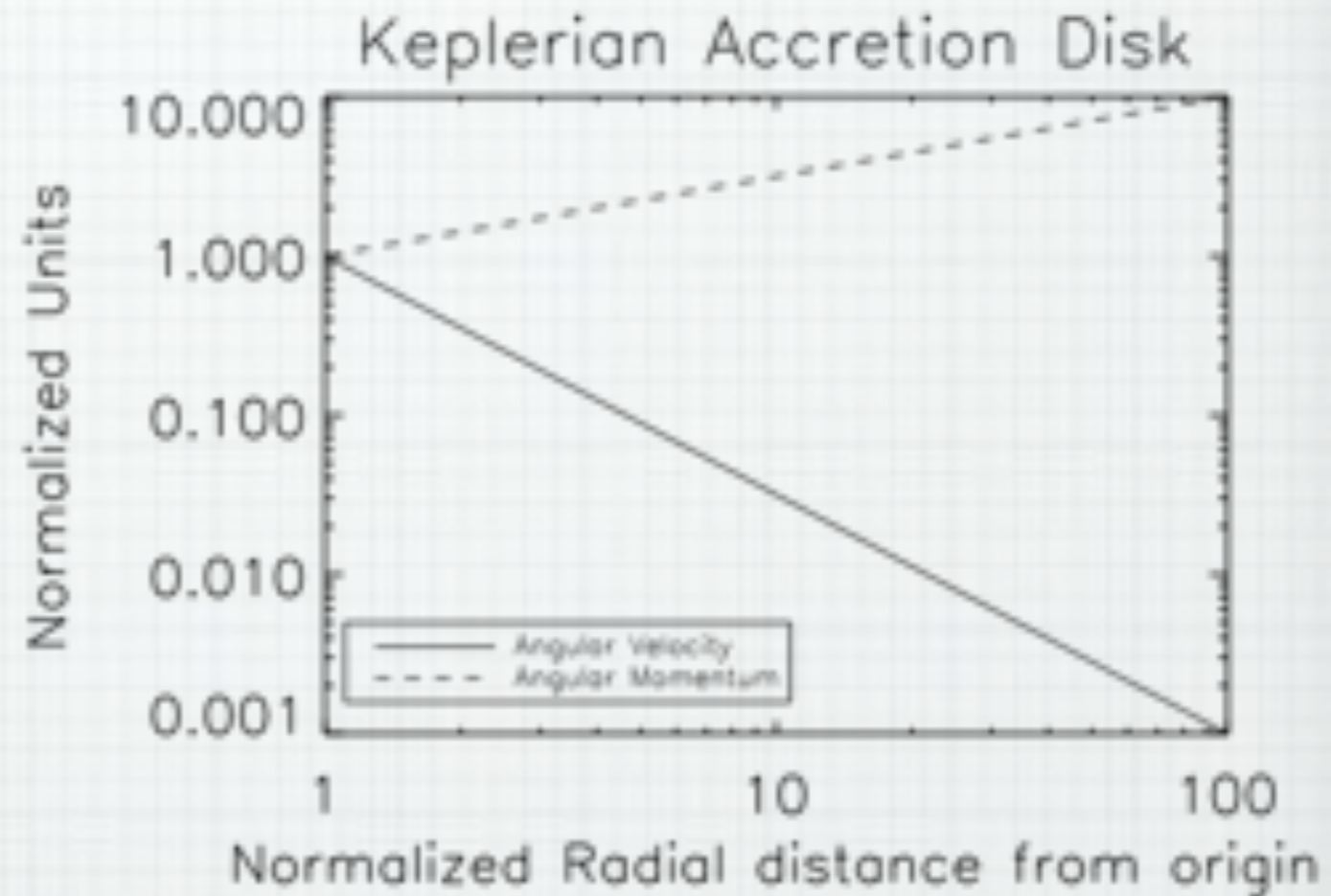
# Side Topic

* Three general types of astronomers

    * Theorists

    * Observers

    * Engineers

* Each compliments the other

    * brief description

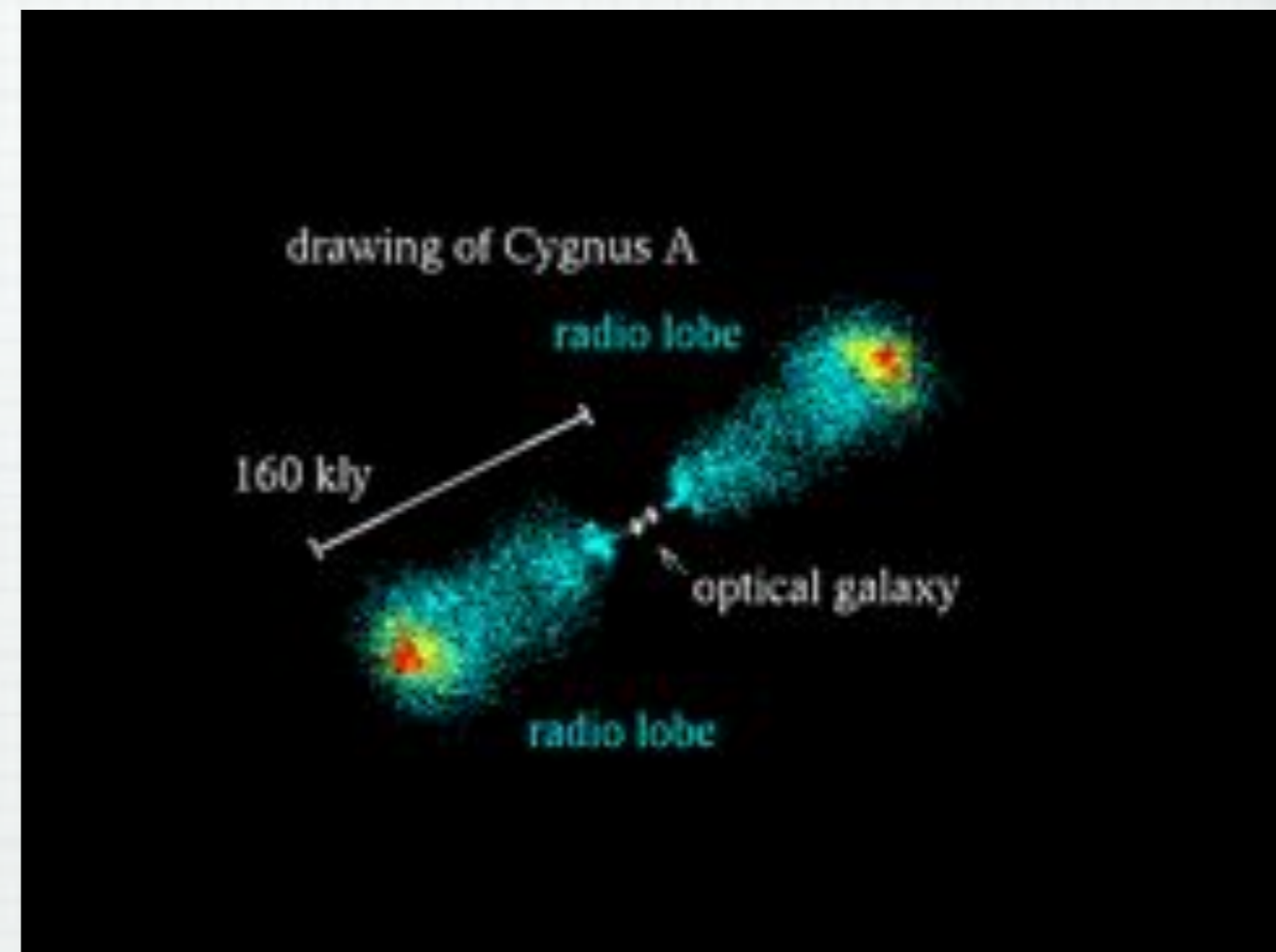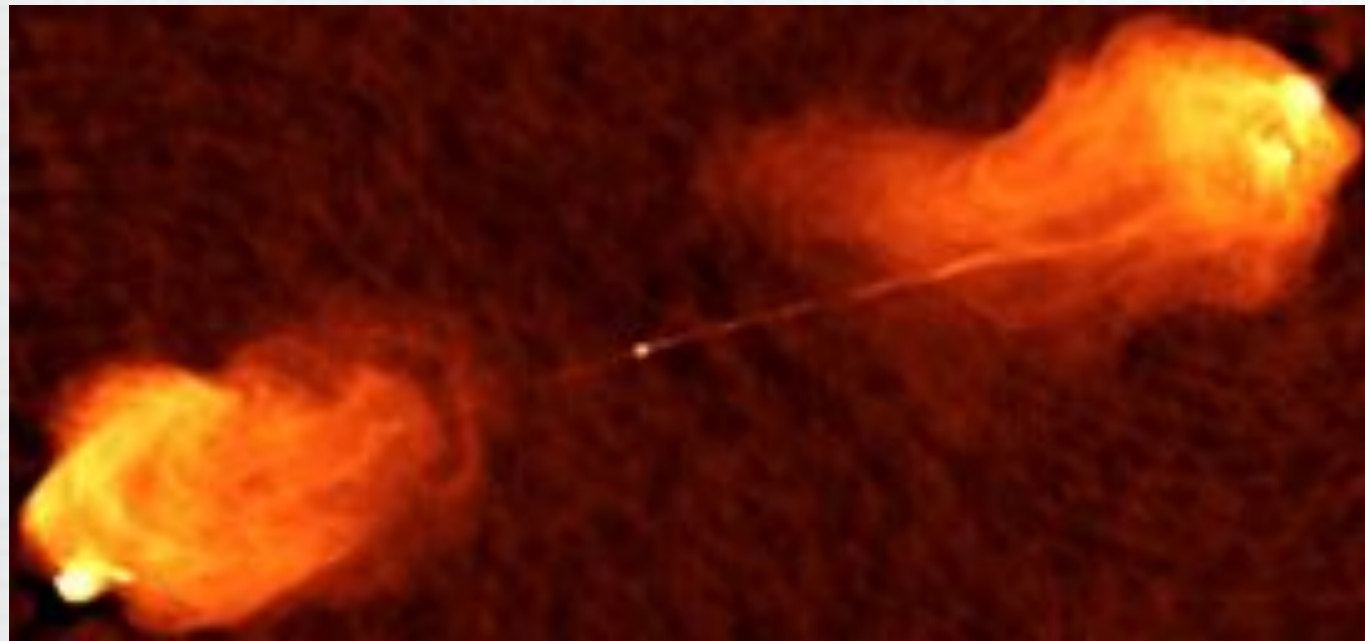# Accretion Disks



Keplerian Accretion Disk
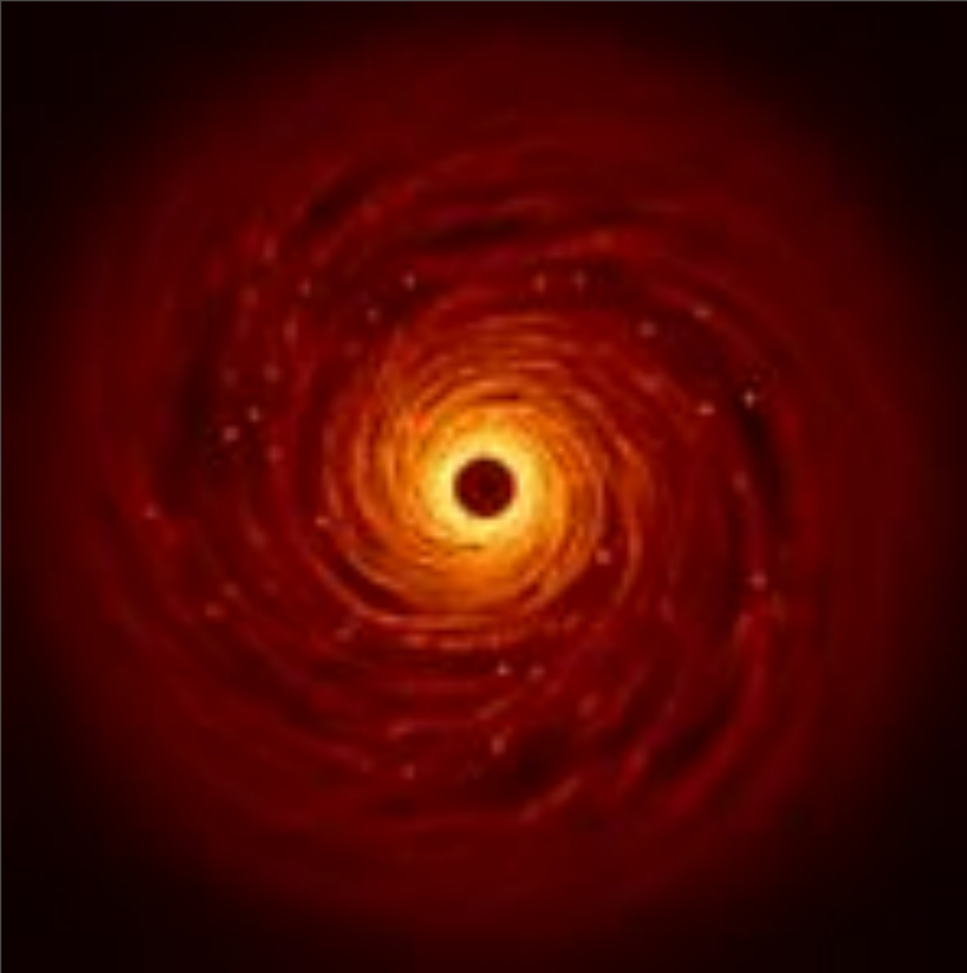
Rotating, weakly magnetized, shear flows of gas that occur in throughout space and are powered

