Stellar Astrophysics

Lecture 2

Reading questions?

- What's opacity?
- Bring reading questions to class from now on.

Temperature From Luminosity

- The flux received a distance r away from a star of radius R is

$$F(r) = \sigma T_{eff}^4 (\frac{R_{star}}{r})^2$$

- Which can be inverted to determine an effective temperature

Magnitude

- Kind of silly but more negative is brighter
  - Sun -26.74
  - Full Moon -12.92
  - Jupiter -2.94
  - 3-4 faintest naked eye stars in urban environment
  - 6.5 naked eye limit in good darkness ~9500 stars
  - 7-8 absolute best in darkest skies with great eyes

Magnitude difference

$$m_1 - m_2 = 2.5Log_{10}(\frac{F_2}{F_1})$$

$$F_2 = 10F_1 \rightarrow m_1 - m_2 = 2.5$$

For every 1 order of magnitude one object is 2.5 times brighter/dimmer

How about the sun vs the moon?

$$|m_1 - m_2| = |-26.73 + 12.7| = |-14.03| = 14.03$$

$$10^{14.03/2.5} = 10^{5.612}$$

- We receive about 400,000 times more light from the sun
- Every 5 orders of magnitude equates to 100 times the flux
- The basic equation to be inverted comes from the definition

$$\frac{F_2}{F_1} = 100^{\frac{m_1 - m_2}{5}}$$

In reality

- These are apparent magnitudes, what we see, m = apparent magnitude
- It would be nice to get some benchmark absolute magnitude
- We define an absolute magnitude as that which we would observe if the object were 10 parsecs away by M

Lets try again remembering the definition of flux

$$\frac{F_{10}}{F} = 100^{\frac{m-M}{5}} = (\frac{d_{parsecs}}{10})^2$$

We can invert this to find d if we know the intrinsic luminosity and to relate the apparent and absolute magnitude

$$d = 10^{\frac{m-M+5}{5}} pc$$

$$m - M = 5log_{10}(\frac{d}{10})$$

We can either get the distance the object is from us if we know its absolute magnitude or vice versa

Let's get the sun's absolute magnitude, it should be 4.83ish

In Reality

- No detector measures the flux in all wavelengths
- We generally measure in bands, Blue, Red, Ultraviolet
- There is also selective extinction at different wavelengths due to dust which scatters blue light more than red
- Google Color Index and Bolometric Magnitude for details

How do we know what energy levels/Ionization States a star is in?

- Sort of important
- We need to know the excitation and ionization states the constituents in a star's atmosphere occupy to determine what we expect to see

Enter the Boltzmann and Saha Equations

- But first, a digression
- We will need to have an equation describing the pressure in a star when discussing its structure
- There are pressure due to particles whizzing about and due to radiation
- Particle pressure first

$$P_{gas} = \frac{\rho K_B T}{\mu m_H}$$

$$\rho = \sum_i n_i m_i$$ mass density

$$\mu = \frac{1}{m_h n_{tot}} \sum_i n_i m_i$$ mean molecular weight

In general the pressure of a gas is $P=n_{tot}k_BT$

For a partially ionized hydrogen plasma $P = (n_I+n_{II}+n_e)k_BT$

Boltzmann Equation

- For a system dominated by collisions the proportion of atoms in excited state b to those in excited states a is given by the Boltzmann equation

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{\frac{-(E_b - E_a)}{k_B T}} \qquad \frac{n_i}{n_{ion}} = \frac{g_i}{U_{ion}} e^{\frac{-E_i}{k_B T}}$$

- Where the g's represent the degeneracy of the state
- Remember, for a hydrogen atom there are two spin states
- The degeneracy for hydrogen is then $2n^2$ where n is the energy level
- U_{ion} is a partition function, it will be given

$$E_n = 13.6[1 - \frac{1}{n^2}]eV$$

Let's do an example

- At what temperature does a system have an equal number of atoms in the first and second energy level or

$$1 = \frac{2(2^2)}{2(1^2)} e^{(-13.6eV/2^2 - (-13.6eV/1^2))/k_B T}$$

$$\frac{10.2eV}{k_B T} = ln(4)$$

$T = 8.54 \times 10^4$ K

The Saha Equation

- Or, how to determine relative ionization states
- Z's are partition functions, a way to calculate a weighted sum for the different electronic transitions that result in the same energy level.
- Take stat therm for more

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}(\frac{2\pi m_e k_B t}{h^2})^{3/2} e^{-E_i/k_B T}$$

