Phys 132 Sp 2026

Lecture 6

Mon: Warm Up 3

Tues: Discussion / quiz

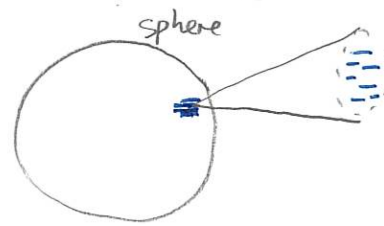
Ex: 31, 36, 38, 39, 40, 41
↑ challenging

Electric field produced by a continuous charge distribution

Coulomb's law provides the fundamental rule for determining the electric field produced by any collection of point charges. Given that all charge originates from subatomic point charges this is in principle enough to calculate any electric field produced by stationary charges.

Quiz 1

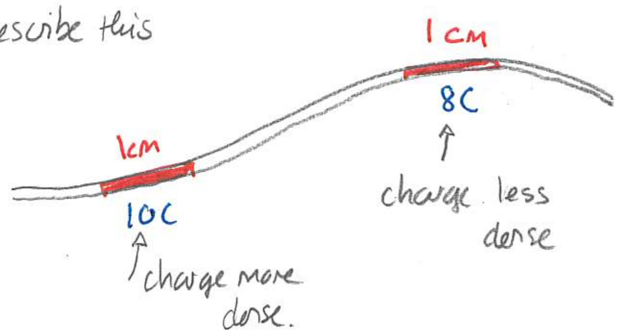
Treating this as a collection of point charges will result in a vast number of electric field vector calculations. In situations like this the number of charges per cubic meters is so large that we could effectively assume that charge is continuously distributed.



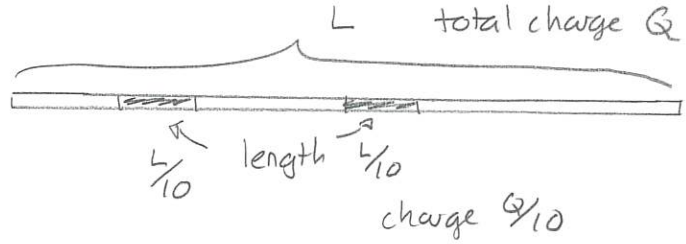
DEMO: Falstad Conducting Planes
- show field vectors.

One dimensional charge distributions

The simplest situation is a one dimensional wire with negligible cross section. The first step will be to describe how the charge is distribution. In general it will not be uniform and we need to describe this and we will use linear charge density to describe how the charge is distributed



Consider a straight wire with length L where the charge is uniformly distributed.



Then any two segments of the same length have the same charge. So the charge per unit length is the same everywhere. In this case we define

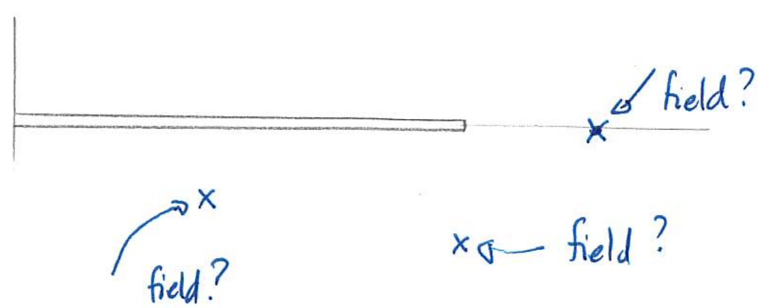
> The linear charge density
>
> $$\lambda = Q/L$$
>
> uniformly distributed
>
> where Q = total charge
> L = total length
>
> units C/m

Quiz 2

Quiz 3

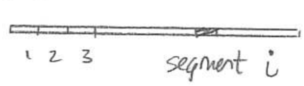
We can calculate the electric field at various locations

We will focus on the field along the axis

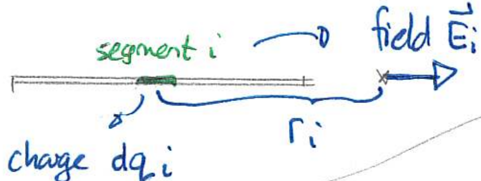


The strategy is:

Break wire into many small segments



(b) Treat each segment as a point charge and use Coulomb's law to determine the field it contributes