* Form in binary systems when one object overflows it roche lobe

* Also forms in early galaxies

* Results in jets, novae, etc

* we may cover in more depth later

# Cygnus A





drawing of Cygnus A

radio lobe

160 kly

optical galaxy

radio lobe
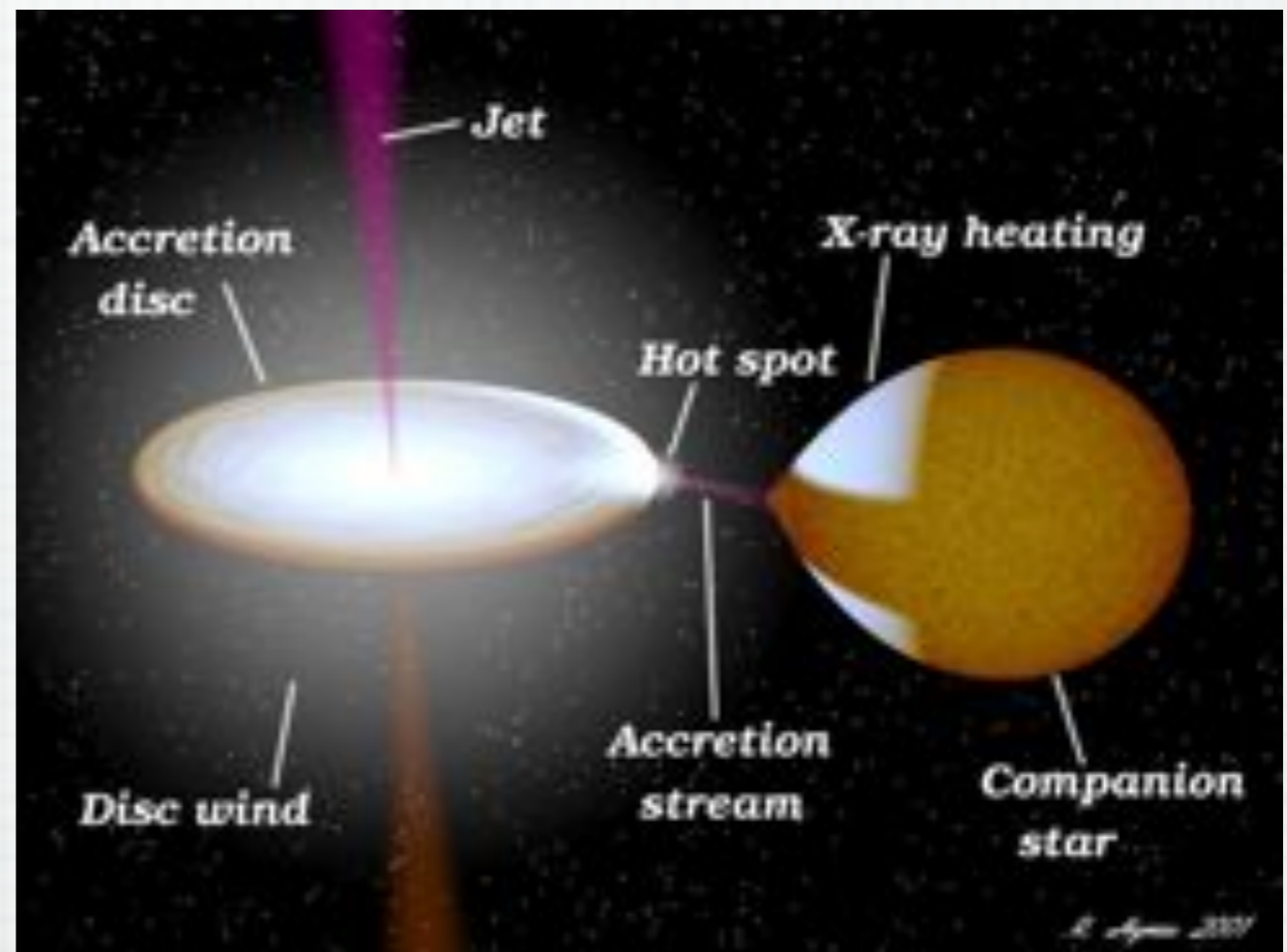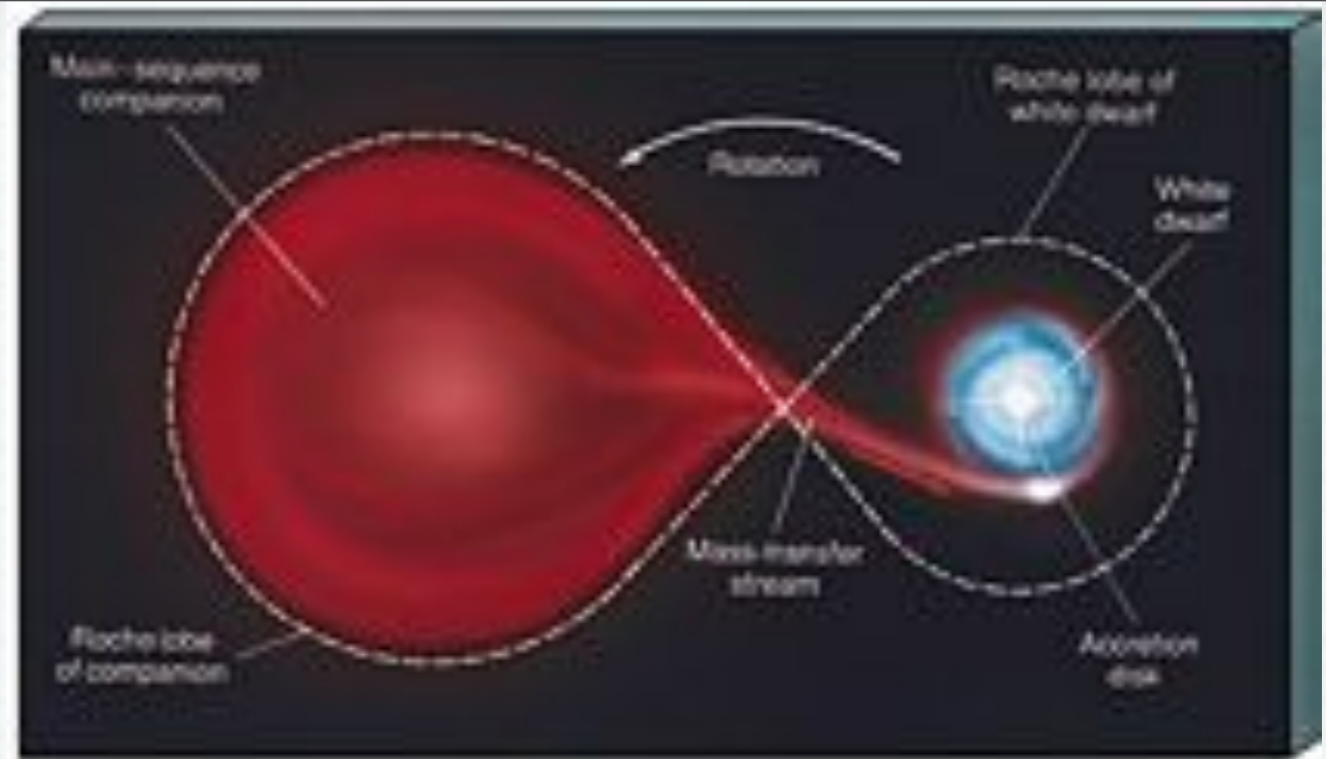
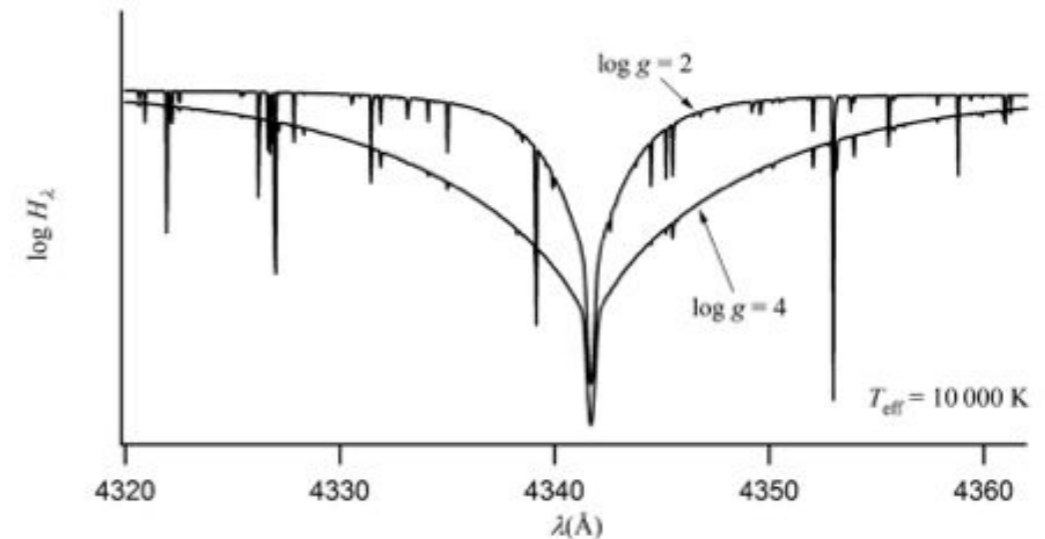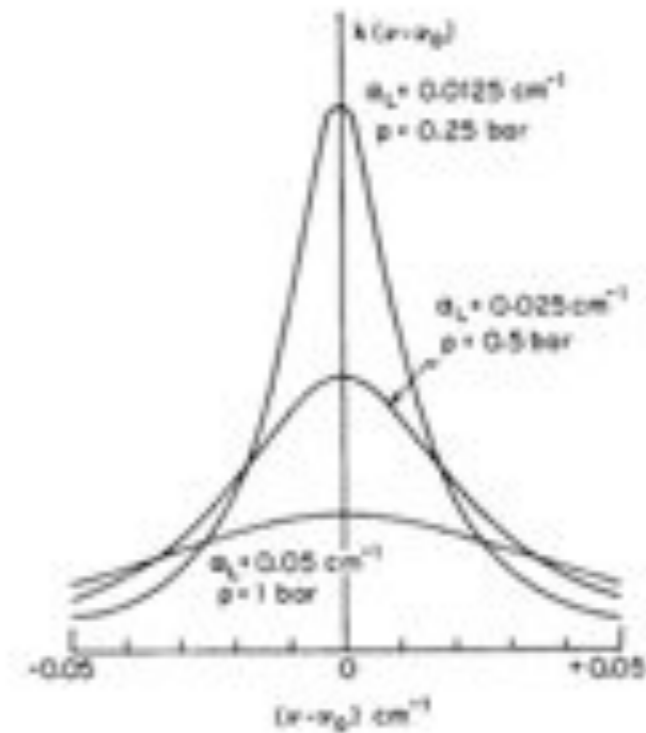# We apply the above line profiles modified to include GR
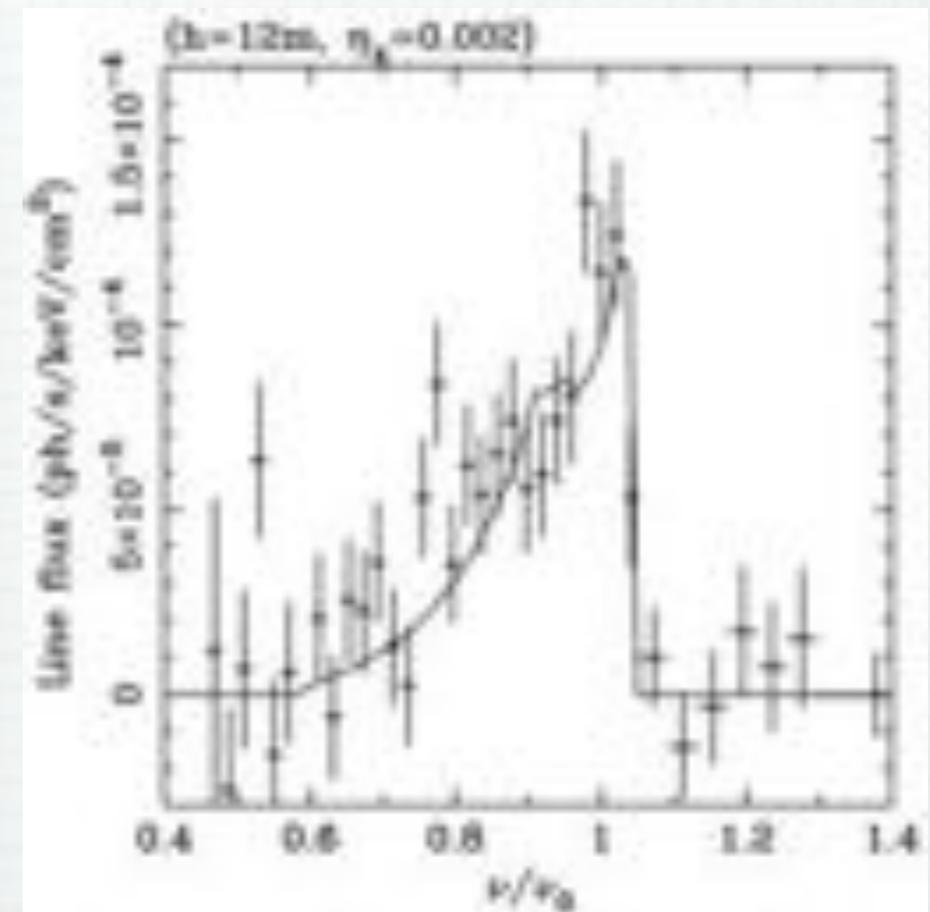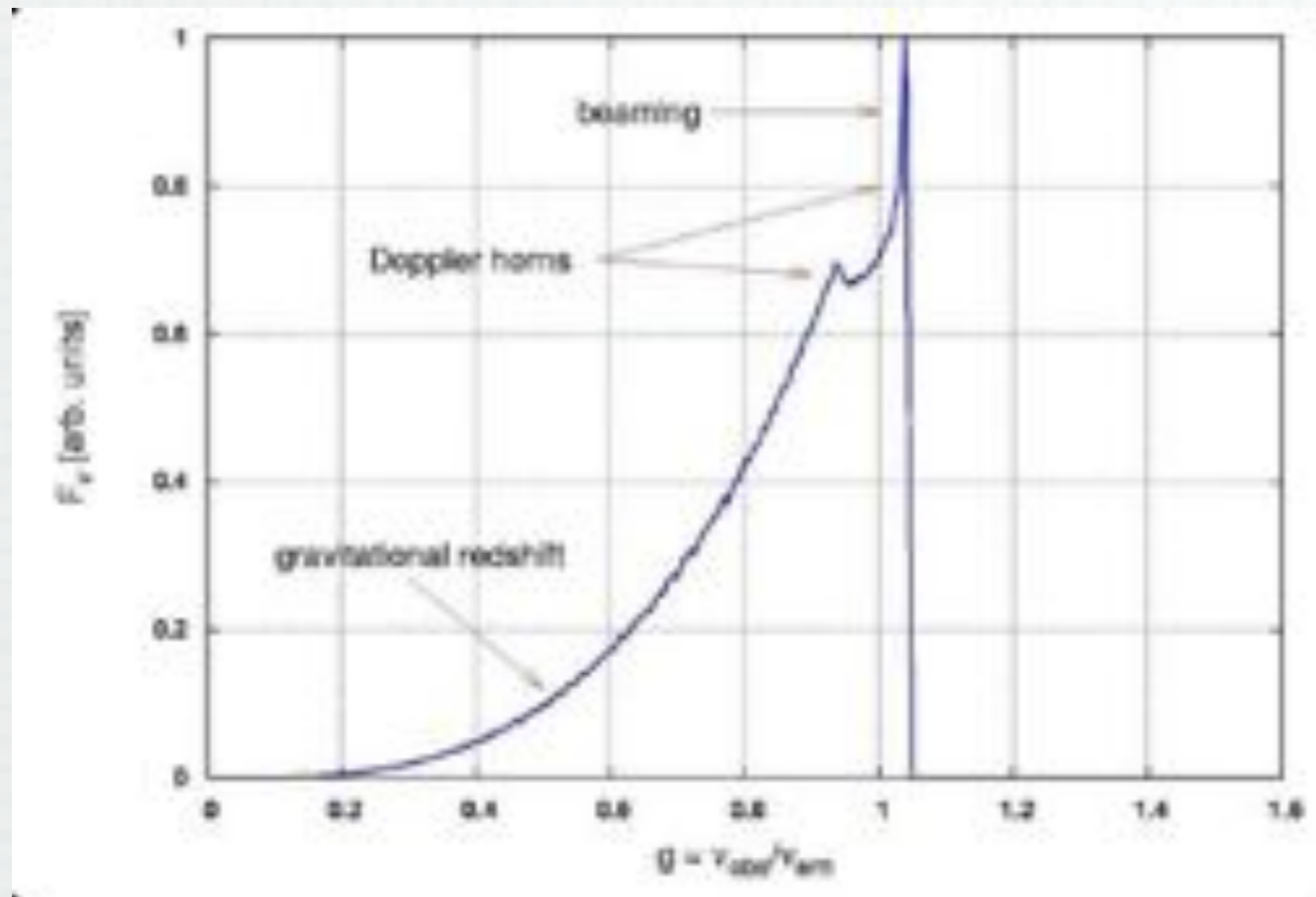