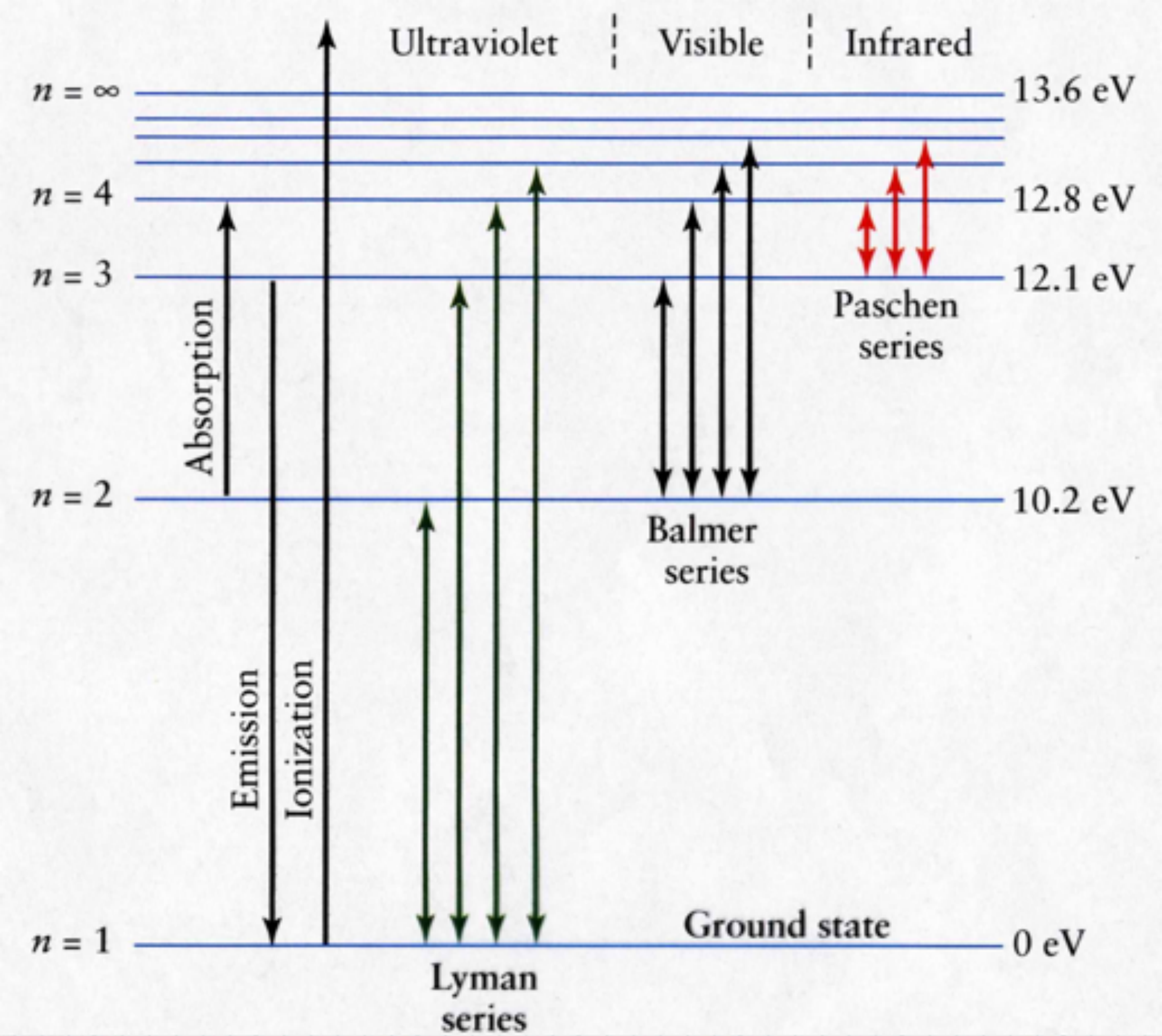
Problem 11

Homework Hint

- Electron pressure = $n_e k_B T$
- Rewrite the Saha using this or use this afterwards
- Let's work problem 10

Spectral classification

- Let's start with non-ionizing hydrogen transitions



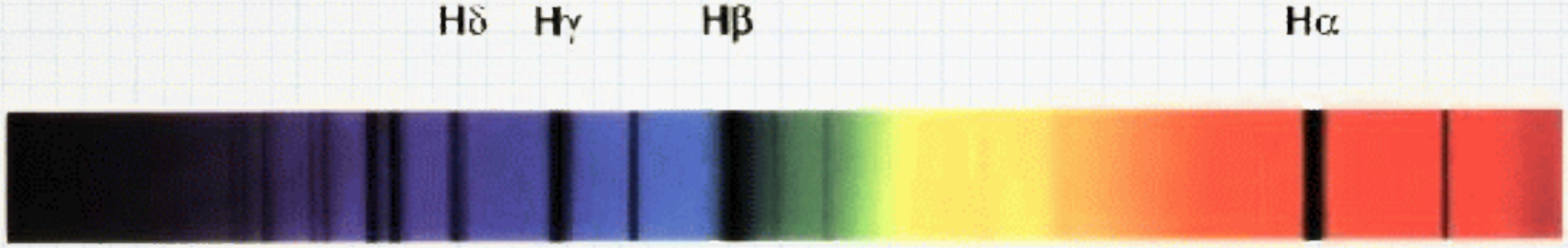
$$\Delta E_{mn} = \frac{hc}{\lambda_{n\to m}}$$

