Stellar Astrophysics

Lecture 3

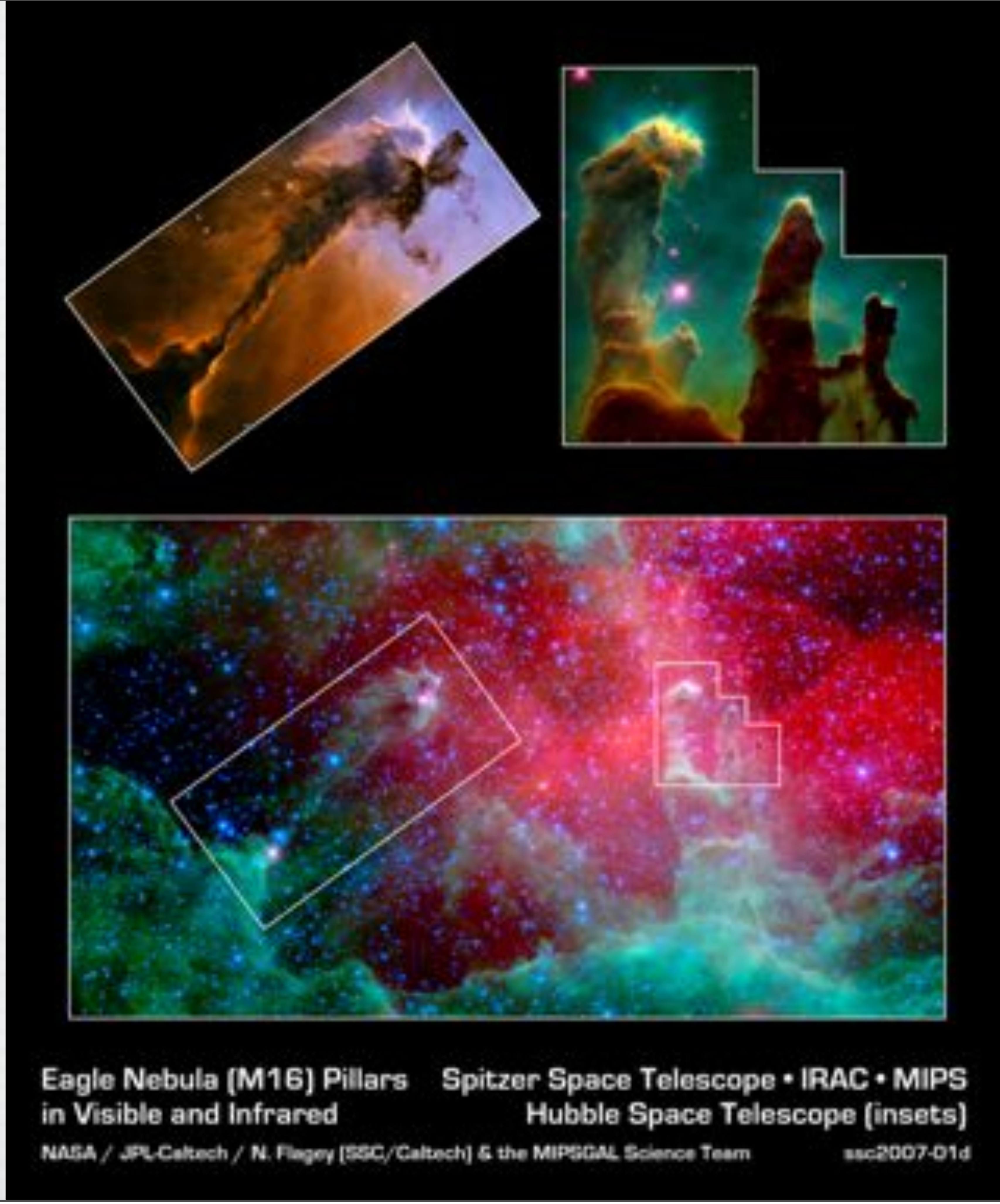
Stellar Formation

* Hydrostatic Equilibrium
* Virial Theorem
* Jean's Criterion
* Time Scales

But First Pictures

Part of Carina Nebula

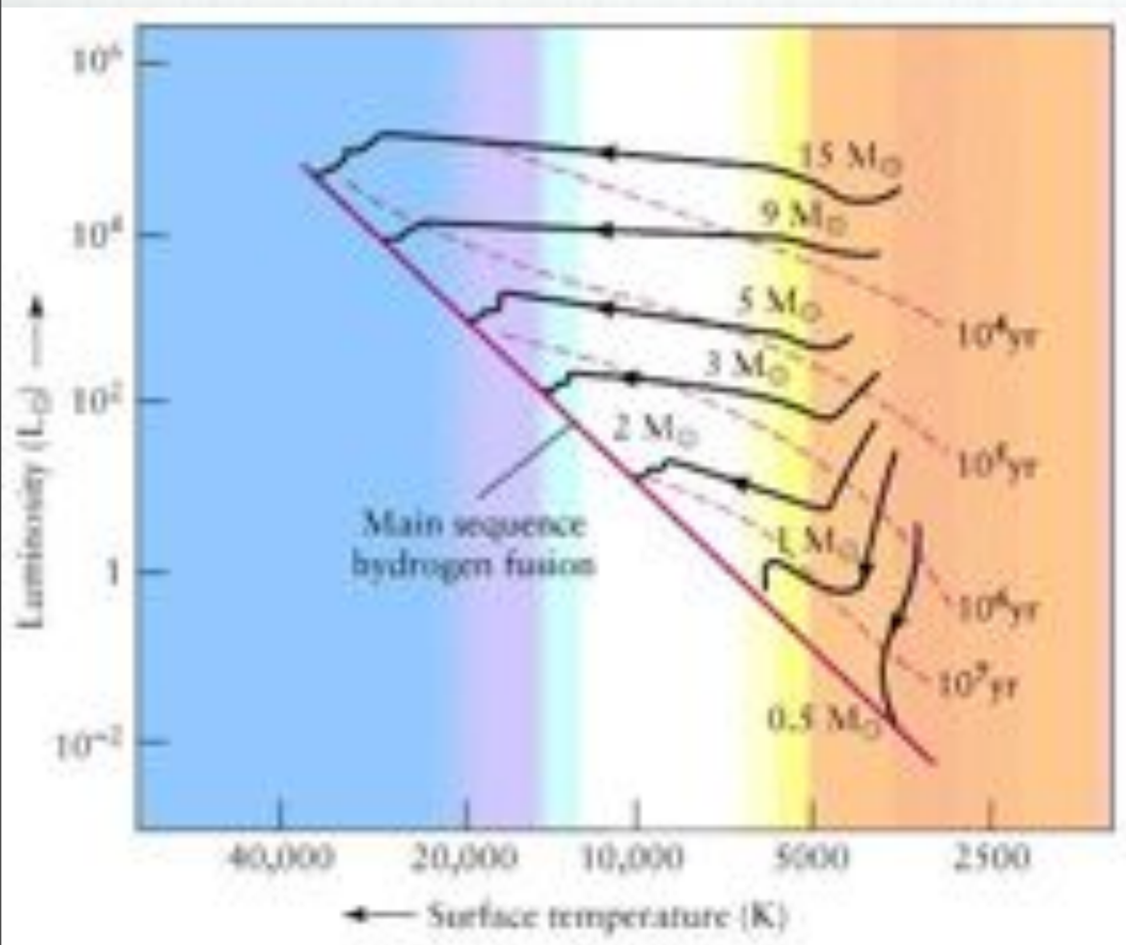
Eagle Nebula

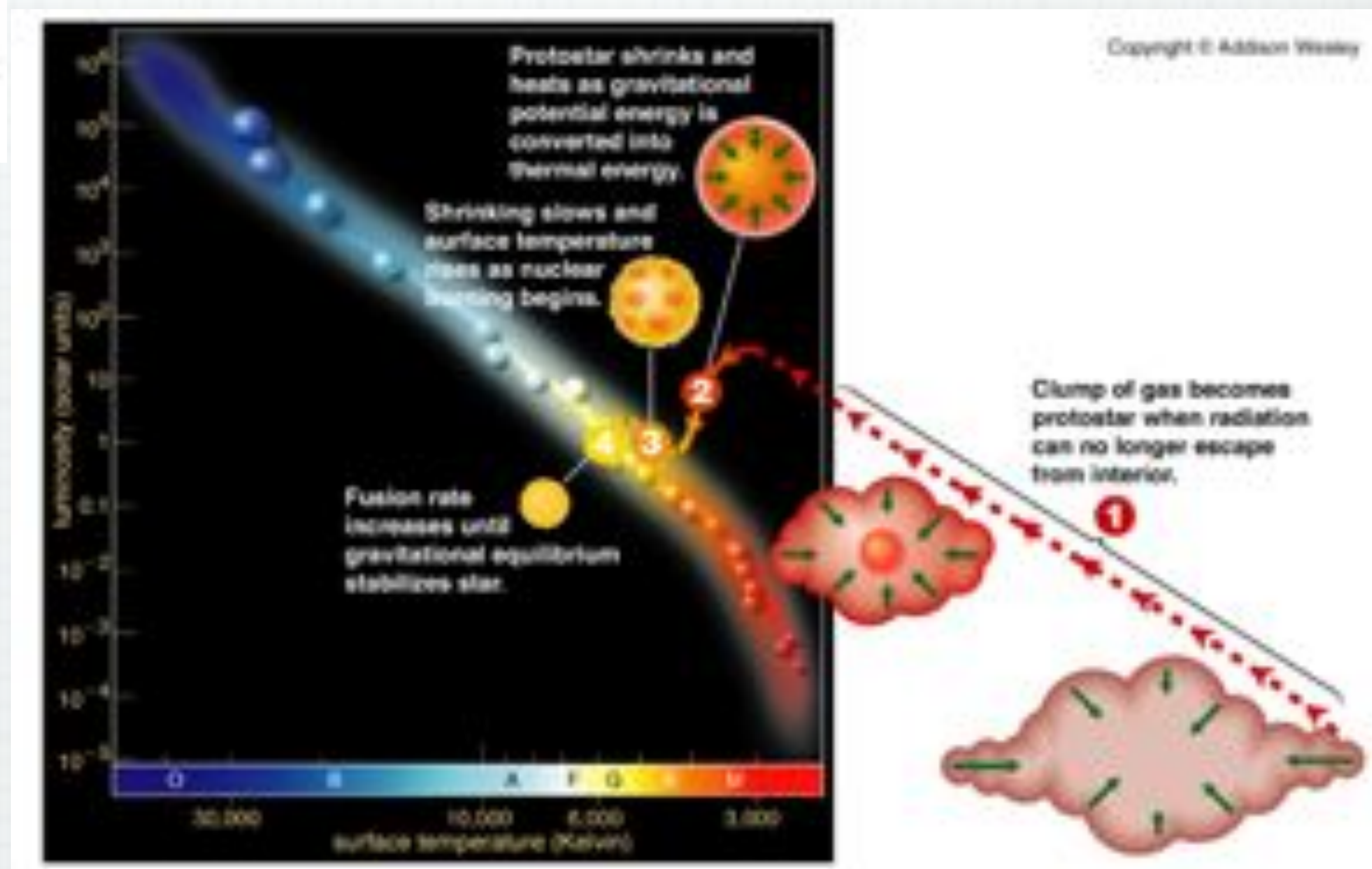


In Orion

Pre Main Sequence

* Jean's collapse sets in
* Cloud contracts
* Cloud heats up
* Rotation rate increases due to conservation of angular momentum
* Star heats due to gravitational energy being converted to thermal energy