Allows us to probe GR effects, determine mass, rotation, etc.

# Back on topic
# Pressure Broadening

* Above processes only include electric potential

* In a star, collisions also come into play

* Electric field of colliders introduces Stark effect and van Der Waals effect



**Figure 4.11** The surface flux within the $H_\gamma$ line at the surface of atmospheres with $T_{eff} = 10\,000$ K but with different surface gravities typical of main-sequence (log $g = 4$) and supergiant (log $g = 2$) stars. Other atomic lines from various metals are also seen within the $H_\gamma$ line.

# Zeeman Effect
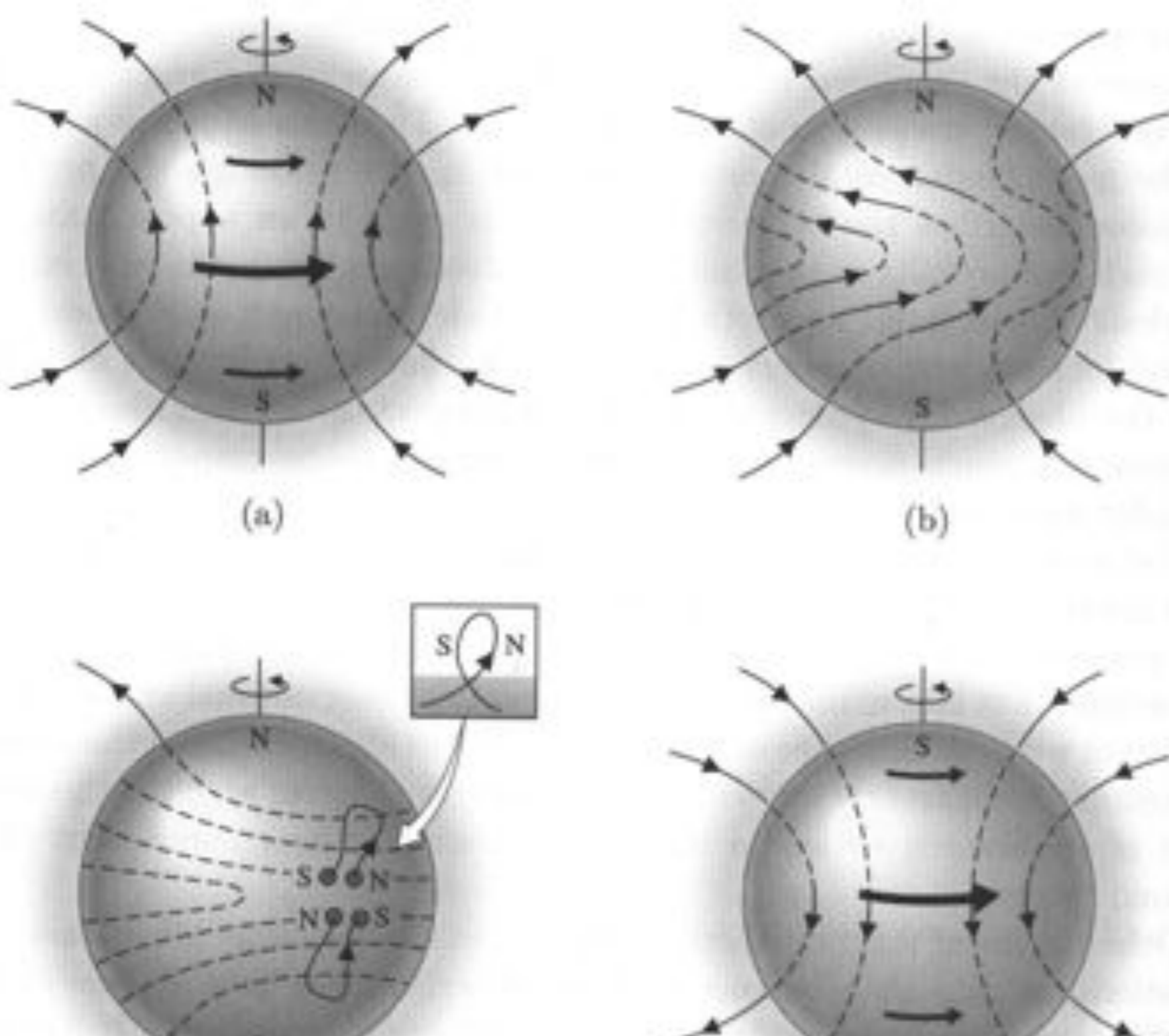


* Fields generated by dynamo action

* Twisted field lines lead to reconnection - magnetic energy goes into thermal energy