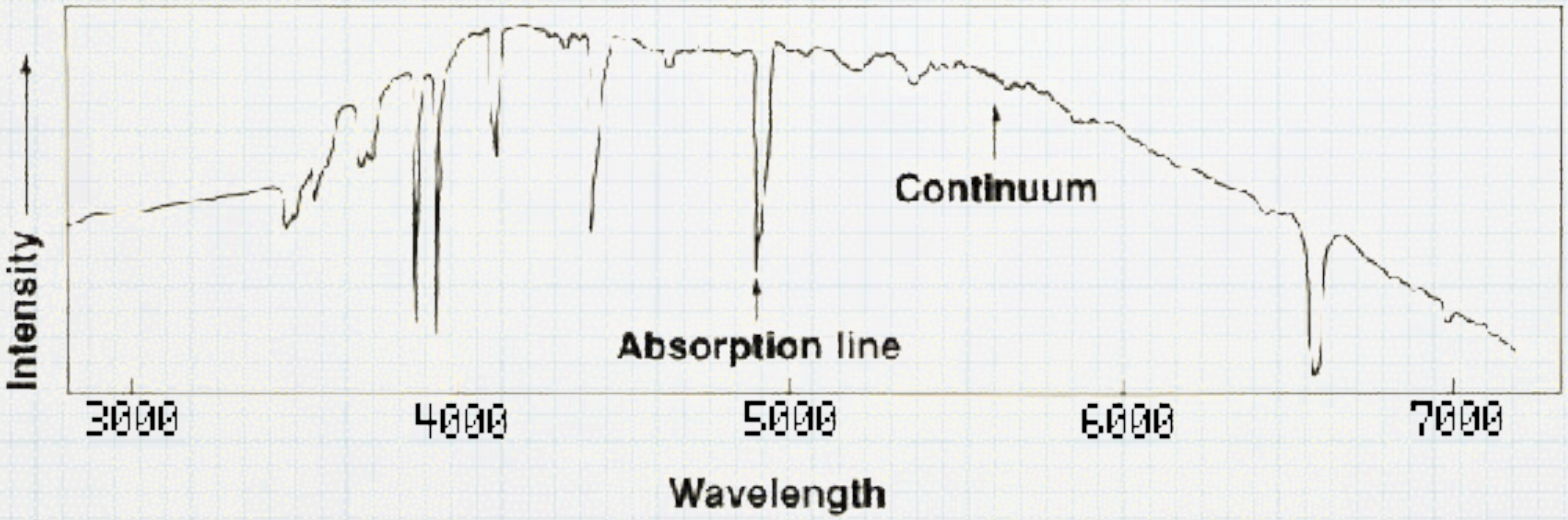
$\alpha = 1$

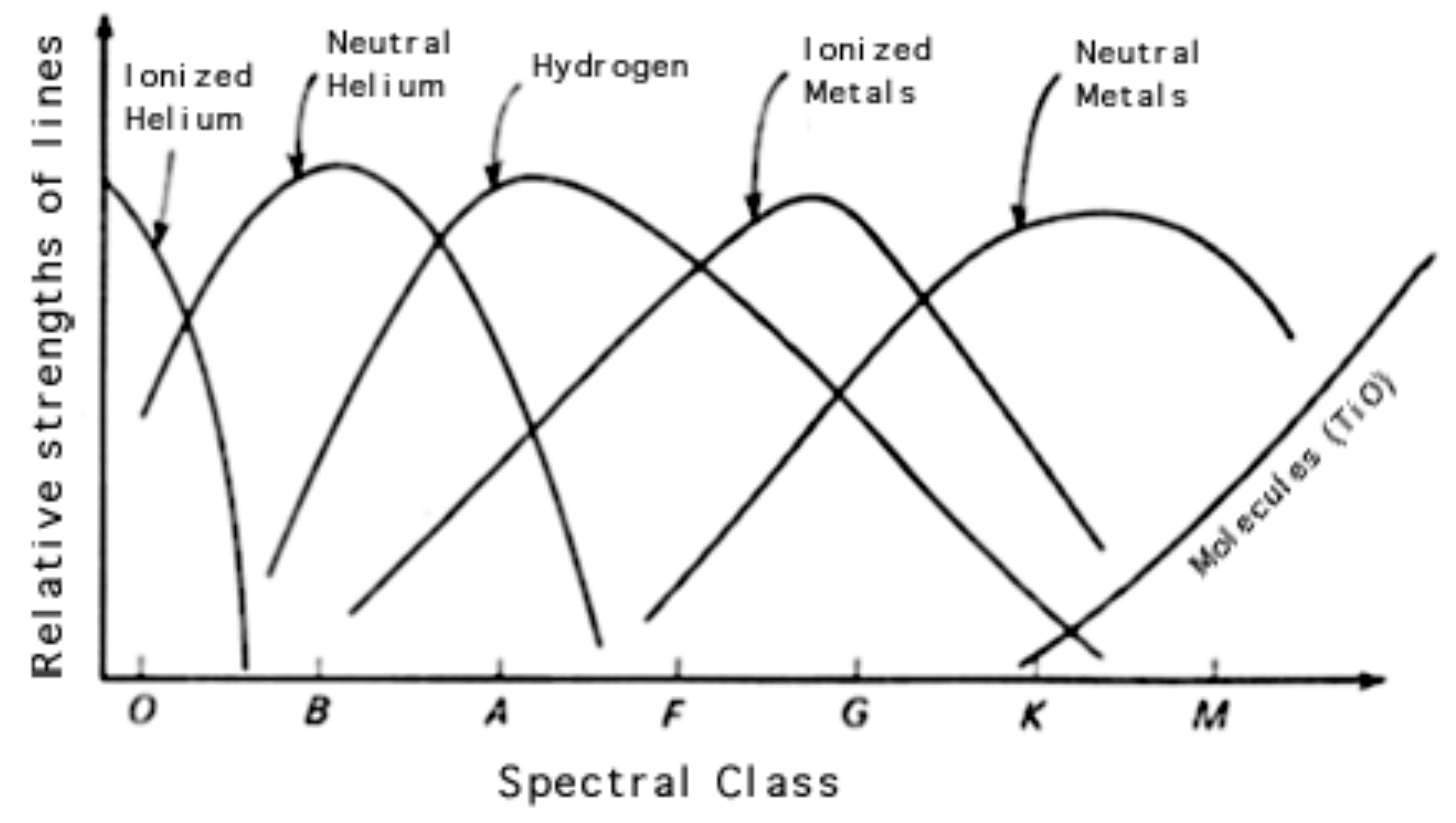
$\beta = 2$

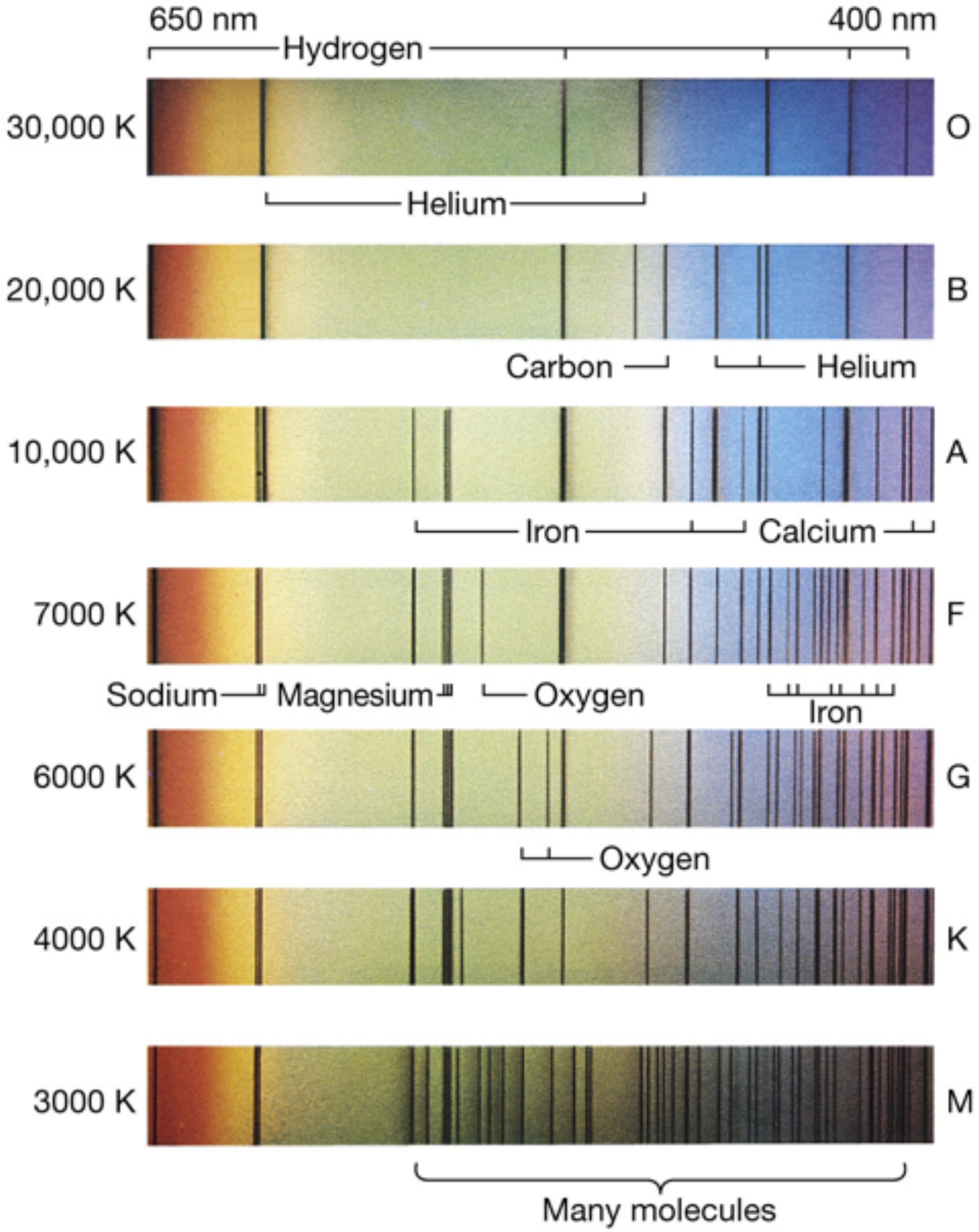
$\gamma = 3$

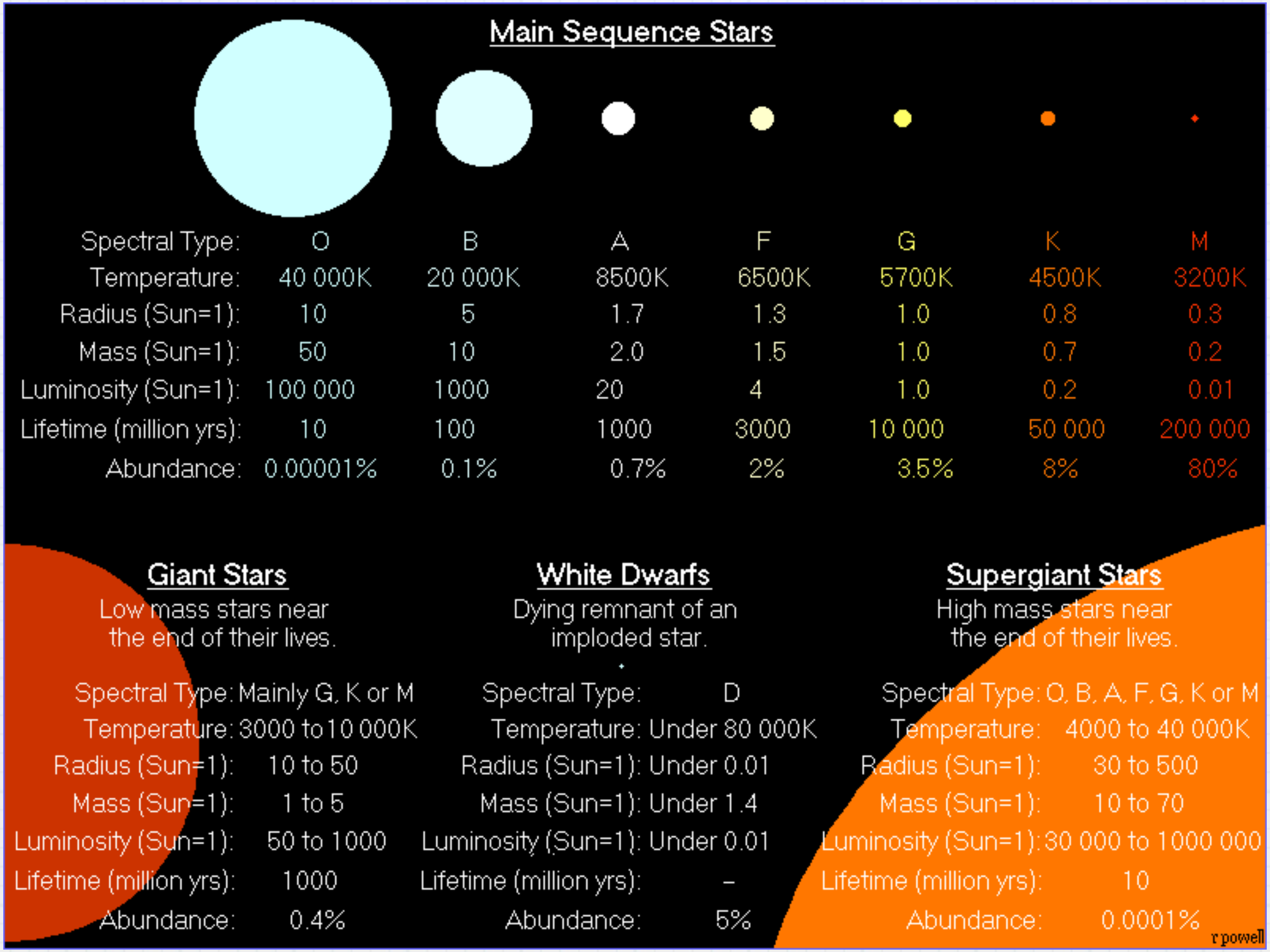
etc


Hδ
Hγ
Hβ
Hα


Intensity
Continuum
Absorption line
3000
4000
5000
6000
7000
Wavelength


Relative strengths of lines
Ionized Helium
Neutral Helium
Hydrogen
Ionized Metals
Neutral Metals
Molecules (TiO)
O
B
A
F
G
K
M
Spectral Class


650 nm
400 nm
Hydrogen
30,000 K
O
Helium
20,000 K
B
Carbon
Helium
10,000 K
A
Iron
Calcium
7000 K
F
Sodium
Magnesium
Oxygen
Iron
6000 K
G
Oxygen
4000 K
K
3000 K
M
Many molecules



Main Sequence Stars

Spectral Type:	O	B	A	F	G	K	M
Temperature:	40 000K	20 000K	8500K	6500K	5700K	4500K	3200K
Radius (Sun=1):	10	5	1.7	1.3	1.0	0.8	0.3
Mass (Sun=1):	50	10	2.0	1.5	1.0	0.7	0.2
Luminosity (Sun=1):	100 000	1000	20	4	1.0	0.2	0.01
Lifetime (million yrs):	10	100	1000	3000	10 000	50 000	200 000
Abundance:	0.00001%	0.1%	0.7%	2%	3.5%	8%	80%

Giant Stars

Low mass stars near the end of their lives.

