Statistical and Thermal Physics: Final Exam

12 May 2020

Name: ____________________________ **Total:** /60

Instructions

- There are 8 questions on 11 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$R = 8.31\,\text{J/mol K} \qquad N_A = 6.02\times10^{23}\,\text{mol}^{-1} \qquad 1\,\text{atm} = 1.01\times10^{5}\,\text{Pa}$$

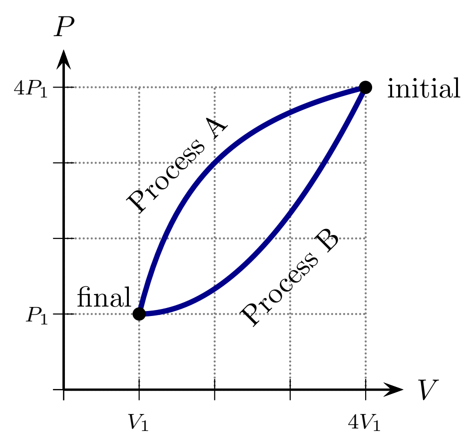
$$k = 1.38\times10^{-23}\,\text{J/K} = 8.61\times10^{-5}\,\text{eV/K} \qquad e = 1.60\times10^{-19}\,\text{C} \qquad \hbar = 1.05\times10^{-34}\,\text{Js}$$

$$N! \approx N^N e^{-N}\sqrt{2\pi N} \qquad \ln N! \approx N\ln N - N \qquad \sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \qquad e^x \approx 1 + x \quad \text{if } x \ll 1$$

$$\int_{-\infty}^{\infty} e^{-ax^2}\mathrm{d}x = \sqrt{\frac{\pi}{a}} \qquad \int_0^{\infty} x e^{-ax^2}\mathrm{d}x = \frac{1}{2a} \qquad \int_0^{\infty} x^2 e^{-ax^2}\mathrm{d}x = \sqrt{\frac{\pi}{4a^3}} \qquad \int_0^{\infty} x^3 e^{-ax^2}\mathrm{d}x = \frac{1}{2a^2}$$

Question 1

A diatomic ideal gas initially undergoes one of the two indicated compressions between the same initial and final states. In each of the following explain your answers.



a) Is the work done on the gas in process A larger than, smaller than or the same as that done on the gas in process B?

b) Is the heat added to/removed from the gas in process A larger than, smaller than or the same as that for the gas in process B?

c) Is the change in entropy in process A larger than, smaller than or the same as that for the gas in process B?

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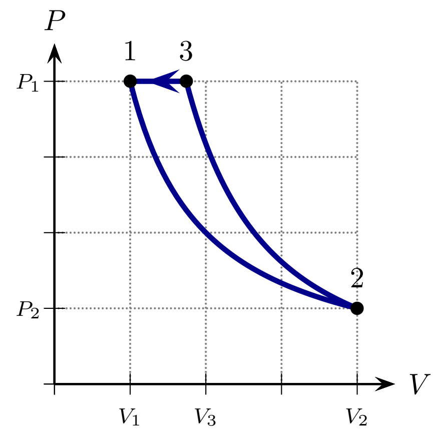
Question 2

A monoatomic ideal gas is initially in a state with pressure P_1 and volume V_1. It undergoes an isothermal expansion to state 2, where $V_2 = 4V_1$. This is followed by an adiabatic compression to state 3, with pressure $P_3 = P_1$. Finally the gas is compressed at constant pressure to its original volume. The diagram illustrates this.



a) Show that $P_2 = P_1/4$.

b) Show that $V_3 = V_1 4^{(\gamma-1)/\gamma}$ where $\gamma = 5/3$ is the adiabatic constant.

Question 2 continued ...

c) Determine the work done, the heat added or removed and the change in energy for the portion of the cycle from state 2 to state 3. Your answers must be an expression in terms of P_1, V_1 and numbers.

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Question 3

Answer either part a) or part b) for full credit for this problem.

a) Starting with the Gibbs free energy, $G = E - TS + PV$, and assuming that the number of particles is fixed, show that

$$\mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}P.$$

Use this to i) describe whether the Gibbs free energy increases, decreases or stays the same during an isothermal expansion of an ideal gas and ii) prove the Maxwell relation

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T.$$

Question 3 continued ...

b) A small quantity of metal, with mass m and heat capacity c_{metal}, is initially at temperature 500 K. It is placed into a large bath of water with temperature initially at 300 K. The bath is so large that its temperature does not change; the metal reaches equilibrium at the initial temperature of the bath; the process occurs at constant pressure. Assume that the heat capacity of the metal is independent of temperature. Determine expressions for the change in entropy of the metal and the change in entropy of the water (in terms of m and c_{metal}).

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Question 4

Consider two systems of spin-1/2 particles. Each spin-1/2 particle has magnetic dipole moment of magnitude μ and is in the same external magnetic field B. System A has 4 particles and system B has 6 particles. System A initially has energy $-2\mu B$ (i.e. three particles have spin up) and system B initially has energy $2\mu B$ (i.e. two particles have spin up). The two systems are placed into contact and can exchange energy with each other (but not particles).

a) List the possible macrostates of the combined system after they are in thermal contact. List the energy for each macrostate and the multiplicity of each macrostate.

b) Determine the probability that system A has a larger energy than system B (after they have interacted).

c) Describe the equilibrium state of the combined system.

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Question 5

An Einstein solid contains three oscillators, labeled A, B, C, each with the same frequency. The following table provides three possible microstates of the system, described by the number of energy units that each oscillator has. The only quantity that can be measured for the oscillators is the total energy.

State	q_A	q_B	q_C
State 1	2	0	2
State 2	2	1	2
State 3	1	1	2

a) Describe which of these microstates represent the same macrostate.

b) Let p_1 be the probability with which state 1 might occur and p_3 be the probability with which state 3 might occur. Which of the following is true?

i) $p_1 = p_3$
ii) $p_1 > p_3$
iii) $p_1 < p_3$

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Question 6

A toy particle has two states with energies $0, \epsilon$. Suppose a system of these contains two indistinguishable particles.

a) List all possible microstates of the system if the particles are Bosons. Determine the partition function and mean energy for this system.

b) List all possible microstates of the system if the particles are Fermions. Determine the partition function and mean energy for this system.

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Question 7

Consider N distinguishable identical two-dimensional classical oscillators in contact with a reservoir at temperature T. The energy of any single oscillator is given by

$$E = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega^2\mathbf{r}^2$$

where $\mathbf{r}$ is the two dimensional position vector, $\mathbf{p}$ is the two dimensional momentum vector and m is the mass of the oscillator.

a) Determine the partition function for the system.

b) Determine an expression for the energy of the system and the heat capacity of the system.

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Question 8

Answer either part a) or part b) for full credit for this problem.

a) The states of a hydrogen atom are described by an integer $n = 1, 2, \ldots$ and these have energies $E_n = -13.6\,\text{eV}\dfrac{1}{n^2}$. For $n = 1$ there are two states available to the hydrogen atom and for $n = 2$ there are eight states available. Determine the temperature at which the hydrogen atom must be such that the probability with which $n = 2$ equals that for which $n = 1$ regardless of the particular states within each energy level.

Question 8 continued . . .

b) A particular particle can be in one of three energy states, with energies $-\epsilon, 0$ and $+\epsilon$. Determine an expression expression for the probability with which a single particle is in the state with energy $-\epsilon$. In the limit as $T \to 0$ what does this probability approach?

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