# Zeeman Effect

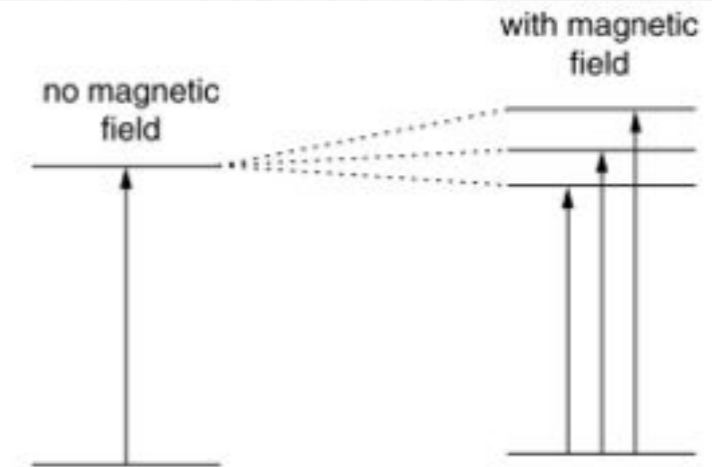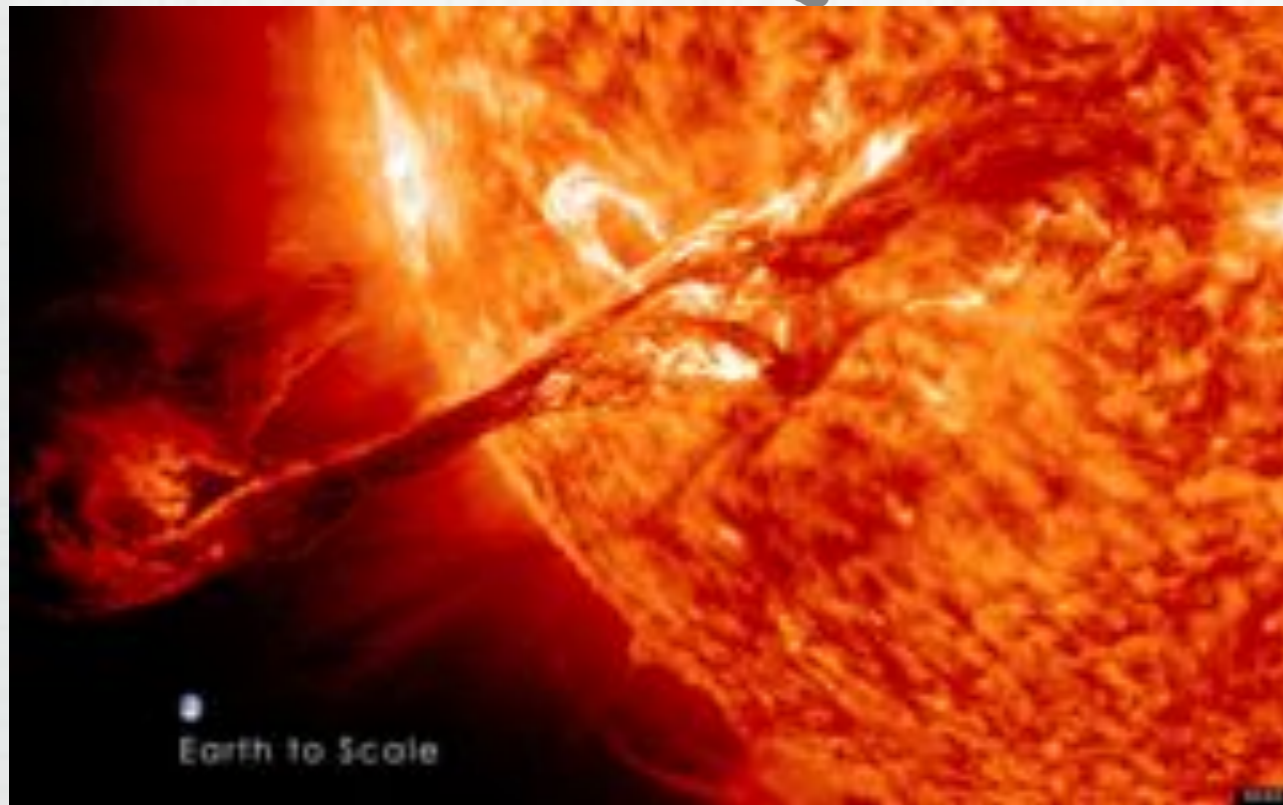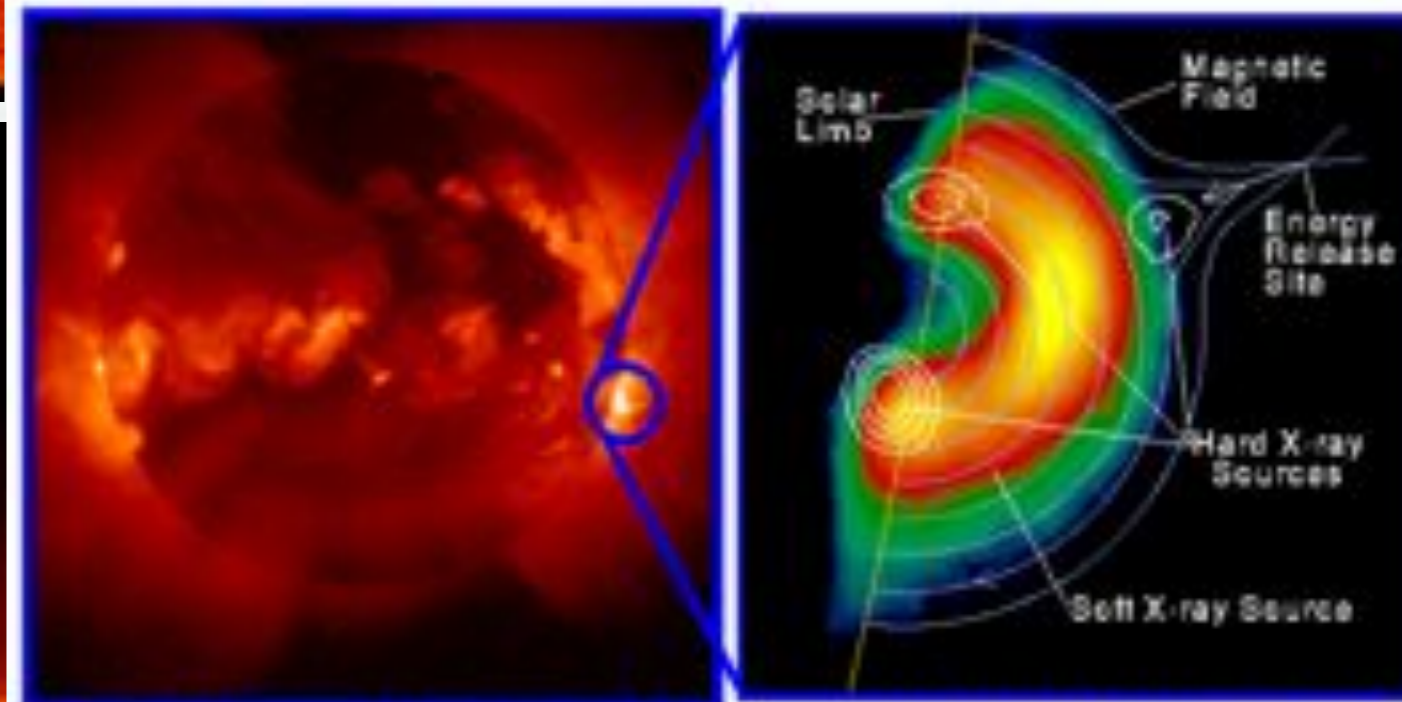* Lines split in the presence of an external magnetic field





**Figure 4.12** Illustration of Zeeman splitting of an atomic energy level and its effect on the atomic transitions.
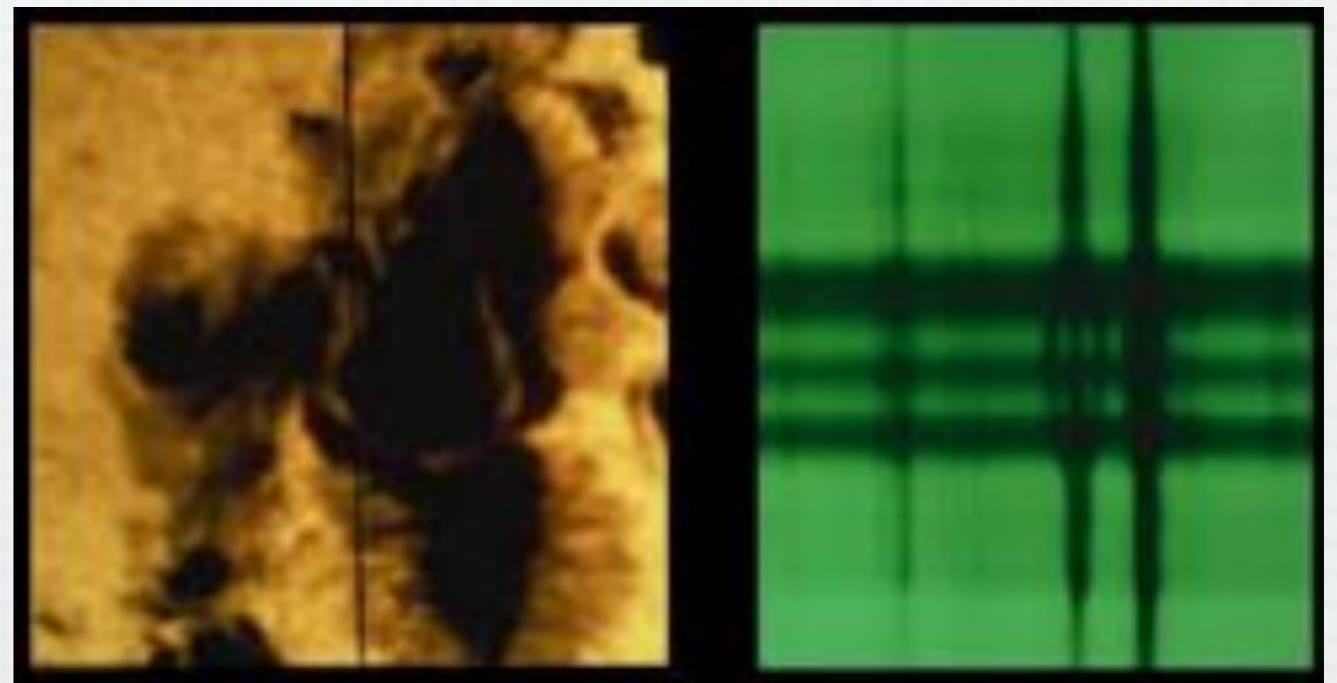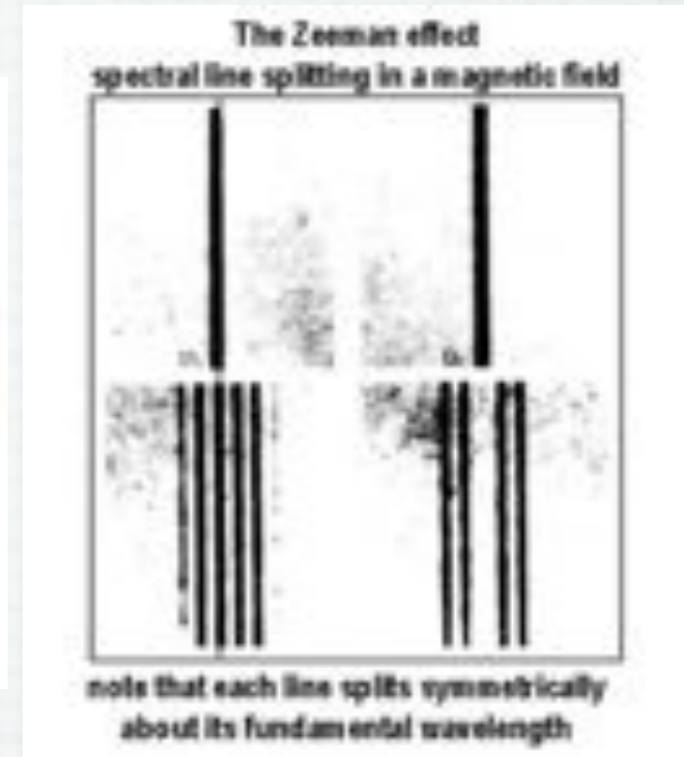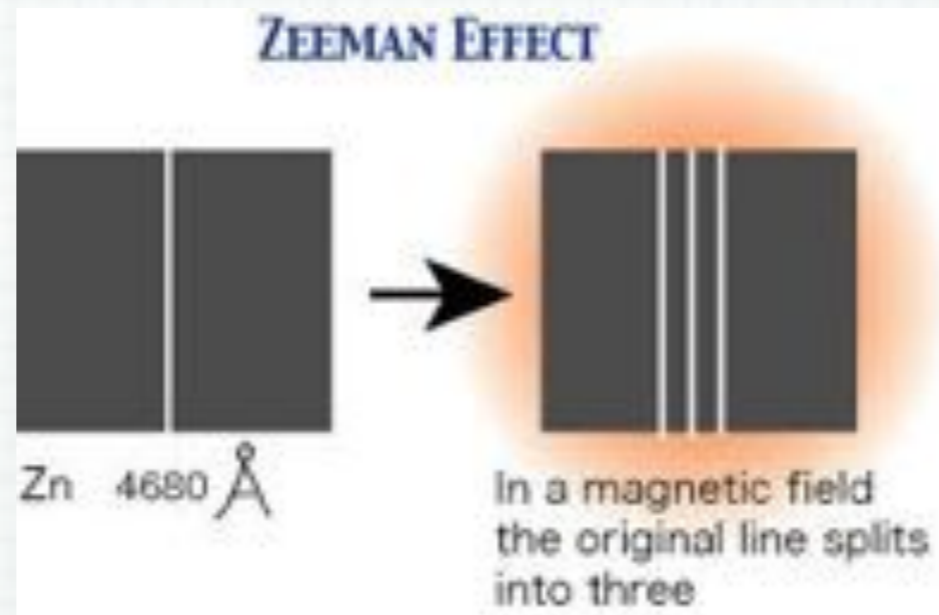


Yohkoh X-ray Image of a Solar Flare, Combined Image in Soft X-rays (left) and Soft X-rays with Hard X-ray Contours (right). Jan 13, 1992.

