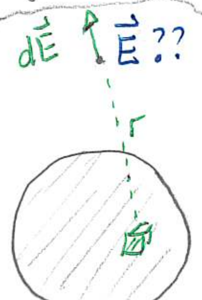
Tues: Discussion / quiz

Ex: 31, 36, 38, 39, 40, 41

Weds: Labs

Electric fields produced by continuous charge distributions

In many situations it is convenient to approximate a charge distribution by a continuous charge distribution, and to calculate the electric field produced by the continuous charge distribution. For example we could consider a uniformly charged sphere



total charge Q

→ volume charge density $\rho = Q/V$

→ Decompose into infinitesimal segments and approximat each as a point charge

→ use

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

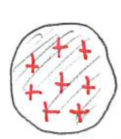
plus direction to get field from segment

→ Integrate over all segments to determine field

$$\vec{E} = \int_{\text{all segments}} d\vec{E}$$

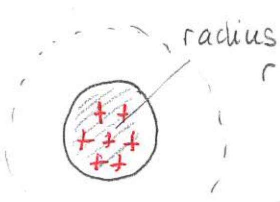
An alternative strategy uses a mathematical law called Gauss' Law. The correct formalism involves vector calculus. However a simple example is:

Suppose that the charge distribution is spherically symmetric, centered at the origin



→ Construct another sphere also centered at origin

Then



Then

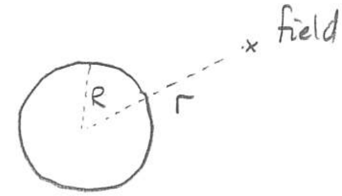
$$E \underbrace{4\pi r^2}_{\text{area sphere}} = \frac{1}{\epsilon_0} \text{ total enclosed charge}$$

Gauss' Law allows us to determine fields in certain cases.

1) <u>Uniformly charged (hollow) sphere</u>

Then the electric field is

$$\vec{E} = \begin{cases} 0 & \text{inside sphere} \\ \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2} \text{ radially out} & \text{outside sphere.} \end{cases}$$



sphere radius R
total charge $Q > 0$

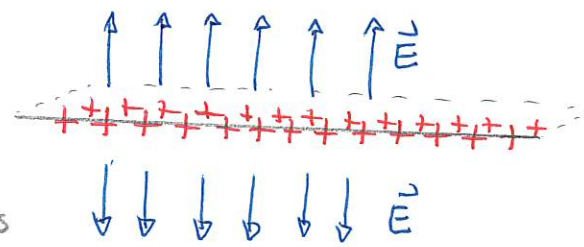
where r is the distance from the center of the sphere

2) <u>Uniformly charged infinite sheet</u>

Consider a flat sheet that is infinite in extent. Let

η = charge per unit area

(surface charge density). Symmetry dictates that the field must point perpendicular to the sheet and is away from the sheet for positive charge density and toward for negative charge density



Warm Up 1

Using Gauss' law or summation of fields gives.

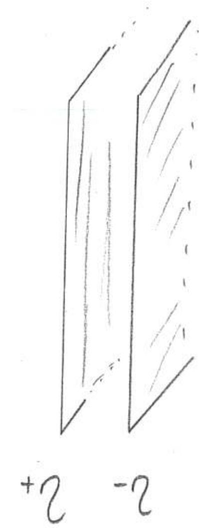
For an uniformly charged infinite sheet the electric field is uniform on either side with

1) magnitude $E = \eta/2\epsilon_0$

2) direction $= \begin{cases} \text{away from sheet (positive sheet)} \\ \text{toward sheet (negative sheet).} \end{cases}$

Parallel Plate Capacitor

A parallel plate capacitor consists of two identical infinite sheets that are parallel. They are charged so that they carry equal and opposite charges. The surface charge densities have the same magnitudes. Then the electric field at any point is



$$\vec{E} = \vec{E}_{\text{from left plate}} + \vec{E}_{\text{from right plate}}.$$

Quiz 2

This gives:

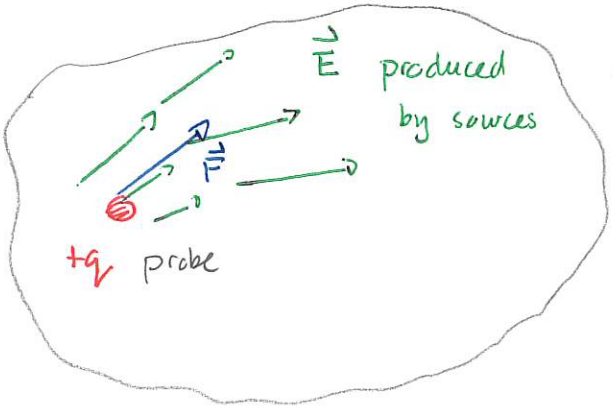
$$\vec{E} = \begin{cases} \dfrac{\eta}{\epsilon_0} & \text{from positive to negative plate} \\ 0 & \text{outside.} \end{cases}$$

Motion of charges in fields

Consider a probe charge in an electric field produced by (other) sources. How will the probe move

DEMO: PhET Electric Field Hockey

We can use Newton's mechanics to answer.



→ Force exerted by field $\vec{F}_{elec} = q_{probe}\vec{E}$

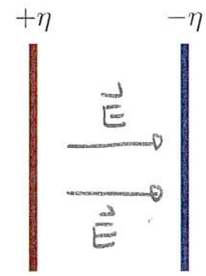
→ Newton's 2nd Law $\vec{F}_{net} = m\vec{a}$

If no other forces $\vec{F}_{elec} = m\vec{a}$

→ Get acceleration → Use kinematics to decide motion

Warm Up 2

46 Proton between parallel capacitor plates



A parallel plate capacitor has plates separated by 0.0020 m. The plates are uniformly charged as illustrated with magnitude $\eta = 6.0 \times 10^{-12}\,\mathrm{C/m^2}$. They are large enough that the field between the plates can be regarded as being produced by infinitely large plates. A proton is released from rest midway between the plates. Ignore gravity in this problem.

a) Describe the trajectory of the proton.

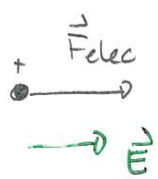
b) Determine how long it takes for the proton to hit a plate after release.

Answer: a) The field points right and is uniform. Thus the force is constant and the acceleration is constant in the direction from the positive to negative plate

b) Strategy

Newton's 2nd Law → Acceleration → Constant acceleration kinematics

① Newton's 2nd Law



$$\vec{F}_{net} = m\vec{a}$$

$$F_{net\,x} = m a_x \quad \Rightarrow \quad q_{proton} E_x = m a_x$$

Now $E_x = \eta/\epsilon_0 \quad \Rightarrow \quad q\,\eta/\epsilon_0 = m a_x \quad \Rightarrow \quad a_x = \dfrac{q\eta}{m\epsilon_0}$

This gives acceleration $a_x = \dfrac{1.6\times10^{-19}\,\mathrm{C} \times 6.0\times10^{-12}\,\mathrm{C/m^2}}{1.67\times10^{-27}\,\mathrm{kg}\ \ 8.85\times10^{-12}\,\mathrm{C^2/Nm^2}} = 6.5\times10^{7}\,\mathrm{m/s^2}$

Now rest ⊚ - - - - - - - - - o

$t=0$ $\quad$ $t=?$

$x_i=0$ $\quad$ $x_f = 0.0020m/2 = 0.0010m$

$V_{ix}=0$ $\quad$ $V_{fx}=?$

$a_x = 6.5\times10^7 m/s$.

$$x_f = \cancel{x_i} + \cancel{V_{ix}} t + \frac{1}{2} a_x t^2 \Rightarrow \frac{2x_f}{a_x} = t^2 \Rightarrow t = \sqrt{\frac{2x_f}{a_x}}$$

Thus $t = \sqrt{\frac{2\times 0.0010m}{6.5\times10^7 m/s}} \Rightarrow t = 5.5\times10^{-6}s$

Note that the speed upon reaching the further plate is determined by

$$V_{fx}^2 = \cancel{V_{ix}^2} + 2a\Delta x$$

$$V_{fx}^2 = 2\times6.5\times10^7 m/s \times 0.0010m \Rightarrow V_{fx} = 360 m/s$$

This is much less then the speed of light. $\boxed{\Sigma}$

Quiz 3