Spectral Type:	Mainly G, K or M
Temperature:	3000 to 10 000K
Radius (Sun=1):	10 to 50
Mass (Sun=1):	1 to 5
Luminosity (Sun=1):	50 to 1000
Lifetime (million yrs):	1000
Abundance:	0.4%

White Dwarfs

Dying remnant of an imploded star.

Spectral Type:	D
Temperature:	Under 80 000K
Radius (Sun=1):	Under 0.01
Mass (Sun=1):	Under 1.4
Luminosity (Sun=1):	Under 0.01
Lifetime (million yrs):	–
Abundance:	5%

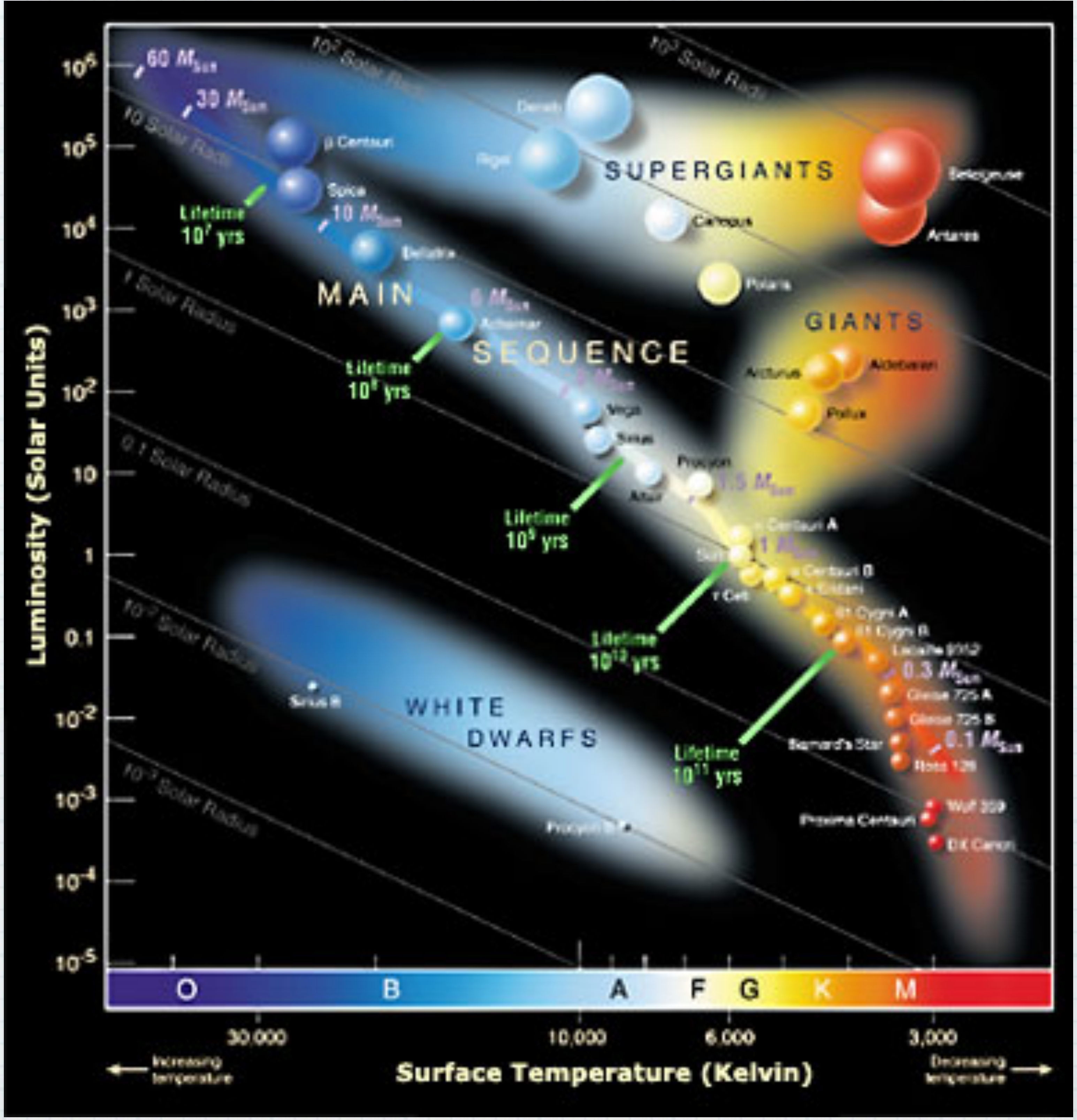
Supergiant Stars

High mass stars near the end of their lives.

Spectral Type:	O, B, A, F, G, K or M
Temperature:	4000 to 40 000K
Radius (Sun=1):	30 to 500
Mass (Sun=1):	10 to 70
Luminosity (Sun=1):	30 000 to 1000 000
Lifetime (million yrs):	10
Abundance:	0.0001%