# Zeeman Effect

* Generally the split is too small to be resolved but still results in broadened lines which may be used to estimate field strength

* Solar magnetic field 50-400 micro Tesla

* Neutron stars magnetic field 108 Teslas

* Earth's magnetic field 25-65 micro Tesla

* Sunspot $10^{-1}$ Tesla

* NMR 23.5 Tesla



ZEEMAN EFFECT

Zn   4680 Å

In a magnetic field the original line splits into three



The Zeeman effect spectral line splitting in a magnetic field

note that each line splits symmetrically about its fundamental wavelength

# Einstein coefficients

* Stimulated absorption $B_{ij}$

  * Excites an electron, a photon comes in and is absorbed

* Spontaneous emission $A_{ji}$

  * Happens naturally, an electron de-excites and a photon is released

* Stimulated emission $B_{ji}$

  * Lasers/Masers
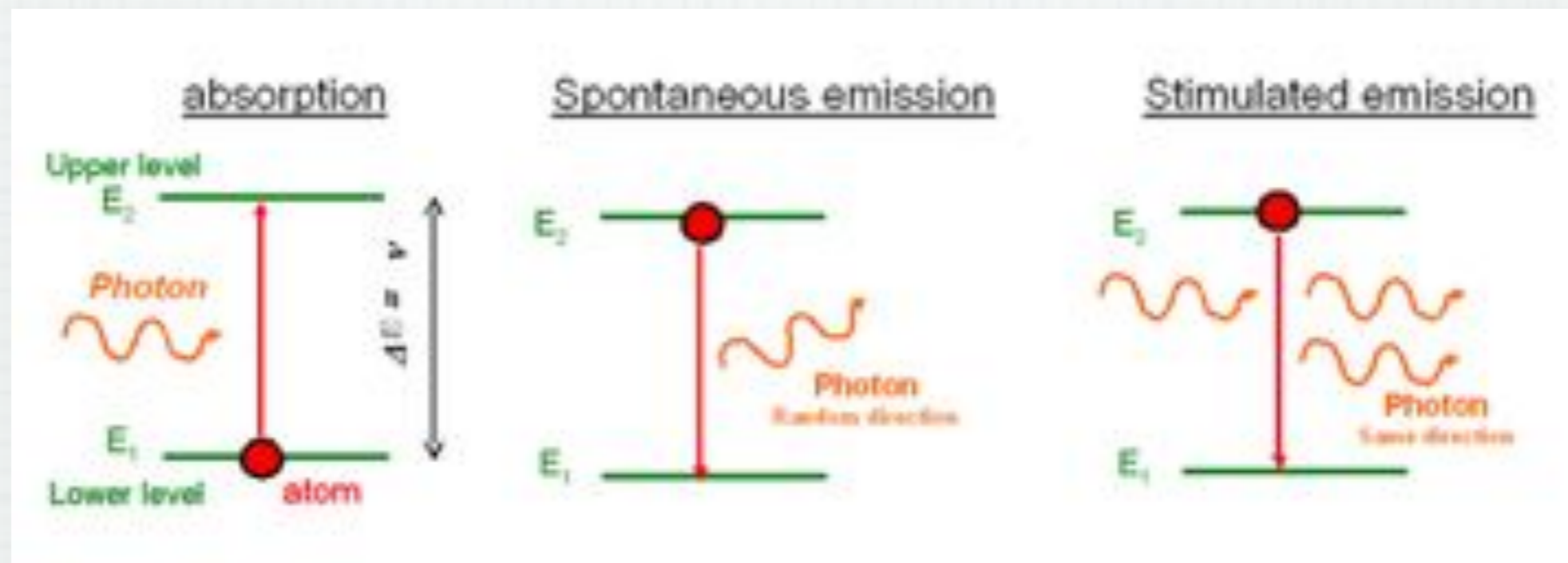
* **Absorption contributes to opacity**

* **Emission to emissivity**

* **Stimulated emission to lasing or masing**

* **Spontaneous emission is isotropic**

* **Stimulated emission is not and depends on specific intensity of radiation field**

# Stimulated emission

* Results in coherent light of same wavelength being emitted

* When a photon of the same wavelength as the energy between two levels passes an excited atom it induces stimulated emission of the excited atom
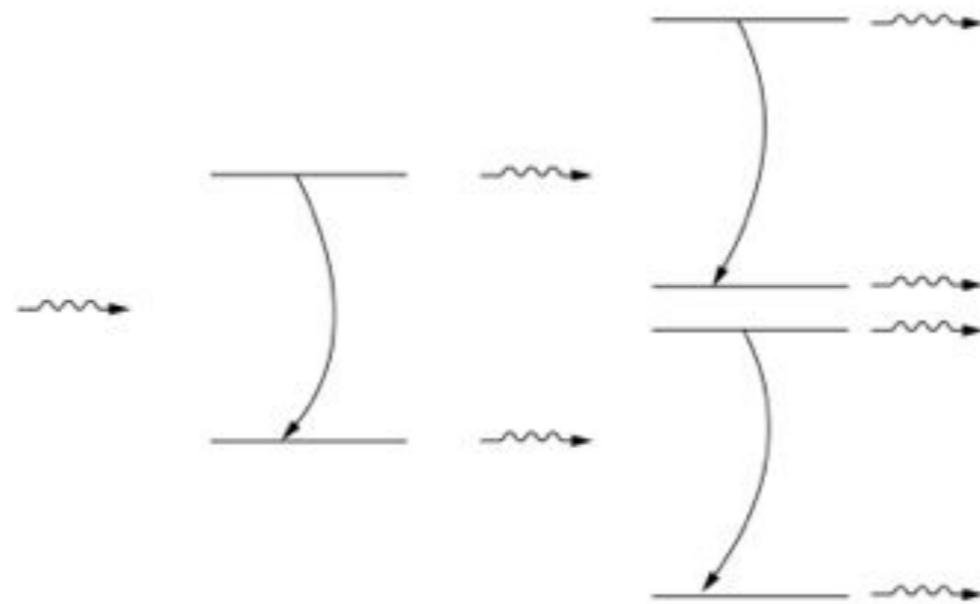


**Figure 4.13** Illustration of stimulated (or induced) emission of radiation from a bound–bound atomic transition. First, a photon induces an emission from an excited atom that cascades to a lower level. Thereafter, the two photons can in turn induce two other excited atoms to emit two additional photons. If there exists a sufficient number of exited atoms, the number of photons in the beam of radiation may grow exponentially.
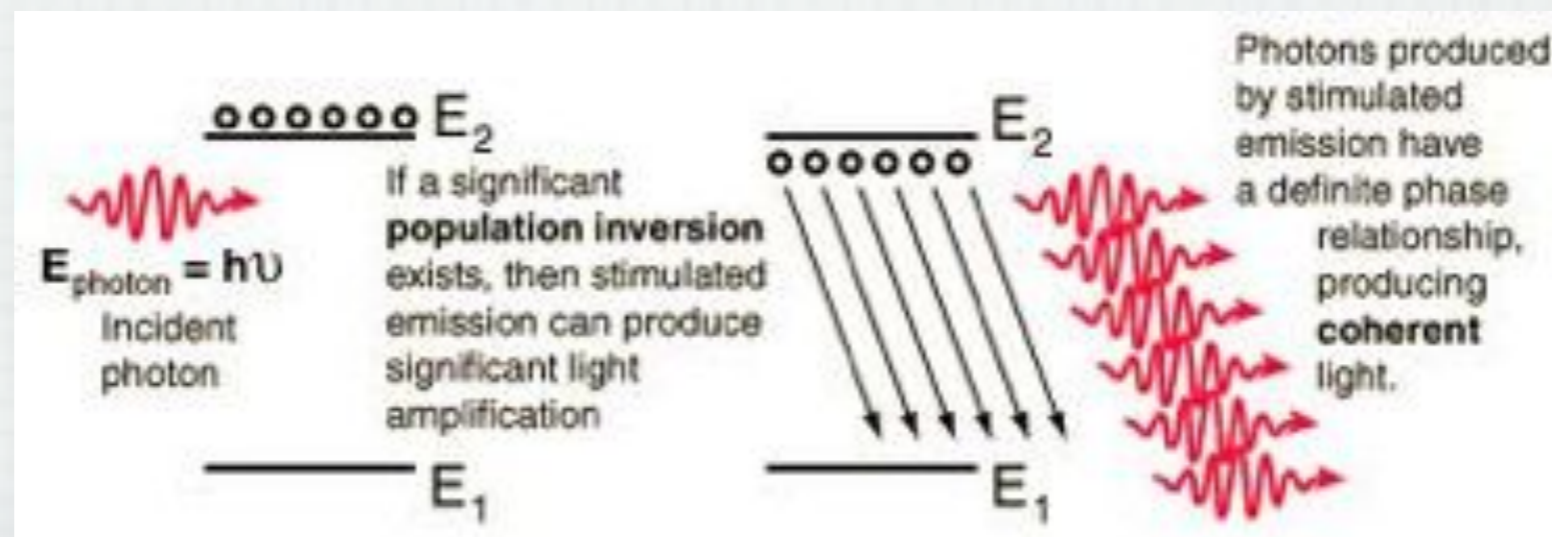
# From Quantum theory

$$k_\nu \rho = \frac{\pi e^2}{m_e c} f_{ij} \psi_\nu n_i \left(1 - e^{\frac{-h\nu_0}{k_B T}}\right)$$

The term in parenthesis is valid only for LTE else it equals

$$\left(1 - \frac{n_j g_i}{n_i g_j}\right)$$

In NLTE the second term above may be greater than one due to a population inversion

This leads to a negative opacity or an exponential increase in emission

- On earth, we use electric fields to get a population inversion (batteries)