15 M☉
9 M☉
5 M☉
3 M☉
2 M☉
1 M☉
0.5 M☉
Main sequence
hydrogen fusion
10^4 yr
10^5 yr
10^6 yr
10^7 yr
10^6
10^4
10^2
1
10^{-2}
Luminosity (L☉) ⟶
40,000
20,000
10,000
5000
2500
⟵ Surface temperature (K)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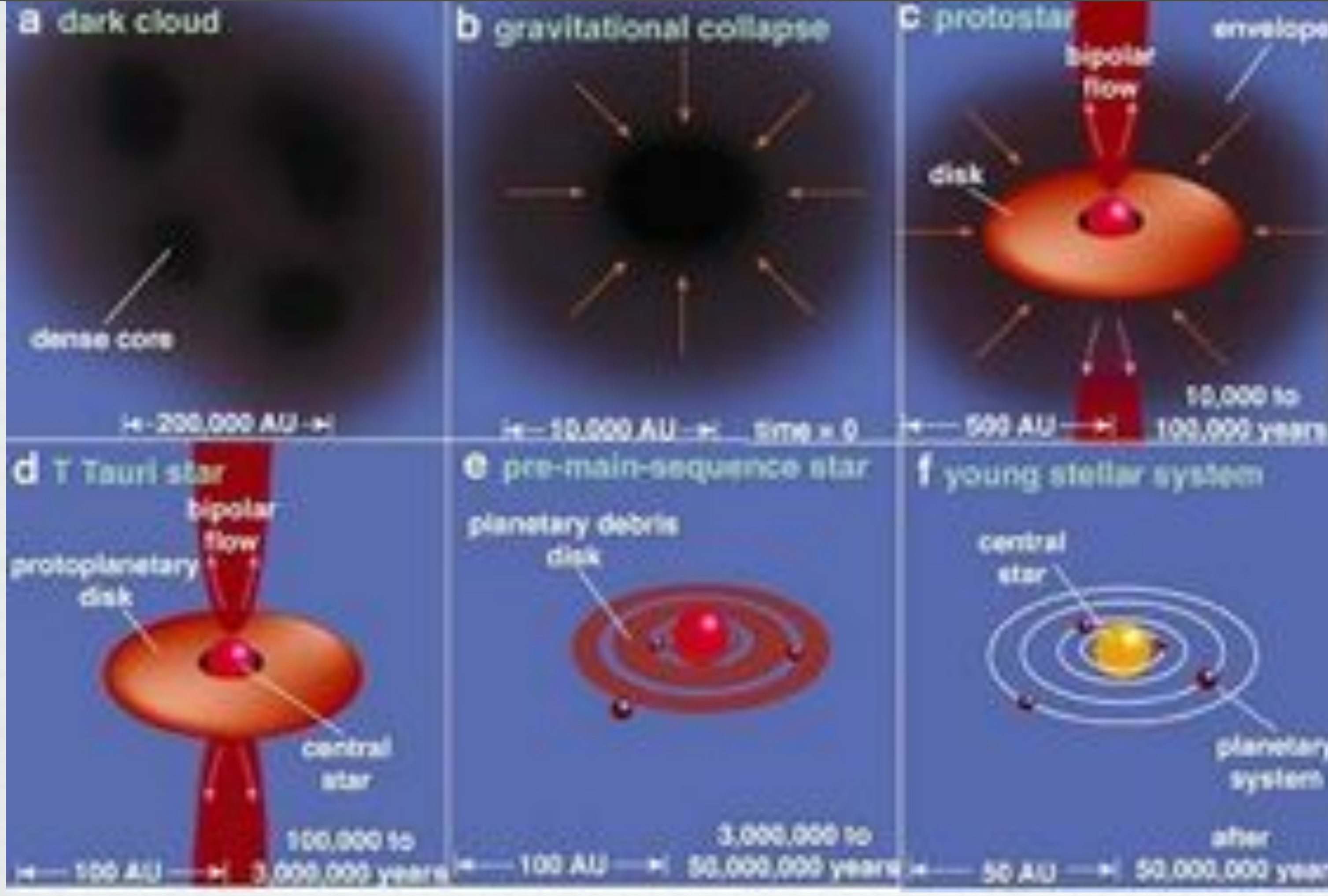
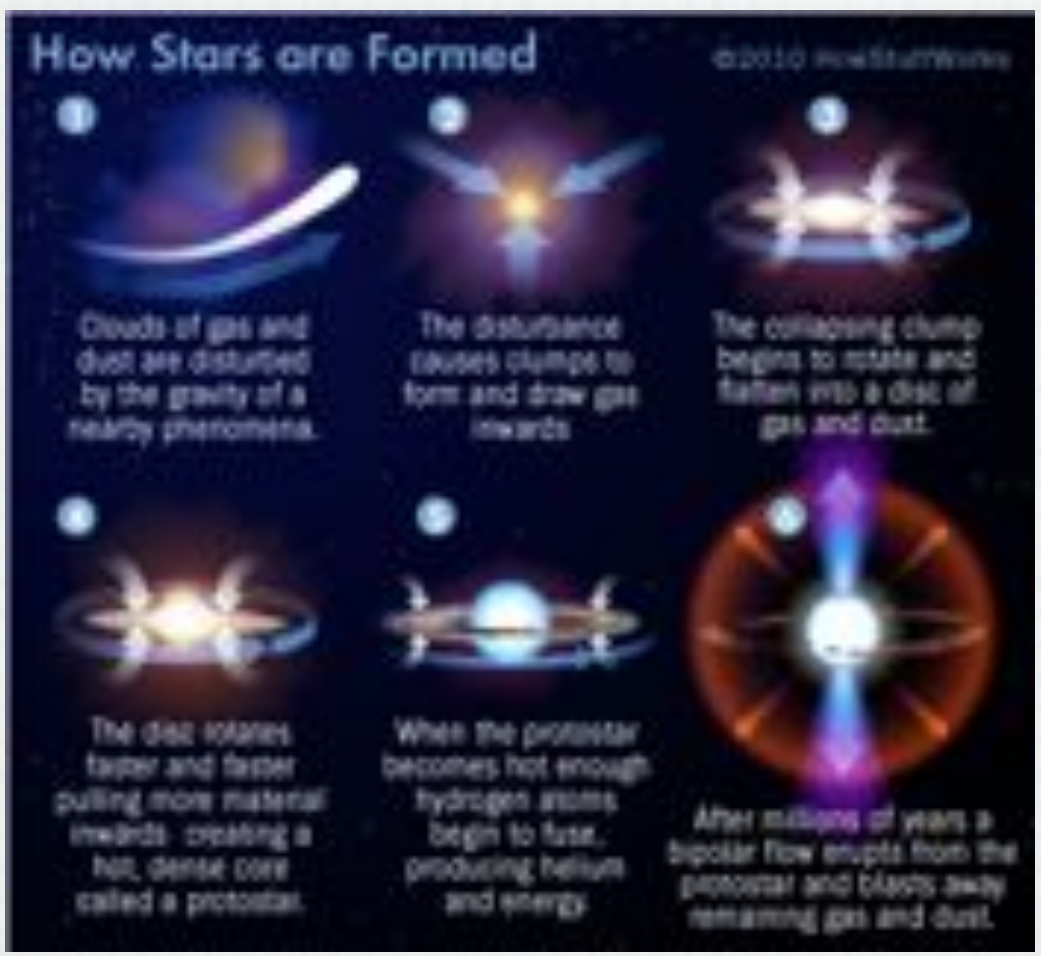