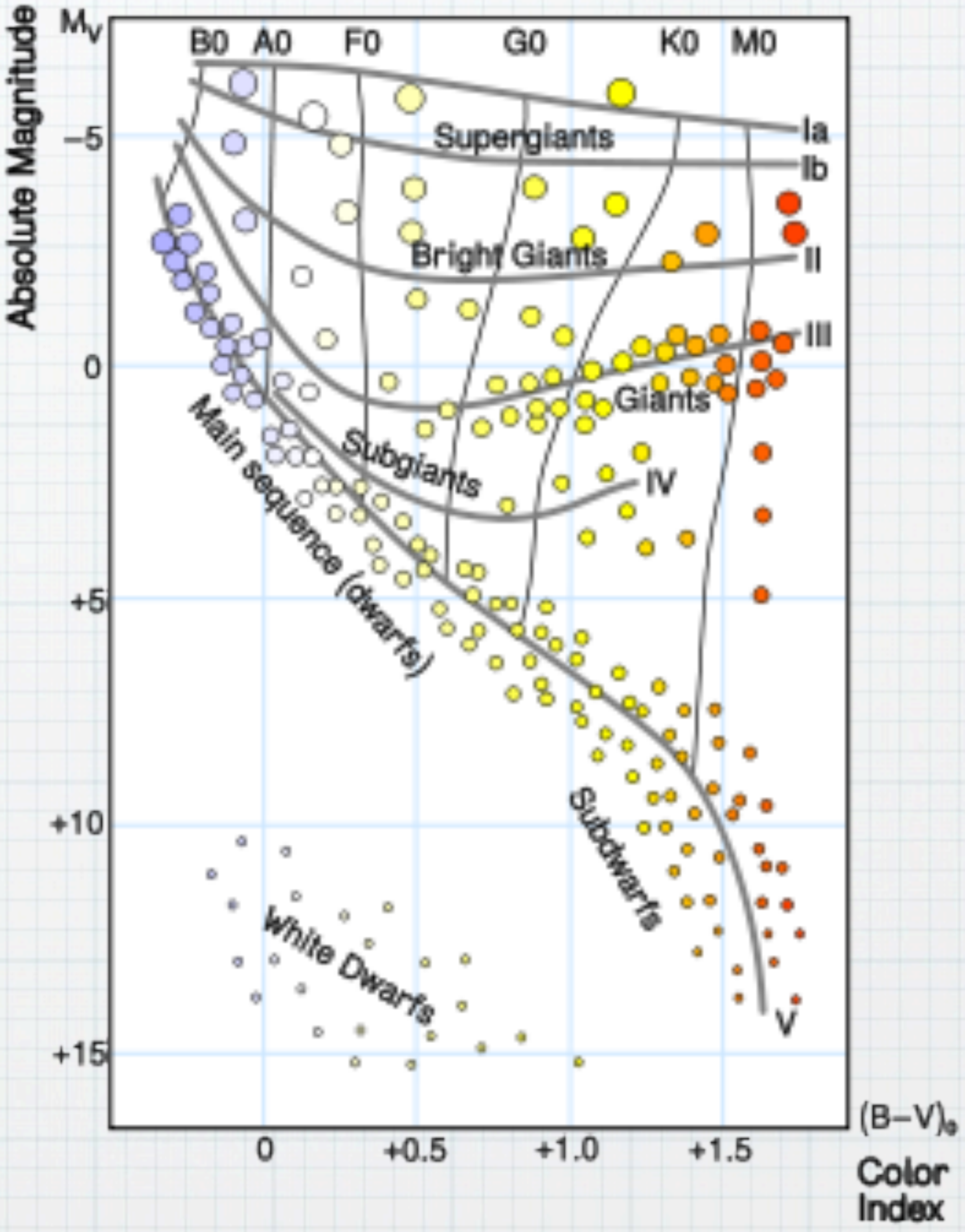
r powell

HR Diagram

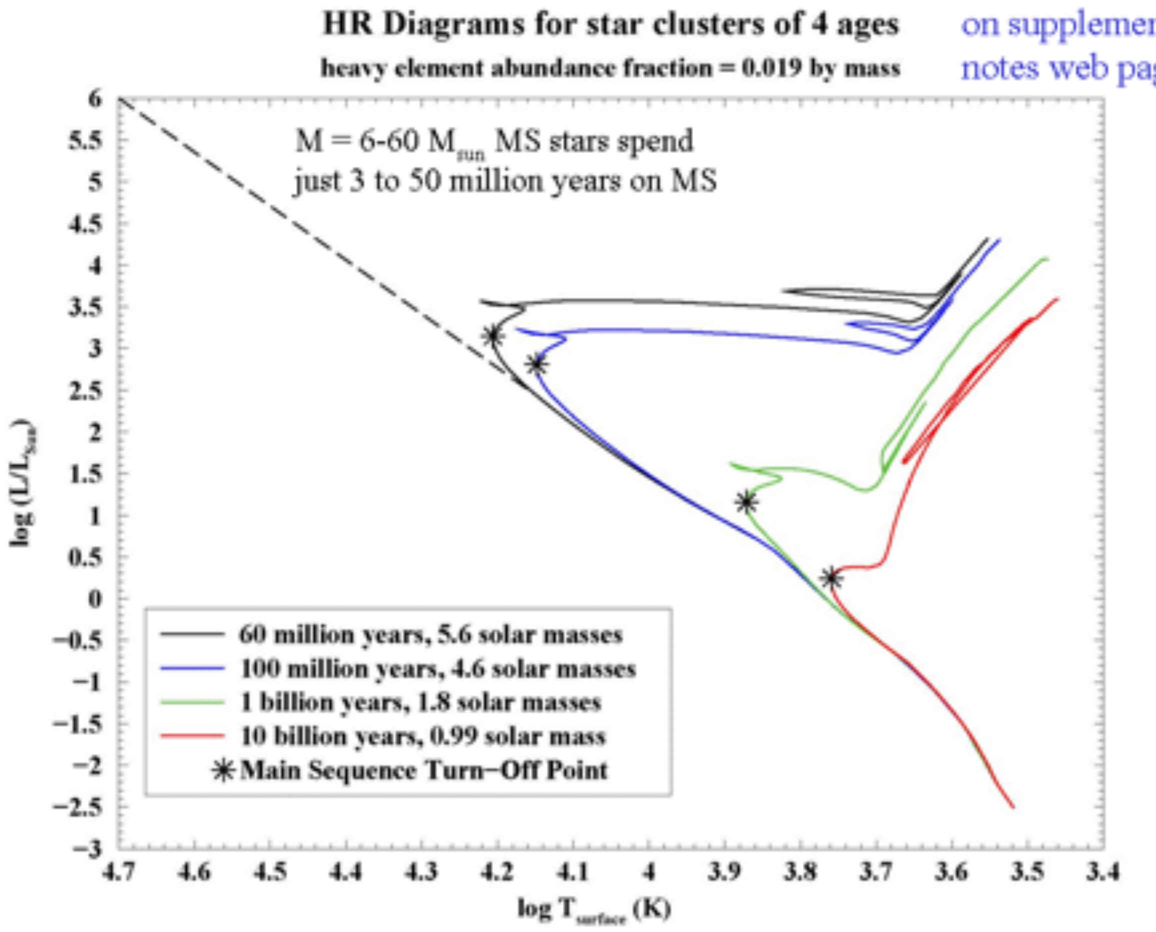

Luminosity (Solar Units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^3 Solar Radii
10^2 Solar Radii
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10^{-2} Solar Radius
10^{-3} Solar Radius
60 M_{Sun}
30 M_{Sun}
β Centauri
Spica
10 M_{Sun}
Bellatrix
Lifetime 10^7 yrs
MAIN
6 M_{Sun}
Achernar
SEQUENCE
Lifetime 10^8 yrs
3 M_{Sun}
Vega
Sirius
Altair
Procyon
1.5 M_{Sun}
Lifetime 10^9 yrs
α Centauri A
Sun
1 M_{Sun}
α Centauri B
τ Ceti
ε Eridani
Lifetime 10^{10} yrs
61 Cygni A
61 Cygni B
Lacaille 9352
0.3 M_{Sun}