- In space, NLTE effects do it for us

- See Masers, just lasers in the microwave

- Water in distant quasars estimated at $10^{14}$ times that in all Earth's oceans
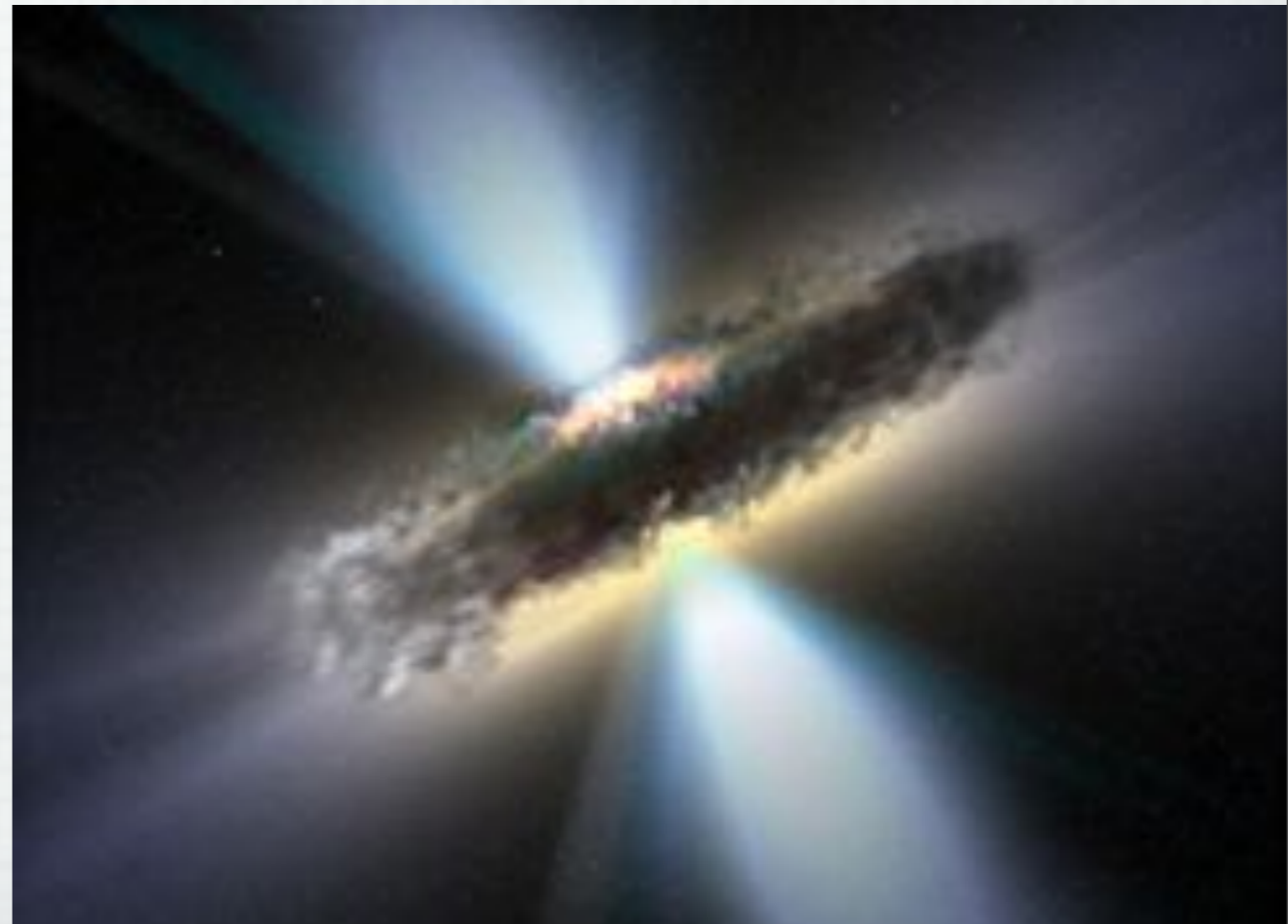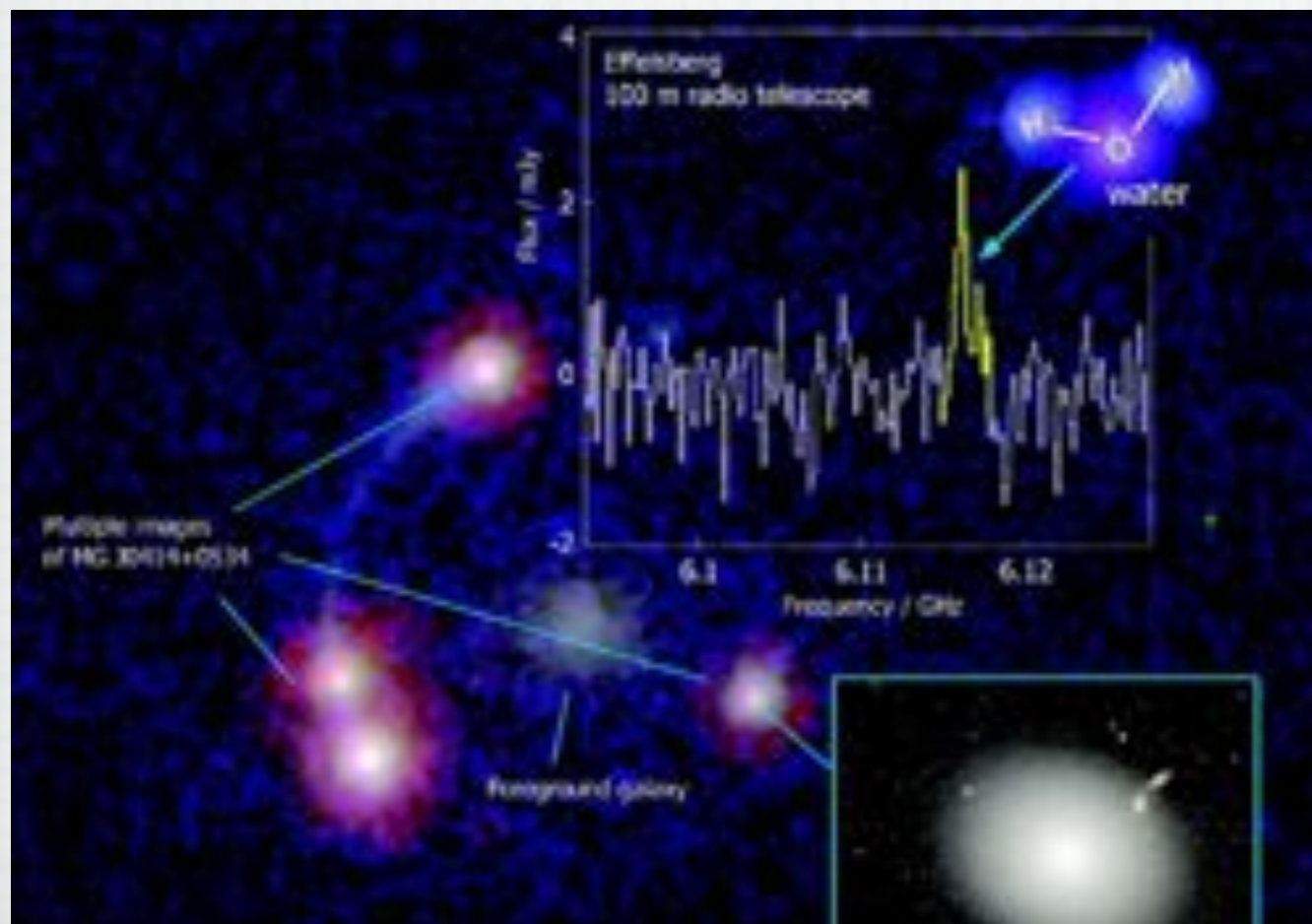
# Einstein Coefficients

* Assume LTE and detailed balance

$$n_i B_{ij} I_\nu = n_j A_{ji} + n_j B ji I_\nu$$

Isolate specific intensity

$$I_\nu = \frac{A_{ji}}{B_{ji}} \frac{1}{\frac{n_i B_{ij}}{n_j B ji} - 1}$$

In LTE the level populations and specific intensity are given by

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{\frac{-h\nu_0}{k_b T}} \quad \text{and} \quad B_{\nu_0} = \frac{2h\nu_0^3}{c^2} \frac{1}{e^{\frac{h\nu_0}{k_B T}} - 1}$$

Using the above in the specific intensity yields

$$\frac{A_{ji}}{B_{ji}} = \frac{2h\nu_0^3}{c^2} \quad \text{and} \quad \frac{g_i B_{ij}}{g_j B_{ji}} = 1$$

# Universal Values

* Calculate one, know all three

* From quantum theory $B_{ij}\dfrac{h\nu_0}{4\pi} = \dfrac{\pi e^2}{m_e c} f_{ij}$

* So $\quad \kappa_\nu \rho = B_{ij}\dfrac{h\nu_0}{4\pi}\psi_\nu n_i\left(1 - e^{\frac{-h\nu_0}{k_B T}}\right)$

* This is the opacity for a line for an i-j transition

* and $\quad \Gamma_{ij} = \displaystyle\sum_{k<j} A_{jk} + \sum_{k<i} A_{ik}$

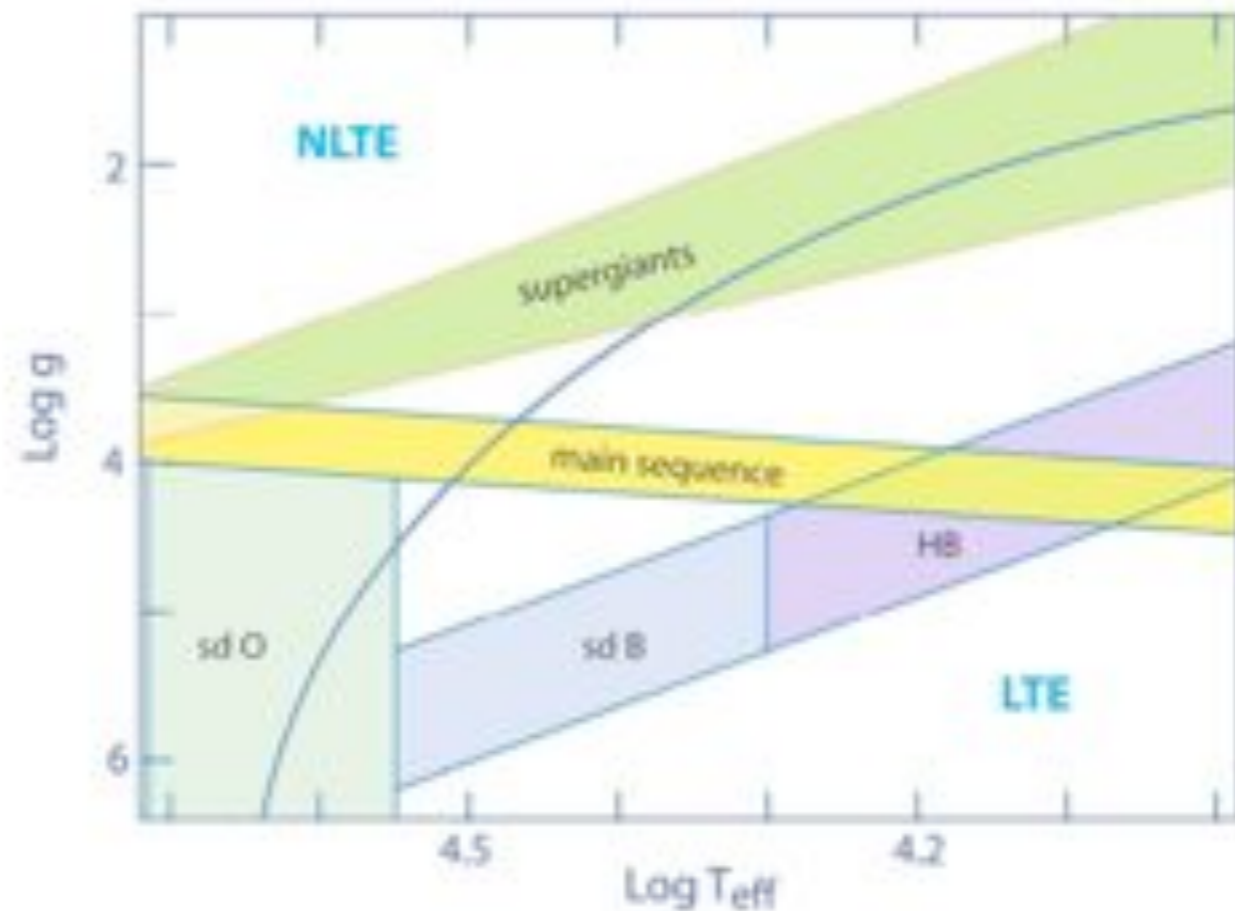* for lyman alpha = $A_{21}$

* for lyman beta = $A_{31} + A_{32}$

# Validity?

* To start, we are talking about individual transitions, must account for thousands

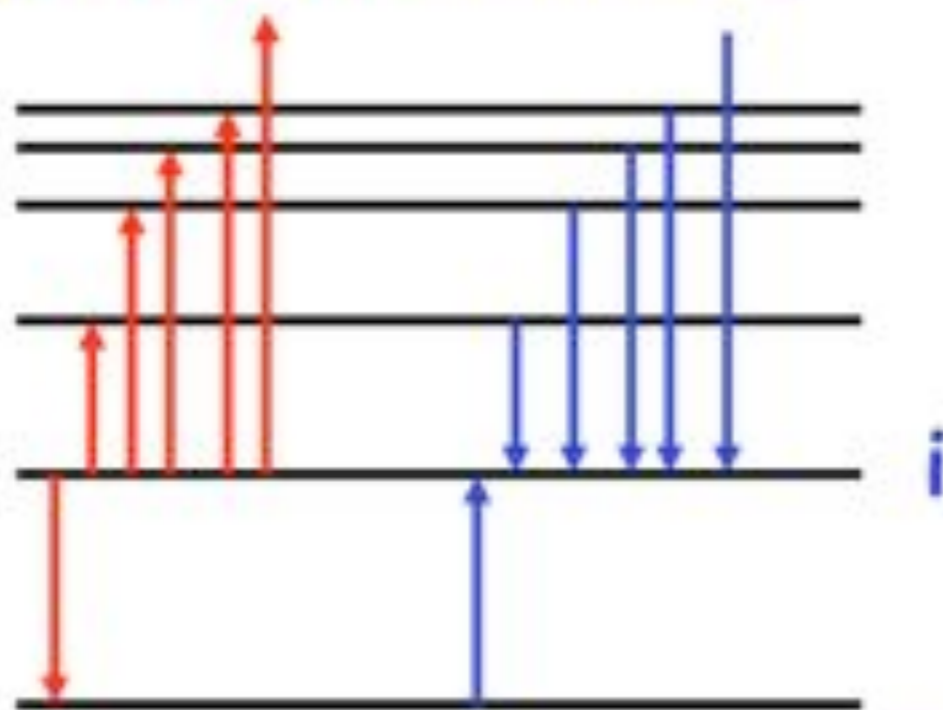* Fails to work when in NLTE, lower densities