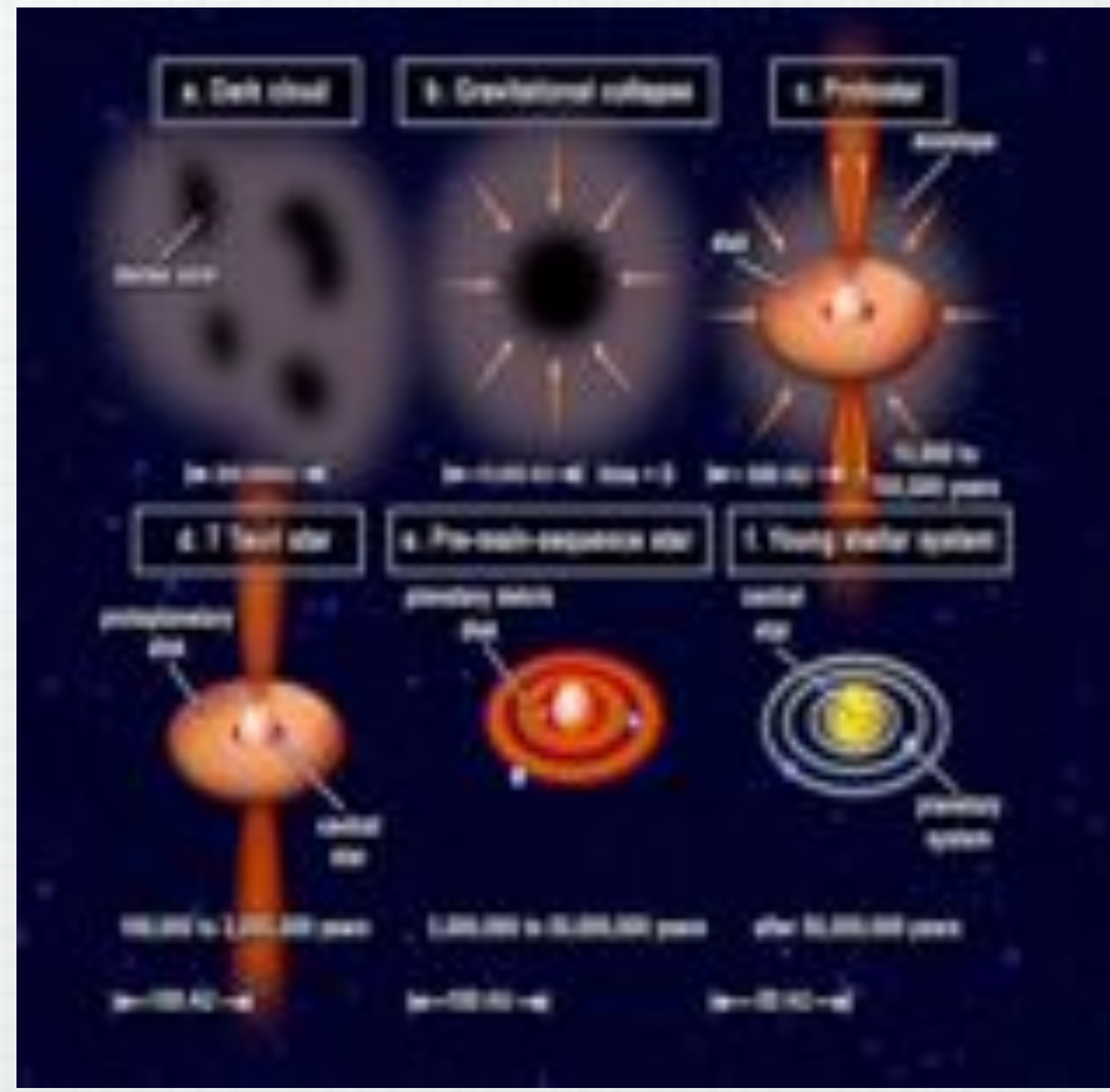
Protostar shrinks and heats as gravitational potential energy is converted into thermal energy.
Shrinking slows and surface temperature
as nuclear
begins.
Fusion rate increases until gravitational equilibrium stabilizes star.
Clump of gas becomes protostar when radiation can no longer escape from interior.
1
2
3
4