→ Add contributions from all segments

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$$

$$= \sum_{\text{all segments}} \vec{E}_i$$

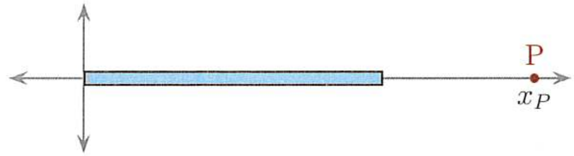
approximation

→ Improve approximation by

* increasing number of segments (smaller size)
* replace addition with integration

37 Field of a uniformly charged rod

A uniformly charged rod is oriented as illustrated. The length of the rod is L and the total charge (positive) is Q. *(132S22 Class)*



a) As an example, suppose that the length of the rod is 2.0 m and the total charge is 60 C. Determine the charge density and use it to determine the charge in any segment of the rod with length 0.010 m. Repeat this for a segment with length 0.0050 m.

b) Consider a segment from $x \to x+\mathrm{d}x$ where x is some point in the rod. Write expressions for the length of the segment and the charge it contains (in terms of $\mathrm{d}x$ and λ).

c) Indicate the field produced by this segment and determine an expression for its components (in terms of $x_p, x, \mathrm{d}x$ and λ).

d) Write an expression (eventually in terms of calculus) for the components of the net electric field produced by all segments.

Answer: a) $\lambda = Q/L = \dfrac{60C}{2.0m} = 30C/m.$ $\lambda = 30C/m$

For segment of length Δx the charge is Δq and

$$\frac{\Delta q}{\Delta x} = \lambda \Rightarrow \Delta q = \lambda \Delta x$$

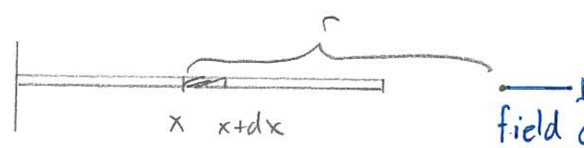
So for $\Delta x = 0.010m \Rightarrow \Delta q = 30C/m \times 0.010m = 0.30C$

$\Delta x = 0.005m \Rightarrow \Delta q = 30C/m \times 0.005m = 0.15C.$

b) the length is $x + dx - x = dx$

the charge is $dq = \lambda dx.$

c) Here



field $d\vec{E}$ or $\vec{E}_i$

$$\vec{E}_i = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{\imath}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{r^2}\hat{\imath}$$

But $r = x_p - x$. Thus

$$E_{ix} = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{(x_p - x)^2}dx$$

$$E_{iy} = 0$$

d) $E_x = \sum_{\text{all segments } i} E_{ix}$ $\Rightarrow E_x = \int_{x=0}^{x=L} \underbrace{\frac{1}{4\pi\epsilon_0}\frac{\lambda}{(x_p-x)^2}dx}_{\text{segment at } x \to x+dx}$

$E_y = 0$

So

$$\boxed{E_x = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(x_p-x)^2}}$$

The integral is done using a "u-substitution". Here $u = x_p - x$

$\Rightarrow du = -dx$

$x=0 \Rightarrow u = x_p$
$x=L \Rightarrow u = x_p - L$

Thus

$$E_x = \frac{-\lambda}{4\pi\epsilon_0}\int_{x_p}^{x_p-L}\frac{du}{u^2} = +\frac{\lambda}{4\pi\epsilon_0}\frac{1}{u}\Big|_{x_p}^{x_p-L}$$

$$\Rightarrow E_x = +\frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{x_p-L} - \frac{1}{x_p}\right] = \frac{\lambda}{4\pi\epsilon_0}\frac{x_p-(x_p-L)}{x_p(x_p-L)}$$

$$\Rightarrow E_x = \frac{\lambda}{4\pi\epsilon_0}\frac{L}{x_p(x_p-L)}$$

Then $\lambda = Q/L$ gives

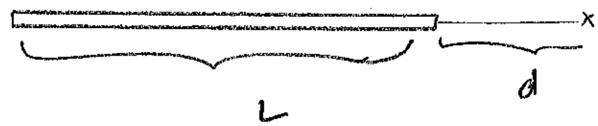
$$E_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{x_p(x_p-L)} \Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x_p(x_p-L)}\hat{\imath}}$$

We can check by considering the field point very far from the wire.
Then $x_p \gg L \Rightarrow x_p - L \approx x_p \Rightarrow \vec{E} \approx \frac{1}{4\pi\epsilon_0}\frac{Q}{x_p^2}\hat{\imath}$

This is exactly the field produced by a source point charge. At far distances the wire would effectively resemble a ~~source~~ point source.

Note that we can write this in terms of distance from the end of the rod since

$$x_p = L + d.$$



Then

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{(L+d)d}\,\hat{\imath}$$