LTE vs NLTE in hot stars

# NLTE

1. f(v) dv remains Maxwellian

2. Boltzmann - Saha replaced by $dn_i / dt = 0$ (statistical equilibrium)

   for a given level $i$  the rate of transitions out = rate of transitions in

## rate out = rate in



$$n_i \sum_{j \neq i} P_{ij} = \sum_{j \neq i} n_j P_{ji}$$

rate equations

$P_{i,j}$ transition probabilities

# Calculation of occupation numbers

## NLTE

1. f(v) dv remains Maxwellian

2. Boltzmann – Saha replaced by $dn_i / dt = 0$ (statistical equilibrium)

   for a given level *i* the rate of transitions **out** = rate of transitions **in**

$$n_i \sum_{j \neq i} P_{ij} = \sum_{j \neq i} n_j P_{ji}$$

$$n_i \underbrace{\sum_{j \neq i} (R_{ij} + C_{ij})}_{\text{lines}} + \underbrace{n_i (R_{ik} + C_{ik})}_{\text{ionization}} = \underbrace{\sum_{j \neq i} n_j (R_{ji} + C_{ji})}_{\text{lines}} + \underbrace{n_p (R_{ki} + C_{ki})}_{\text{recombination}}$$

RATE EQUATIONS

Transition probabilities

radiative

$$R_{ij} = B_{ij} \int_0^{\infty} \varphi_{ij}(\nu) \, J_\nu \, d\nu$$

$$R_{ji} = A_{ji} + B_{ji} \int_0^{\infty} \varphi_{ij}(\nu) \, J_\nu \, d\nu$$

absorption

emission

collisional

$$C_{ij} = n_e \int_0^{\infty} \sigma_{coll}(v) v f(v) dv$$

$$C_{ji} = g_i/g_j e^{E_{ji}/kT} \, C_{ij}$$

## NLTE Atomic Models in modern model atmosphere codes

lines, collisions, ionization, recombination

Essential for occupation numbers, line blocking, line force

Accurate atomic models have been included

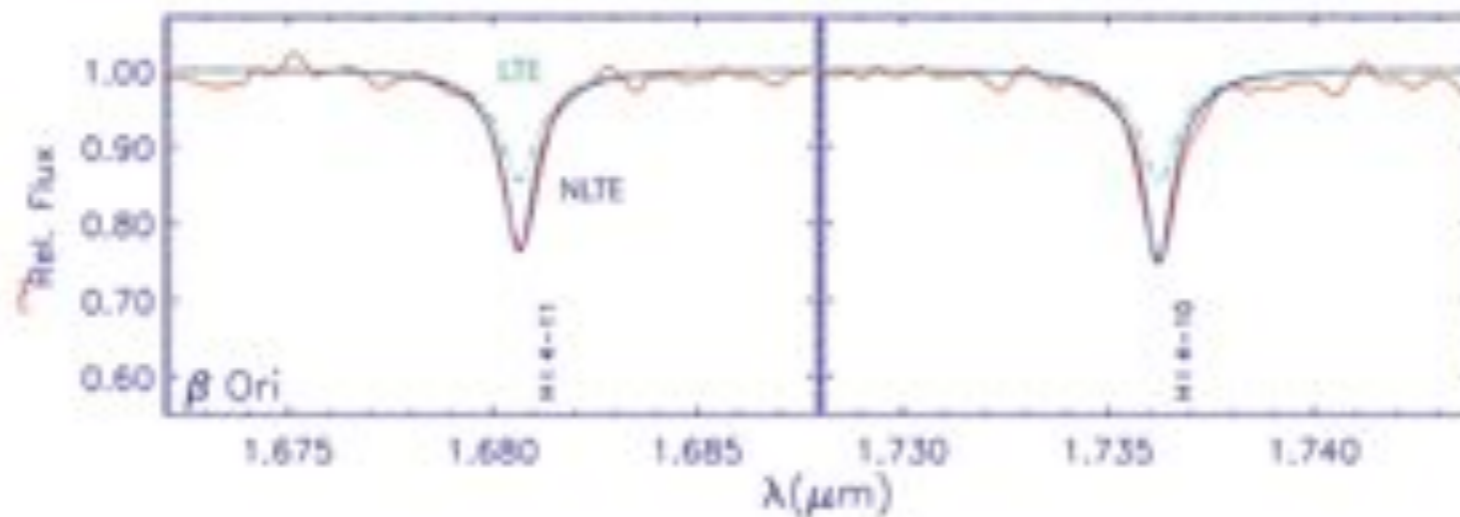| | |
|---|---|
| 26 | elements |
| 149 | ionization stages |
| 5,000 | levels ( + 100,000 ) |
| 20,000 | diel. rec. transitions |
| $4 \cdot 10^5$ | b-b line transitions |
| | Auger-ionization |

recently improved models are based on **Superstructure**

Eissner et al., 1974, CPC 8,270

## LTE vs NLTE: line fits – hydrogen lines in IR



Brackett lines

# Beyond the scope of this course

* Visit here for more detail

* http://www.ifa.hawaii.edu/users/kud/teaching_09/7_Non-LTE.pdf

* http://www.ifa.hawaii.edu/users/kud/teaching_09/8_Line_formation.pdf

* http://cifist.obspm.fr/NLTE/presentations/2-rtesol.pdf

* http://arxiv.org/pdf/astro-ph/0208294v1.pdf

# Equivalent Width

* Or how to measure the depth of an absorption line

* Defined as the width of a hypothetical absorption line of rectangular shape  that absorbs that absorbs all of the radiation within it with the same energy absorbed as the true line
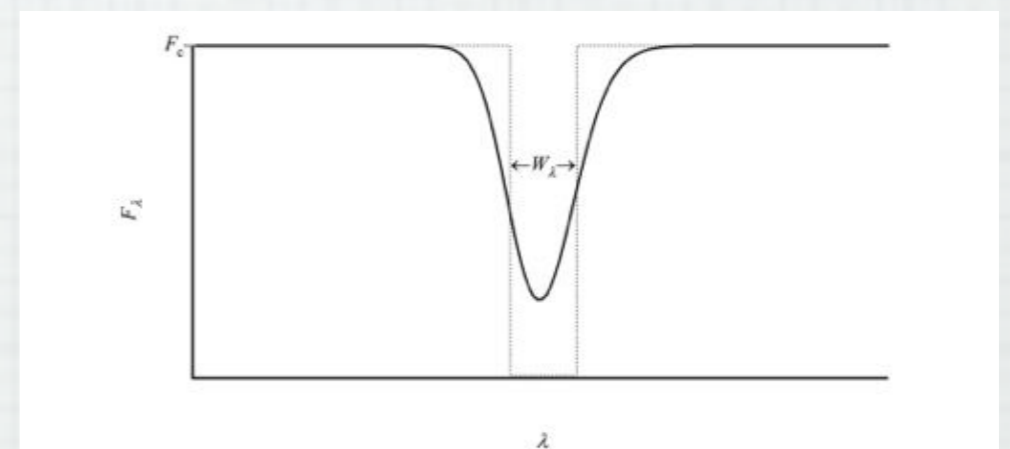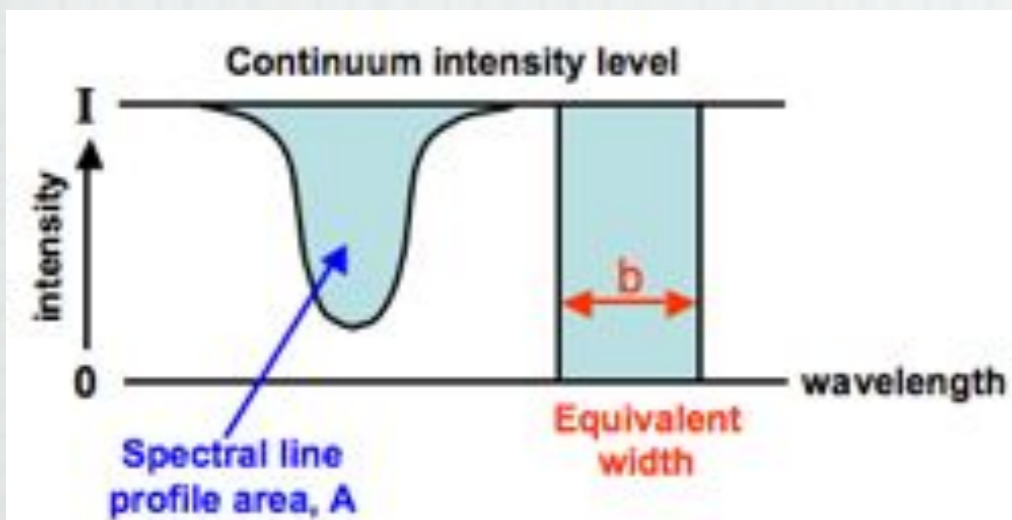
Continuum intensity level

I

intensity

0

wavelength

b

Equivalent width

Spectral line profile area, A

$F_c$

$F_\lambda$

$\leftarrow W_\lambda \rightarrow$

$\lambda$

**Figure 4.14**  Schematic definition of the equivalent width ($W_\lambda$) of an atomic line. The fictitious rectangular line which absorbs all photons within it has a width such that it absorbs the same quantity of energy as the atomic line to which it is associated. The quantity $F_c$ is the flux of the continuum.

$$W_\lambda = \int \left(\frac{F_c - F_\lambda}{F_c}\right) d\lambda$$

$$= \int (1 - R_\lambda) d\lambda = \int A_\lambda d\lambda$$

and A is the line absorption depth

Where

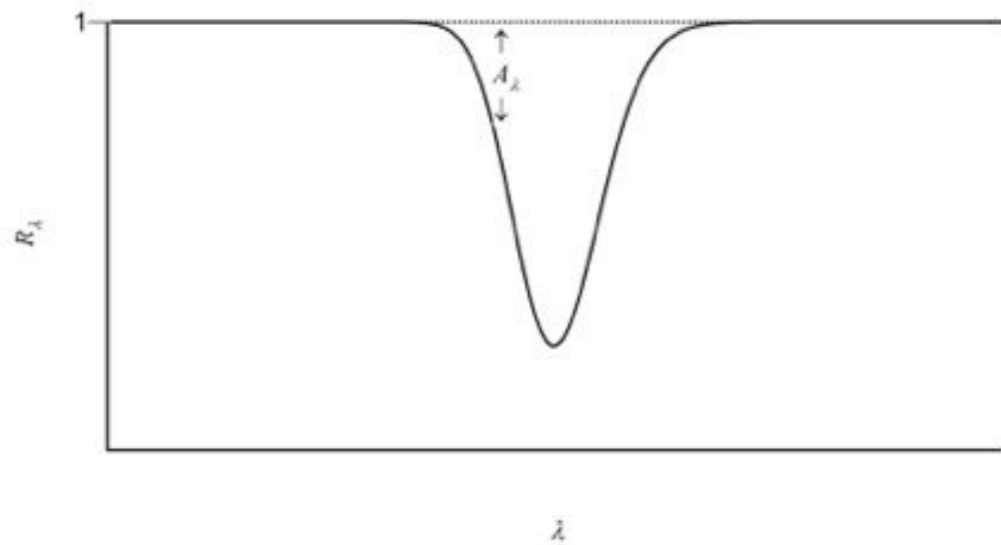$$R_\lambda = \frac{F_\lambda}{F_c} \qquad A_\lambda = 1 - R_\lambda = 1 - \frac{F_\lambda}{F_c}$$



**Figure 4.15** Illustration of the residual intensity ($R_\lambda$) and the line (or absorption) depth ($A_\lambda$).

$$W = \int_{\lambda_1}^{\lambda_2} \frac{F_c - F_\lambda}{F_c} d\lambda$$

# Line Formation

* The intensity in the continuum we see originates from a more or less isotropic radiation field at an optical depth of ~ 2/3