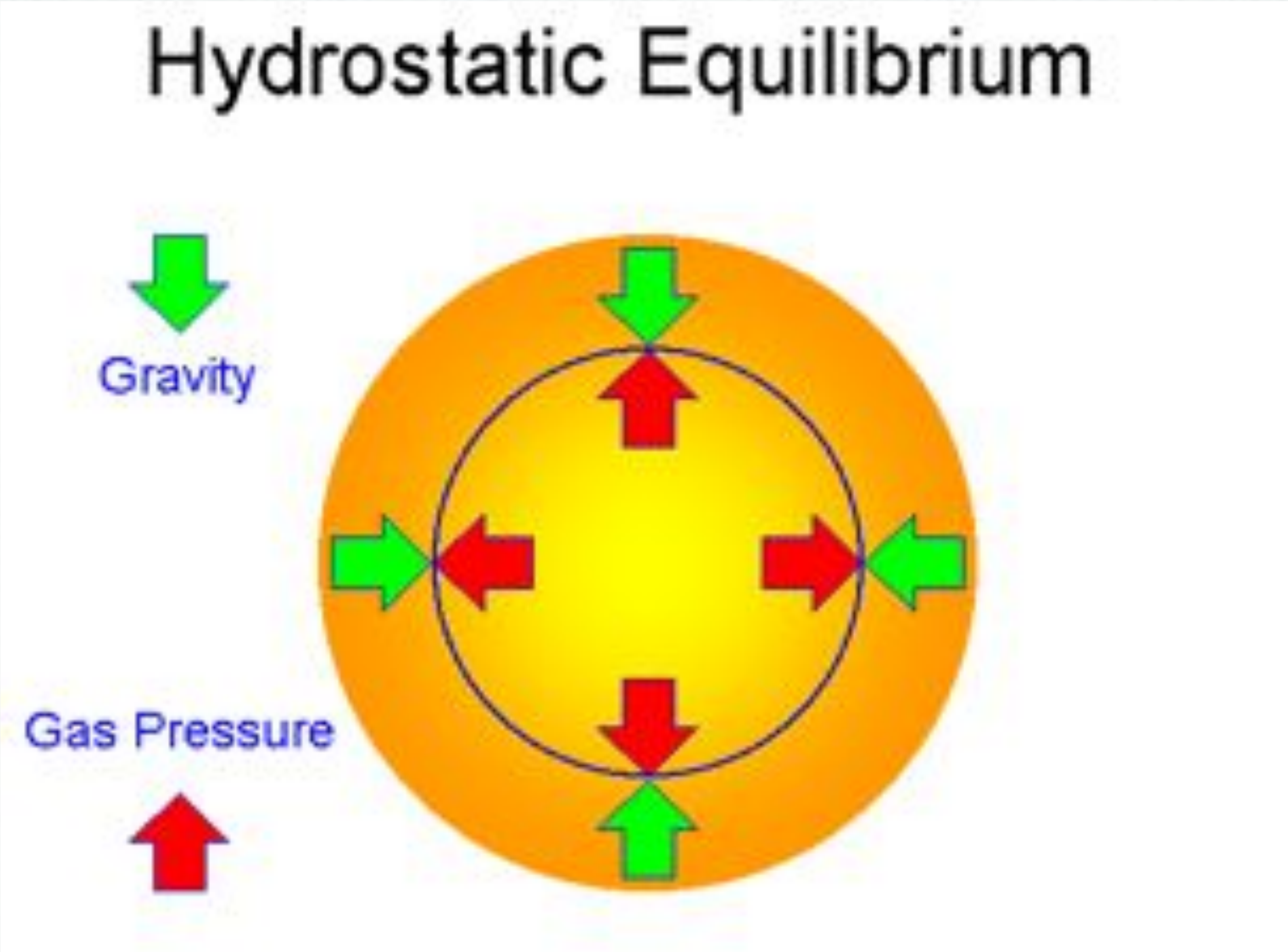

a dark cloud
dense core
200,000 AU
b gravitational collapse
10,000 AU
time = 0
c protostar
bipolar flow
envelope
disk
500 AU
10,000 to 100,000 years
d T Tauri star
bipolar flow
protoplanetary disk
central star
100 AU
100,000 to 3,000,000 years
e pre-main-sequence star
planetary debris disk
100 AU
3,000,000 to 50,000,000 years
f young stellar system
central star
planetary system
50 AU
after 50,000,000 years