Gliese 725 A
Gliese 725 B
Barnard's Star
0.1 M_{Sun}
Ross 128
Lifetime 10^{11} yrs
Wolf 359
Proxima Centauri
DX Cancri
Deneb
Rigel
SUPERGIANTS
Betelgeuse
Antares
Canopus
Polaris
GIANTS
Aldebaran
Arcturus
Pollux
Sirius B
WHITE DWARFS
Procyon B
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
Increasing temperature
Surface Temperature (Kelvin)
Decreasing temperature

Terminology

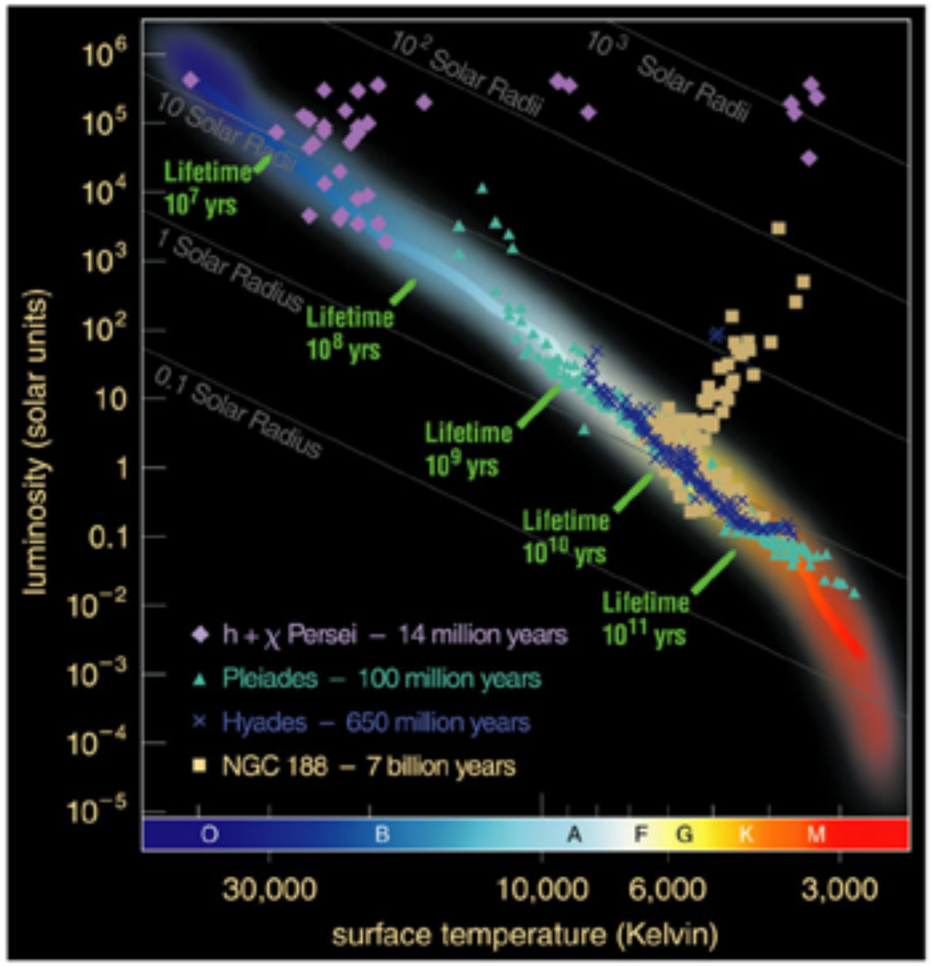
- Letter - Spectral Class
- Number 0-9, lower is hotter
- Roman Numeral
    - I,II,III,IV,V
- Sun is G2V



Cluster Identification



on supplementary notes web page



How Many of Each Type?