* The line emission is from a shallower optical depth so the emissivity is less

* It's all in emission, just some is in less emission
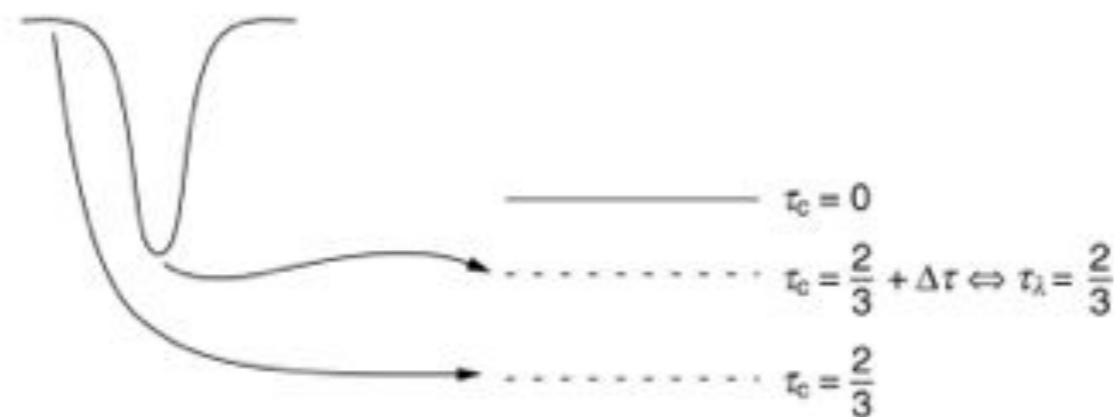


**Figure 4.16** Illustration showing the depth at which the continuum is formed ($\tau_c = 2/3$) and where the atomic line under consideration is formed ($\tau_c = 2/3 + \Delta\tau$) where $\Delta\tau < 0$. The depth $\tau_c = 2/3 + \Delta\tau$ corresponds to the depth $\tau_\lambda = 2/3$.
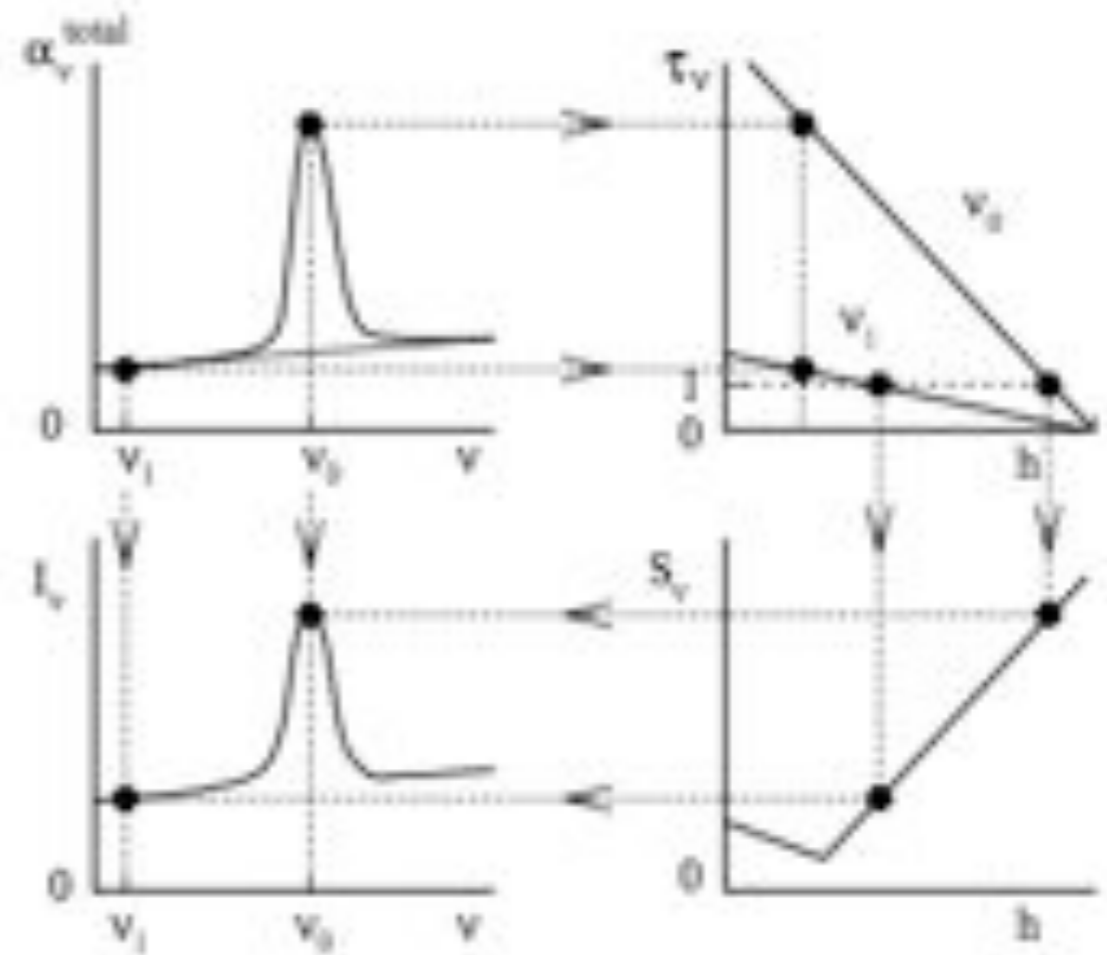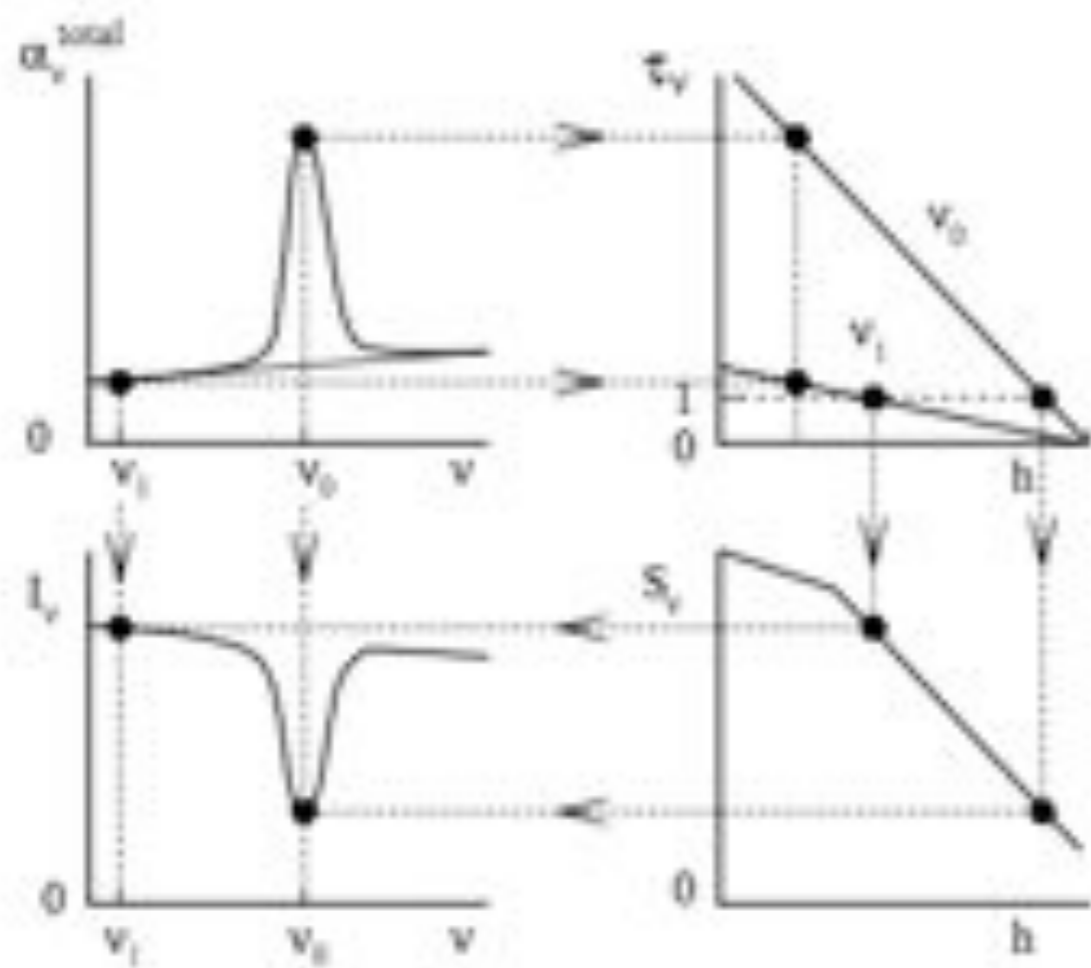
See the book for details
Essentially We want to define A in terms of line versus
continuum opacity

$$A_\lambda \sim \frac{2}{3} \frac{k_l}{k_c} \frac{dlnB_\lambda}{d\tau_c} \Big|_{\tau_c = 2/3}$$

The continuum opacity is more or less independent of elemental
abundances whereas the line opacity is directly related to
elemental abundances
The equivalent width is therefor a measure of abundances of
certain species

# Curve of Growth

* Or, a way to determine abundances based on equivalent width
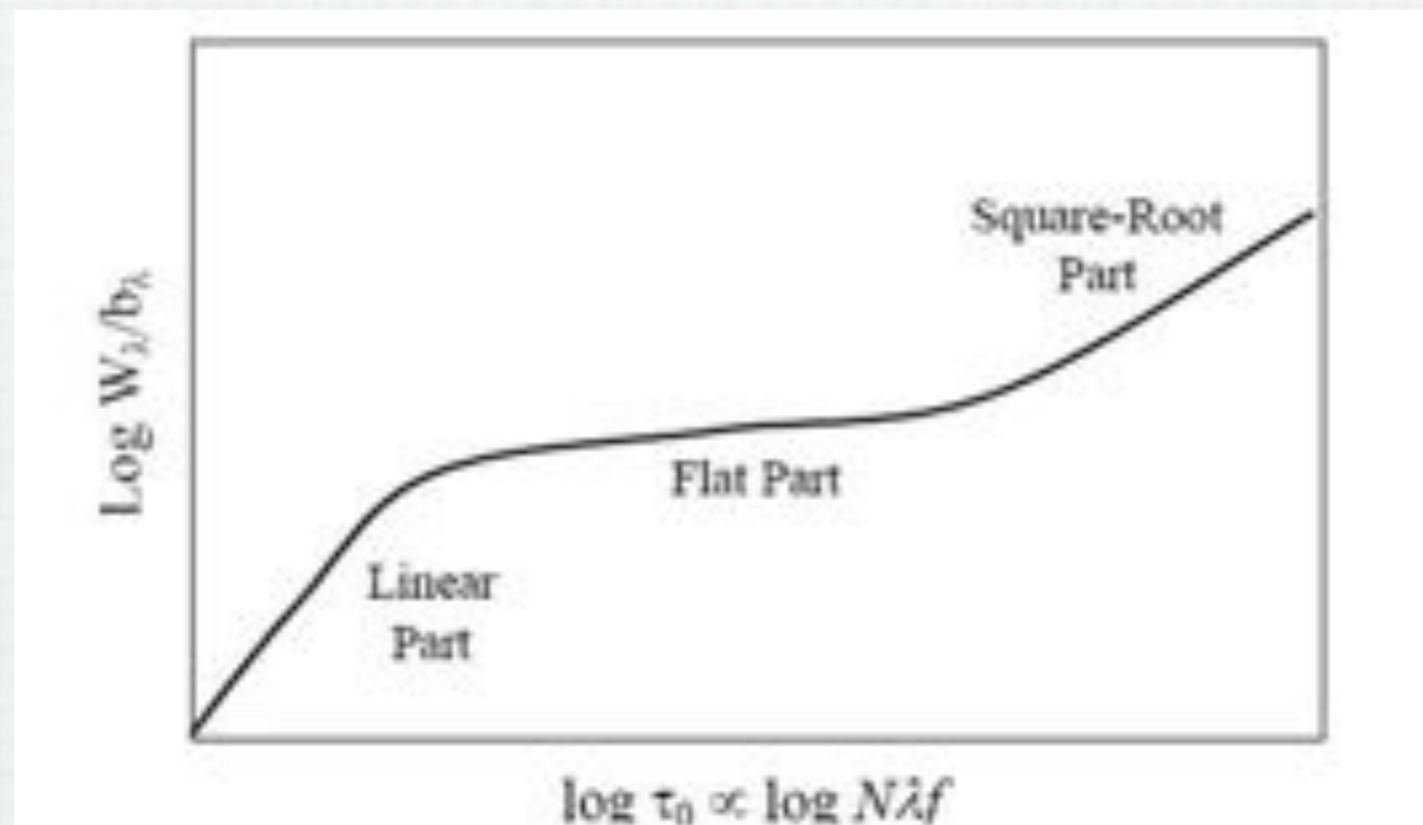
* Prior to line saturation the equivalent width grows proportional to the number of absorbers or is ˜ n

* After this the wings grow as ˜ log(n)

* When pressure broadening increases the broadening grows as the sqrt(n)

* By examining the curve of growth we may estimate the number of absorbers

# Hw

* Skip the last two problems