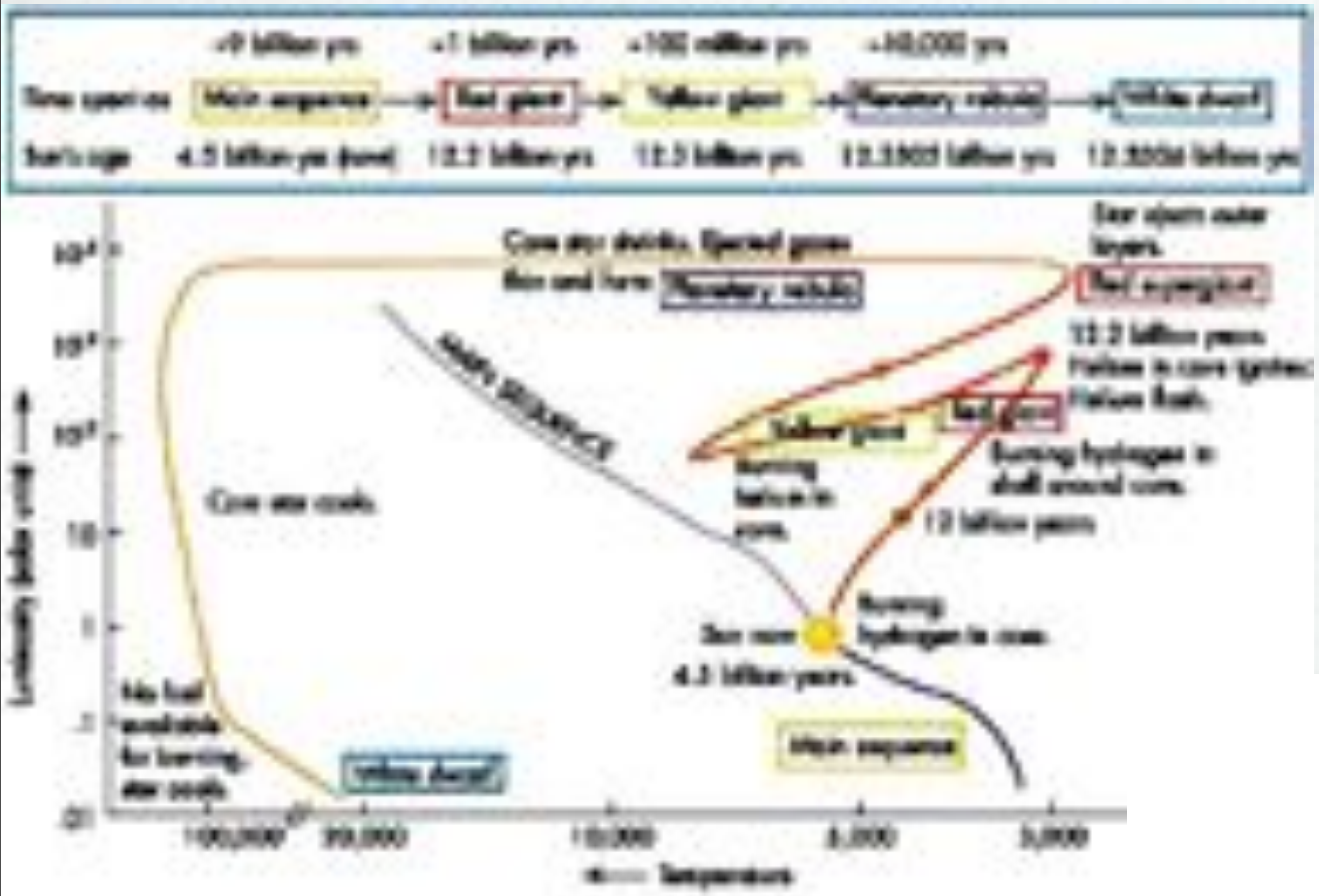

How Stars are Formed
Clouds of gas and dust are disturbed by the gravity of a nearby phenomena.
The disturbance causes clumps to form and draw gas inwards
The collapsing clump begins to rotate and flatten into a disc of gas and dust.
The disc rotates faster and faster pulling more material inwards, creating a hot, dense core called a protostar.
When the protostar becomes hot enough hydrogen atoms begin to fuse, producing helium and energy.
After millions of years a bipolar flow erupts from the protostar and blasts away remaining gas and dust.

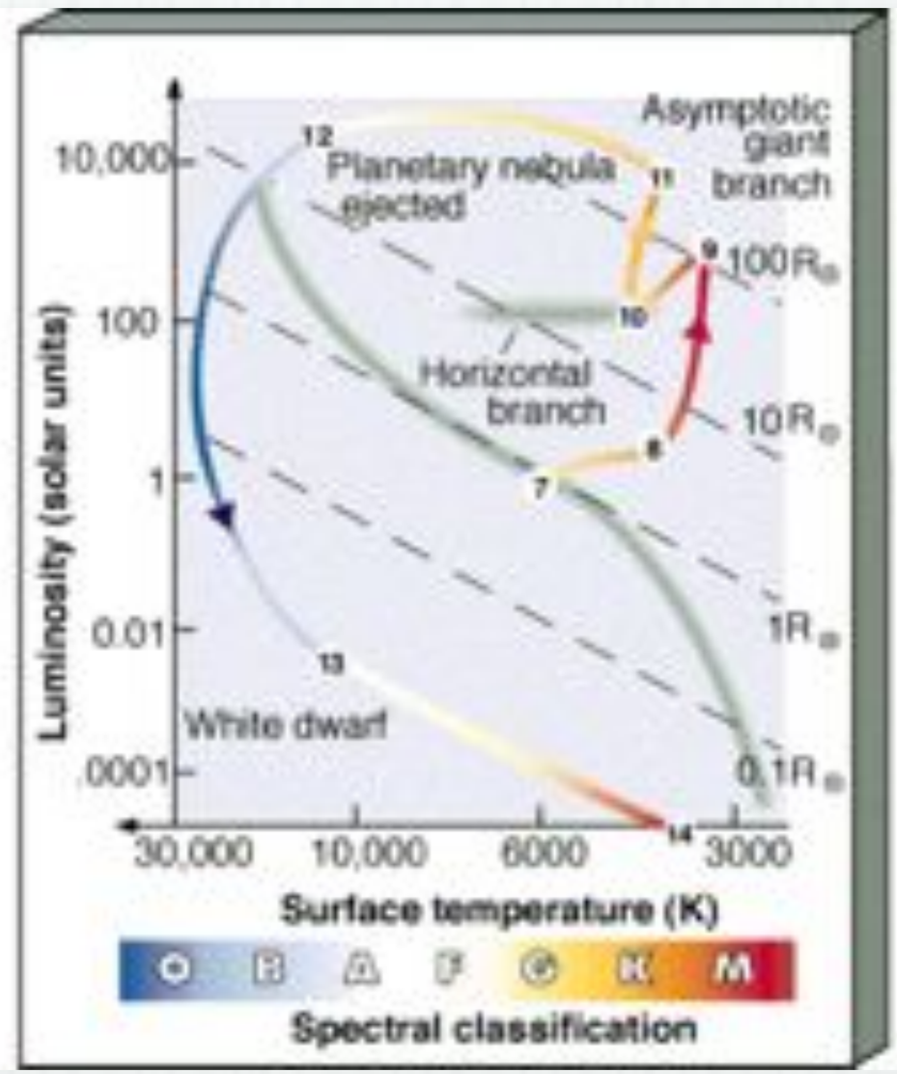

a. Dark cloud
b. Gravitational collapse
c. Protostar
d. T Tauri star
e. Pre-main-sequence star
f. Young stellar system

Hydrostatic Equilibrium

* Or, a star has hit the main sequence




Main sequence
Red giant
Yellow giant
Planetary nebula
White dwarf
Temperature


Asymptotic giant branch
12
Planetary nebula ejected
11
9
100R⊙
10
Horizontal branch
10R⊙
8
7
1R⊙
0.1R⊙
13
White dwarf
14
10,000
100
1
0.01
.0001
Luminosity (solar units)
30,000
10,000
6000
3000
Surface temperature (K)
O B A F G K M
Spectral classification

Issues?

* Star must be able to cool off or else thermal pressure will halt the collapse
* Star must be able to shed angular momentum or it spins up too much