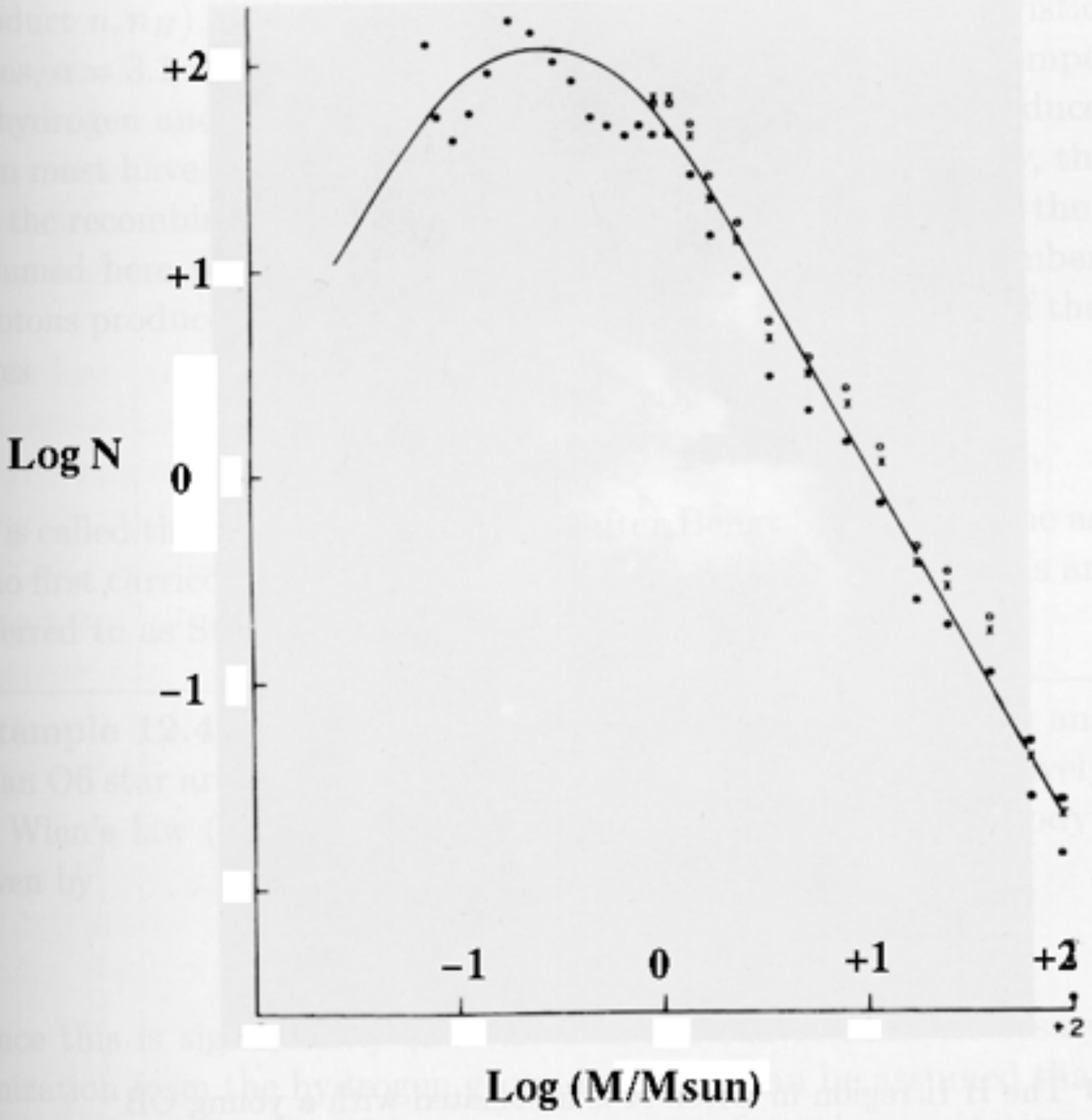
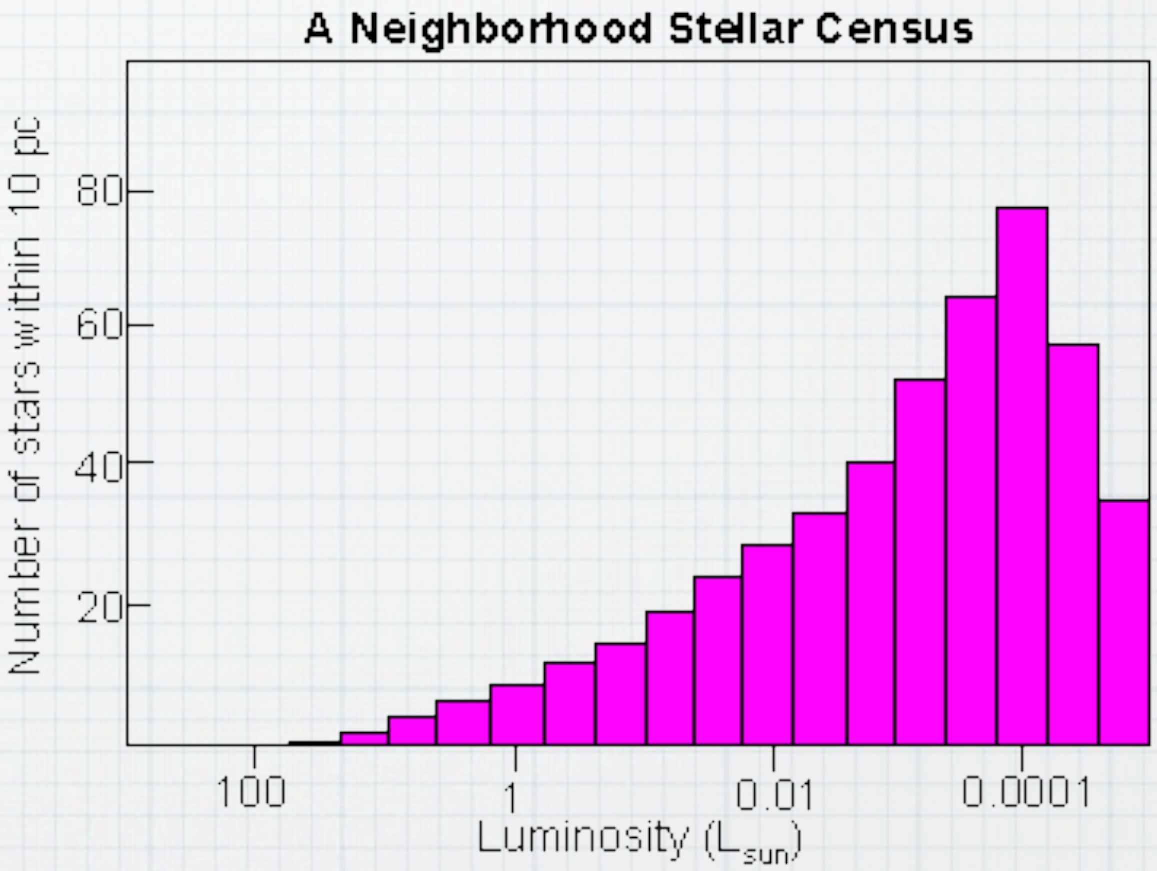


Figure 12.9 The initial mass function, ξ, shows the number of stars per unit area of the Milky Way's disk per unit interval of logarithmic mass that is produced in different mass intervals. Masses are in solar units. (Figure adapted from Rana, *Astron. Astrophys.*, *184*, 104, 1987.)



Homework

- 1 - calculate total luminosity from earth, assume T=300K
- 2 Extra Credit
- 6 -must do integral numerically
- 9 Results in a quadratic $U_1=2$ $U_2=1$, $N_e = N_{II}$
- skip 11,14