Statistical and Thermal Physics: Homework 3

Due: 30 January 2026

1 Thermal expansion

The linear thermal expansion coefficient describes the increase in length of a homogenous material as its temperature changes and is defined by

$$\alpha_1 := \frac{1}{L}\frac{\partial L}{\partial T}$$

where L is the length of the material. *The subscript in α_1 is not standard notation; it is included to distinguish the linear thermal expansion coefficient from the volume thermal expansion coefficient.*

a) For copper, $\alpha_1 = 1.65\times10^{-5}\,\mathrm{K}^{-1}$. Determine the amount by which a copper rod of length 10 m will expand if it is heated from -10° C to 30° C, assuming that α is independent of temperature.

b) A rectangular sheet of any material will expand as its temperature increases. Here the coefficient of area expansion is defined by

$$\gamma := \frac{1}{A}\frac{\partial A}{\partial T}$$

where A is the area of the material. Show that

$$\gamma = 2\alpha_1.$$

Determine the amount by which the area of a rectangular copper roof, whose sides are 10 m and 4 m increases if it is heated from -10° C to 30° C.

2 Thermal expansion coefficient for a van der Waals gas

Determine an expression for the thermal expansion coefficient for a van der Waals gas. *Hint: note that differentiation of V w.r.t. T will be difficult. There is an identity that we encountered in class that will make this easier.*

3 Isothermal compressibility of an ideal gas

Determine an expression for the isothermal compressibility of an ideal gas, in terms of N, P and T. Show that it is positive.

4 Equation of state for a solid

The state of a solid material can be described by the same variables as for a gas. Suppose that the equation of state of the solid is

$$V = V_0 \left(1 + aT - bP\right)$$

where V_0 is a constant equal to the volume when pressure and temperature are zero and a and b are constants that are very small.

a) Determine expressions for the isothermal compressibility and the isobaric expansion coefficient in terms of a, b, T and P.

b) Suppose that a and b are so small that at typical temperatures $aT \ll 1$ and $bP \ll 1$. Determine expressions (that retain the largest non-zero terms), for the isothermal compressibility and the isobaric thermal expansion coefficient in this case. Are they approximately constant or not?