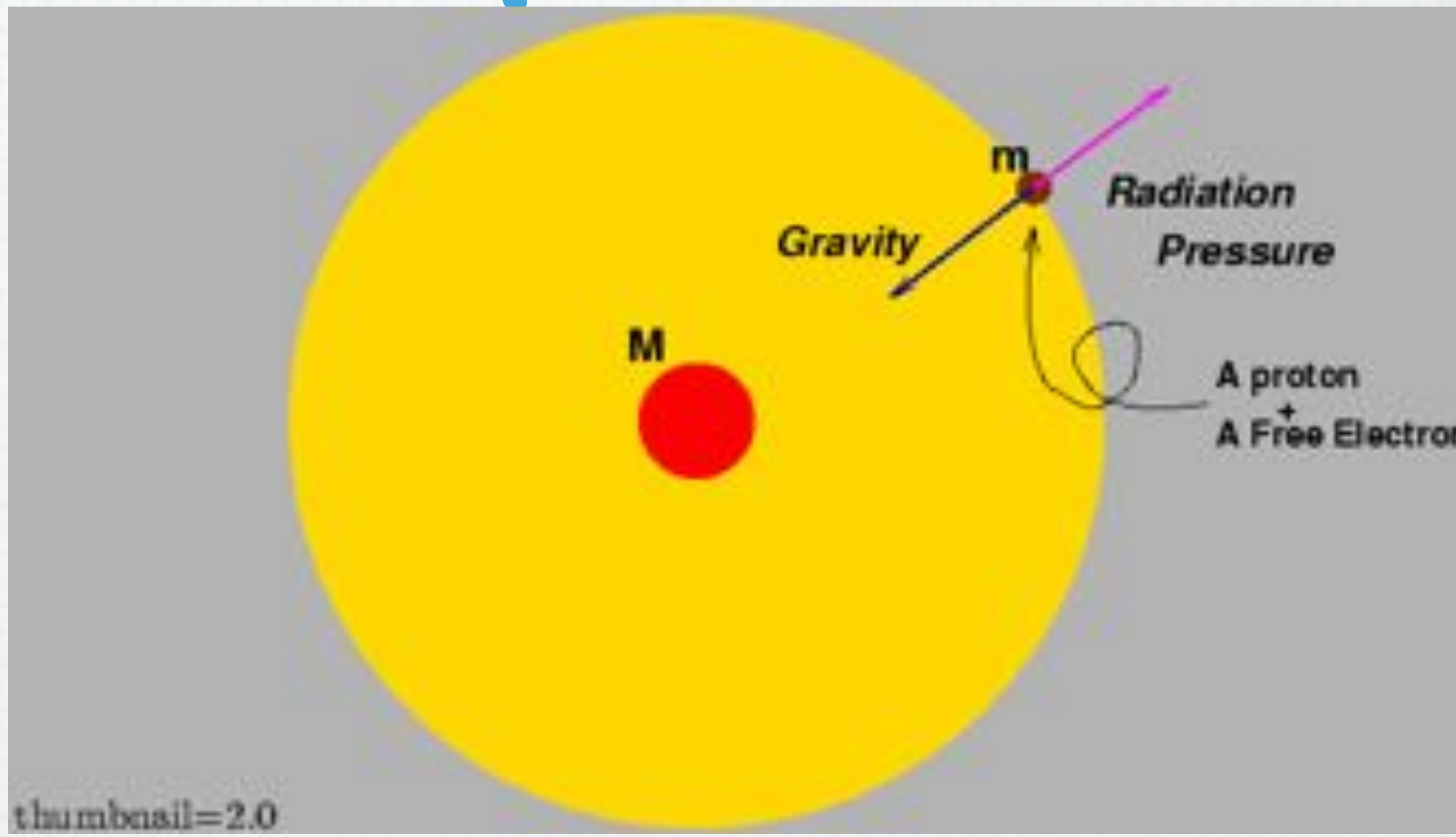
Ok today but not so much early universe

* Today, metals allow for efficient cooling
* In the early universe only molecular hydrogen and helium
* Clouds had to be MUCH bigger to collapse hence larger stars formed

* In the early universe stars could be around 500 times more massive than our sun
* Some died in Pair instability supernovae which completely destroyed the star

Today

* Eddington Luminosity limits the upper size of stars
* Basically the point at which radiation pressure exceeds gravitational binding energy



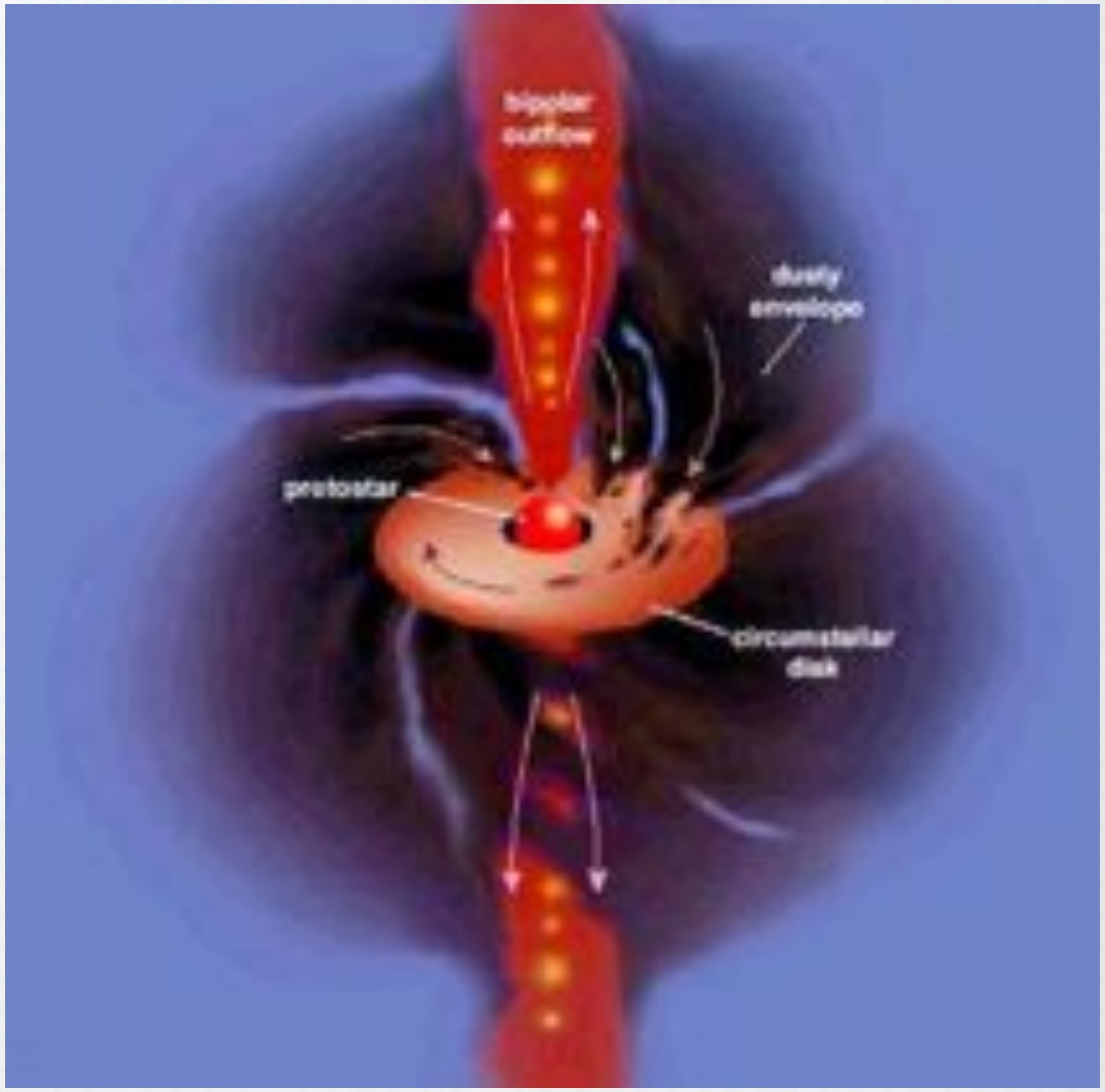
$$L_{\rm Edd} = \frac{4\pi G M m_{\rm p} c}{\sigma_{\rm T}}$$
$$\cong 1.26 \times 10^{31} \left(\frac{M}{M_\odot}\right) {\rm W} = 3.2 \times 10^4 \left(\frac{M}{M_\odot}\right) L_\odot$$

Rotation?

* Conservation of angular momentum spin goes up as radius goes down
* $L = I\omega = (2/5)MR^2$
* $R_1 = 10^{13}$ km $R_2 = 7 \times 10^5$ km
* L_1/L_2 about 10^{16}
* No way!!

Slows Down?

