Modern Physics: Final Exam

19 May 2021

Name: ______________________________ **Total:** **/100**

Instructions

- There are 15 questions on 10 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$c = 3.0 \times 10^8\,\mathrm{m/s}$ $h = 6.63 \times 10^{-34}\,\mathrm{Js}$ $k_B = 1.38 \times 10^{-23}\,\mathrm{J/K}$ $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$

$m_{\mathrm{electron}} = 9.11 \times 10^{-31}\,\mathrm{kg}$ $m_{\mathrm{proton}} = 1.67 \times 10^{-27}\,\mathrm{kg}$ $m_{\mathrm{neutron}} = 1.67 \times 10^{-27}\,\mathrm{kg}$

Question 1

Two light sources emit the same number of photons each second. The light sources have different wavelengths. Which of the following (choose one) is true?

i) The power produced by the source with the larger wavelength is larger than that produced by the source with the smaller wavelength.
ii) The power produced by the source with the larger wavelength is smaller than that produced by the source with the smaller wavelength.
iii) The two sources produce the same power.

Briefly explain your answer.

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Question 2

A fullerene molecule consists of 60 carbon atoms and has mass 1.196×10^{-24} kg. A beam of such molecules is fired with speed 3.000×10^{5} m/s directly toward a barrier that contains a single slit. The particles that pass through the slit emerge on a distant screen. The smallest deflection such that the arrival probability on the screen is zero is 2.000×10^{-9} rad.

a) Determine the width of the slit.

b) Which of the following (choose all that are correct) could change the probability with which particles are deflected at this angle?

i) Firing more particles toward the screen.
ii) Decreasing the particle speed.
iii) Decreasing the slit width.

Explain your answer.

/8

Question 3

Let $z_1 = 3e^{i\pi/3}$ and $z_2 = 2ie^{i\pi/6}$. Let $z = z_1 z_2$. Determine Re[z], i.e. the real part of z.

/6

Question 4

A hydrogen atom emits electromagnetic radiation in a process in which the *final state* of the electron is one of the $n = 1$ energy level states. Determine the extremes of the possible wavelengths that the atom could have emitted in this process.

/6

Question 5

The possible energies of a system are $-4\,\text{eV}$ and $4\,\text{eV}$. The probabilities with which either can occur are 3/4 (for the lower energy state) and 1/4 (for the higher energy state).

a) Determine the mean of the energy of the system.

b) Determine the standard deviation of the energy of the system.

/6

Question 6

A free particle in one dimension is such that the wavefunction that describes its state at one instant is

$$\psi(x) = Be^{-(x-x_0)^2/4a^2}$$

where x_0, a and B are constants. The position of the particle is measured. Which of the following (choose one) is true regarding the outcome of the position measurement?

i) All outcomes are equally likely.
ii) A particular outcome is most likely and this depends on a.
iii) A particular outcome is most likely and this does not depend on a.

/4

Question 7

A particle with mass m is trapped in a one-dimensional infinite well with potential

$$U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise.} \end{cases}$$

The particle is set up so that at one instant it can *only be located in the left half of the well* and its state is described by

$$\psi(x) = \begin{cases} \sqrt{\dfrac{4}{L}} \sin\left(\dfrac{2\pi x}{L}\right) & 0 < x < L/2 \\ 0 & \text{otherwise} \end{cases}$$

This is normalized. Determine the expectation value of position measurements for this particle.

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Question 8

An alpha particle consists of two protons and two neutrons. This particle is trapped in an infinite well with width 0.20 nm. Determine the largest wavelength of the electromagnetic radiation which this particle could emit.

/6

Question 9

A harmonic oscillator can emit electromagnetic radiation. The lowest energy photon that it can emit has energy 5.0 eV. Is it possible for the oscillator to emit a photon with energy 8.0 eV? Explain your answer.

/6

Question 10

A particle with mass m is trapped in a one-dimensional infinite well with potential

$$U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{otherwise.} \end{cases}$$

The particle is set up so that at one instant its state is described by

$$\psi(x) = \begin{cases} \sqrt{\dfrac{30}{L^5}}\, x(x-L) & 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

This is normalized. Determine the expectation value of momentum measurements for this particle.

/9

Question 11

A quantum harmonic oscillator has potential

$$U(x) = \frac{1}{2} m\omega_0^2 x^2.$$

Consider the following as a possible energy eigenstate:

$$\psi(x) = \begin{cases} Ae^{bx} & \text{if } x < 0 \\ Ae^{-bx} & \text{if } x > 0. \end{cases}$$

where $b > 0$ and A is a constant. Verify, by direct substitution, whether or not this is an energy eigenstate (stationary state) for the system.

/10

Question 12

A hydrogen atom is in a state for which the magnitude of the angular momentum is $L = \hbar\sqrt{30}$. Determine the maximum and minimum values of the z-component of the angular momentum for the atom in this state.

/4

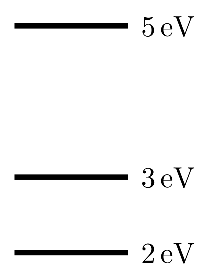
Question 13

A beam of identical particles is fired through a Stern-Gerlach apparatus/experiment. The particles arrive at six approximately equally spaced locations on a screen beyond the magnets. Determine the spin, s, of the particles.

/4

Question 14

A quantum system has the illustrated energy levels for a single particle. In separate situations there are three identical particles with various spins in the system.



a) Determine the minimum total energy for the collection of particles if they are Bosons.

b) Determine the minimum total energy for the collection of particles if they are spin-1/2 Fermions.

c) Determine the minimum total energy for the collection of particles if they are spin-3/2 Fermions.

/6

Question 15

The radial part of the hydrogen atom wavefunction for $n = 3, l = 2, m_l = 0$ is

$$R(r) = \frac{1}{27(3a_0)^{3/2}} \sqrt{\frac{8}{5}} \frac{r^2}{a_0^2} e^{-r/3a_0}$$

where a_0 is the Bohr radius. This is correctly normalized. Determine the expectation value of r.

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