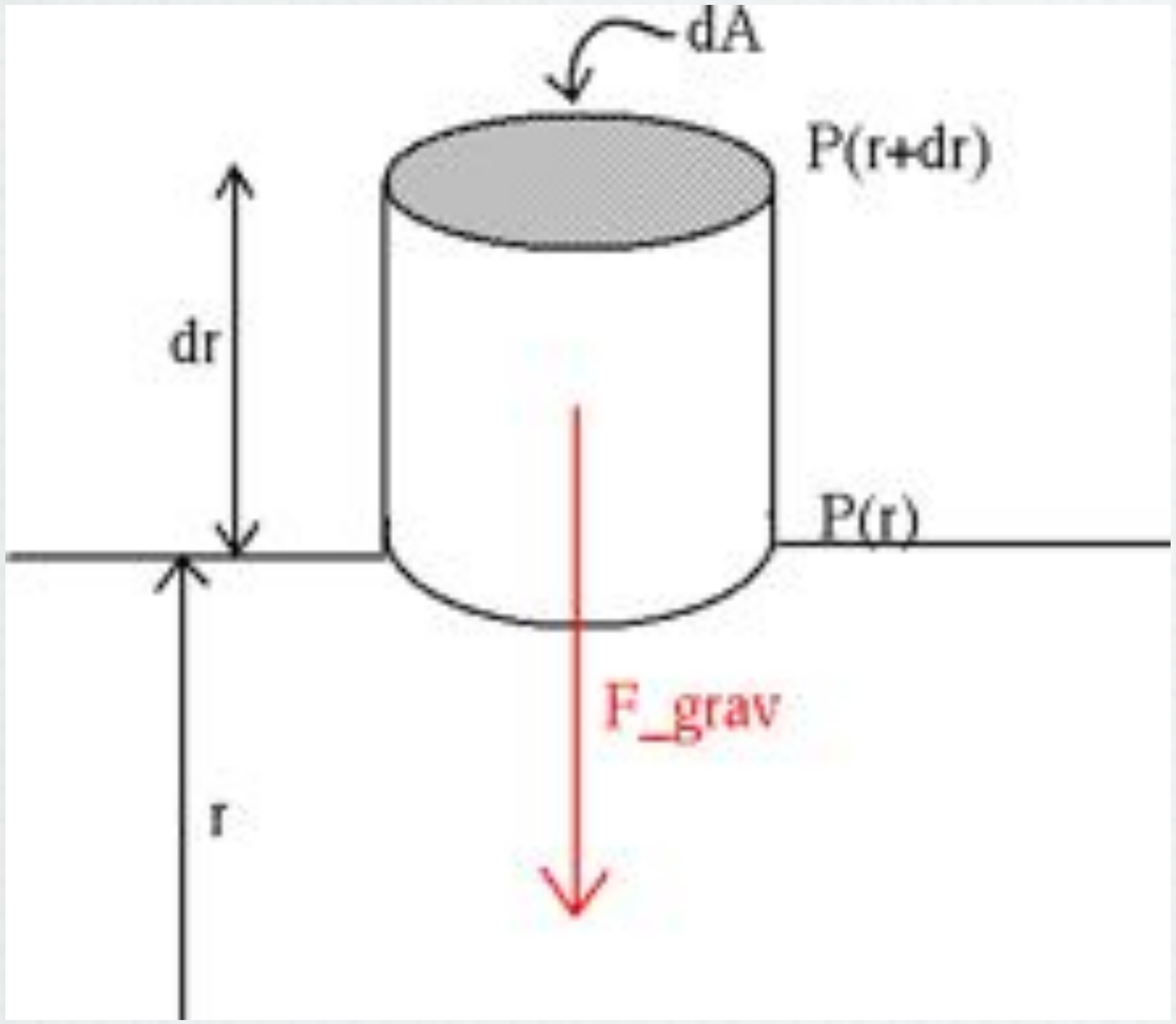
* Yup, has to
* winds, jets, magnetic field instabilities, and waves carry off magnetic fields



Google

* http://en.wikipedia.org/wiki/T_Tauri_star
* http://en.wikipedia.org/wiki/FU_Orionis_star
* http://en.wikipedia.org/wiki/Herbig_Ae/Be_stars

Let's go through some of the pre-main sequence physics



$$dm\ddot{r} = F_G - dAdP$$

$$dP = P(r+dr) - P(r) < 0$$

$$F_G = -G\frac{Mdm}{r^2}$$

For zero acceleration

$$-G\frac{Mdm}{r^2} = dAdP$$

$$dm = \rho dAdr$$

$$G\frac{Mdm}{r^2} = G\frac{M\rho dAdr}{r^2}$$

$$\frac{dP}{dr} = -G\frac{M\rho}{r^2} = -\rho g(r)$$

Central Pressure Estimate?

$$\frac{dP}{dr} \sim \frac{P_s - P_c}{R_s - 0} = -\frac{P_c}{R_{sun}} = -G\frac{M_{sun}\bar{\rho}_{sun}}{R_{sun}^2}$$

* approximately 2.7 x 10^{14} Pascals
* More realistically 2.34 x 10^{16} Pascals or 2.3 x 10^{11} atmospheres
* P = 0 at surface, decreases from r=0 to r=surface

Mass Conservation

$$dM = 4\pi r^2 dr \rho(r)$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

This is different from the column mass described in your book

This is the total enclosed mass in a sphere of radius r

$$\frac{dP}{dM} = \frac{dP}{dr} \times \frac{dr}{dM} = -\frac{GM}{4\pi r^4}$$

For an isothermal atmosphere

$$P(r) = \frac{\rho(r) k_b T}{\mu m_h}$$

Problem 1

Virial Theorem

* Really a statement about how non-equilibrium systems evolve
* For us, we deal with equilibrium

$$\frac{dP}{dM} = \frac{dP}{dr} \times \frac{dr}{dM} = -\frac{GM}{4\pi r^4}$$

$$VdP = \frac{4}{3}\pi r^3 dP = -G\frac{M(r)}{3r}dM$$

$$\int_{P_{center}}^{P(r=0)} Vdp = PV|_{center}^{surface} - \int_0^{V_{star}} PdV$$

$$\int_0^{V_{star}} PdV = \frac{1}{3}\int_0^{M_{Star}} \frac{GM(r)}{r}\, dM$$

$$\Omega = -\int_0^{M_{Star}} \frac{GM(r)}{r}\, dM$$

Where omega is the gravitational binding energy of the star

$$-\Omega = 3\int_0^{V_{Star}} PdV$$

The energy density (ergs/cm^3) of a star with volume V, N particles, and temperature T is given by

$$e = \frac{3N}{2V} k_B T$$

From which the thermal pressure P may be written as

$$P = \frac{Nk_BT}{V} = \frac{2}{3}e$$

From which one may write

$$3\int_0^{V_{Star}} PdV = 2\int_0^{V_{Star}} edV = -\Omega$$

Where the integrated energy density is

$$\int_0^{V_{Star}} edV = U$$

Which results in the simplified form of the virial theorem we will use

$$2U + \Omega = 0$$

This is the statement that, for a star in equilibrium, twice the kinetic energy due to thermal motion plus the negative gravitational binding energy is zero

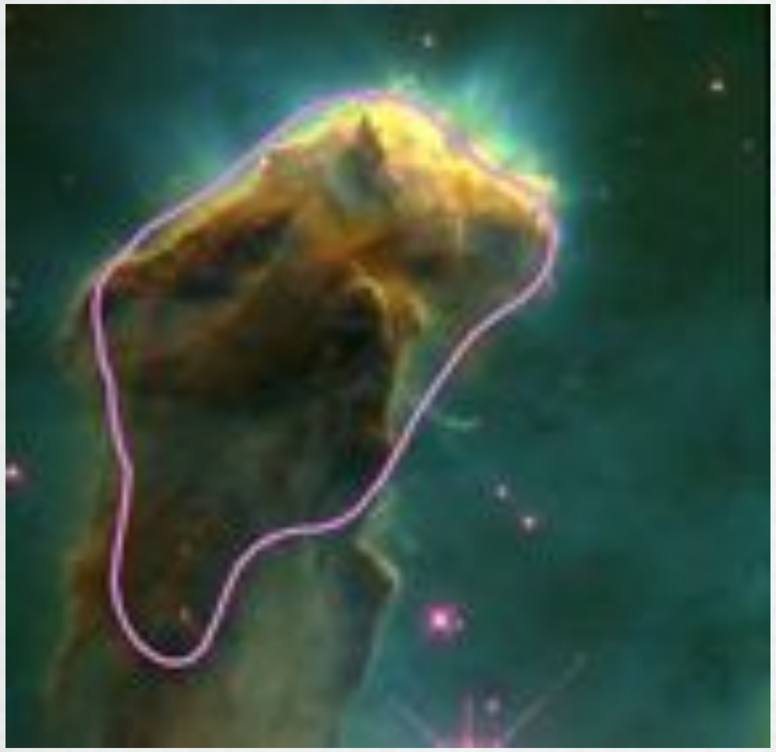
Jean's Length and Jean's Mass

* Stars form from collapsing clouds of gas
* When the self gravity due to overdensities is greater than the thermal pressure, turbulent pressure, and pressure due to the magnetic field an isolated chunk will collapse
* For now we neglect turbulent pressures and the magnetic fields

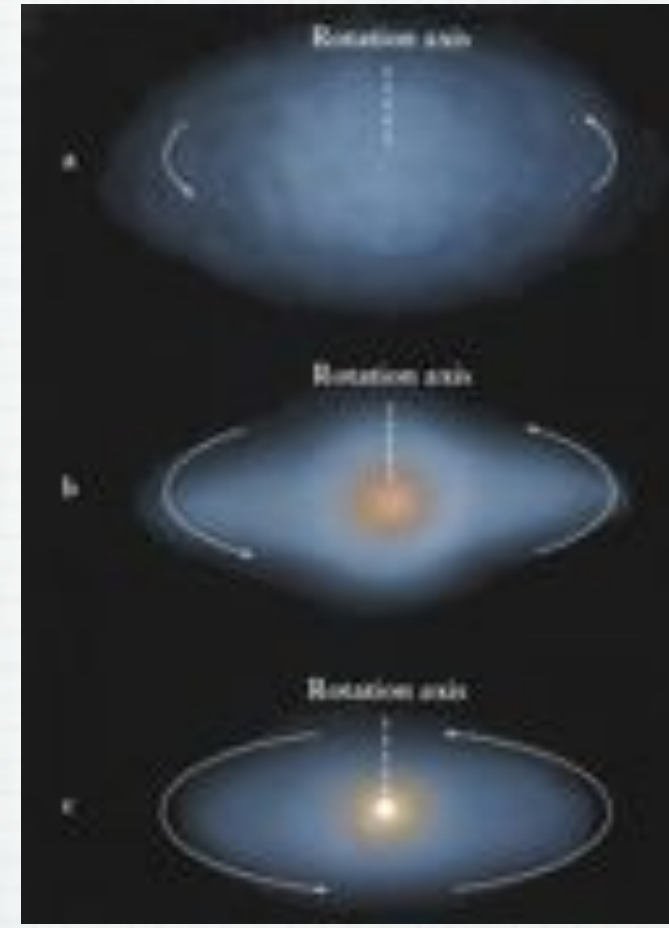
Jean's Length

* Essentially the radius within which the thermal pressure is unable to propagate a sound wave fast enough to overcome gravitational free-fall

Typical Molecular Cloud



* $M = 10 - 10^6\ M_{Solar}$
* $R = .1 - 10$ parsecs
* $T = 10K - 100K$
* $n = 10^4 - 10^6\ H/cm^3$
* Unionized, $\mu = 1$
* $\rho = 1000 m_H = 1.67 \times 10^{-(19\ to\ 21)}\ g/cm^3$



How to calculate the Jean's Length?

* When the cloud is out of virial equilibrium it will contract if it's gravitational energy is greater than its thermal energy
* It will expand when the opposite is true
* For collapse $-\Omega > 2U$

For a gas cloud that is spherical

$$\Omega = -\frac{3}{5}\frac{GM_{cloud}^2}{R_{cloud}}$$

$$U \sim eV_{cloud} = \frac{3}{2}Nk_Bt = \frac{3}{2}\frac{M_{cloud}}{\mu m_H}k_BT$$

$$\frac{3}{5}\frac{GM_{cloud}^2}{R_{cloud}} > \frac{3}{2}\frac{M_{cloud}}{\mu m_H}k_BT \times 2$$

From which R_{jeans}

$$R > \left(\frac{15k_BT}{4\pi\rho\mu m_H G}\right)^{1/2} = R_j$$

Jean's Mass and Density

* Using similar reasoning but equating

$$R_{cloud} = (\frac{M}{\frac{4}{3}\pi\rho})^{1/3}$$

* Leads to

$$M > (\frac{5k_BT}{\mu m_H G})^{3/2}(\frac{3}{4\pi\rho})^{1/2} = M_j$$

Typically of order 1 solar mass

* and, since density is simply mass divided by volume the unstable collapse density is

$$\rho > \rho_j = (\frac{5k_B T}{\mu m_H G})^3 (\frac{3}{4\pi M_{cloud}^2})$$

Time Scales

* Dynamical
* Kelvin Helmholtz
* Nuclear Burning

Dynamical

* Time scale on which a star would expand or contract if its balance between gravity and thermal pressure was disrupted

$$\Delta KE = \frac{1}{2}m\dot{r}^2 = -\Delta PE = GM_{star}m(\frac{1}{r} - \frac{1}{R_{star}})$$

$$\frac{dr}{dt} = -(\frac{2GM_{star}}{r} - \frac{2GM_{star}}{R})^{1/2}$$

Which may be integrated with the change of variables x = r/R

$$\int_0^{t_{dyn}} dt = -(\frac{R^3}{2GM_{star}})^{1/2} \int_0^1 (\frac{x}{1-x})^{1/2} dx$$

$$t_{dyn} = \frac{\pi}{2}(\frac{R^3}{2GM_{star}})^{1/2} = (\frac{3\pi}{32G\rho})^{1/2}$$

Simpler, order of magnitude

* t_{dyn} = (radius of star)/(escape velocity)
* t_{dyn} approximately $R^3/(2GM)^{1/2}$ = 1100 seconds for an existing star
* about 10^5 years for a solar mass star to collapse and form if isothermal
* approximately $1/(G\rho)^{1/2}$

Kelvin Helmholtz time scale

Suppose nuclear reaction were suddenly cut off in the Sun. Thermal time scale is the time required for the Sun to radiate all its reservoir of thermal energy:

$$\tau_{KH} = \frac{U}{L}$$

← Virial theorem: the thermal energy U is roughly equal to the gravitational potential energy

$$\Rightarrow \tau_{KH} \approx \frac{GM^2}{RL} = 3\times10^7 \text{ yr (for the Sun)}$$

Important time scale: determines how quickly a star contracts *before* nuclear fusion starts - i.e. sets roughly the pre-main sequence lifetime.

Nuclear Time Scale

Time scale on which the star will exhaust its supply of nuclear fuel if it keeps burning it at the current rate:

Energy release from fusing one gram of hydrogen to helium is 6 x 10^{18} erg, so:

$$\tau_{nuc} = \frac{qXM \times 6 \times 10^{18} \text{ erg g}^{-1}}{L}$$

...where:

- X is the mass fraction of hydrogen initially present (X=0.7)
- q is the fraction of fuel available to burn in the core (q=0.1)

$$\tau_{nuc} \approx 7 \times 10^{9} \text{ yr}$$

Reasonable estimate of the main-sequence lifetime of the Sun.

Ordering of time scales:

$$\tau_{dyn} << \tau_{KH} << \tau_{nuc}$$

Most stars, most of the time, are in hydrostatic and thermal equilibrium, with slow changes in structure and composition occurring on the (long) time scale τ_{nuc} as fusion occurs.

Do observe evolution on the shorter time scales also:

- Dynamical - **stellar collapse / supernova**
- Thermal / Kelvin-Helmholtz - **pre-main-sequence**

Homework

* All but problems 1 and 7