Phys 396
Fall 2026

Quantum Information: Homework 3

Due: 4 September 2026

1 Stern-Gerlach measurement outcomes and subsequent states.

a) A Stern-Gerlach apparatus is oriented in the direction $\hat{\mathbf{n}} = (\hat{\mathbf{x}} + \hat{\boldsymbol{z}})/\sqrt{2}$. Express, as a superposition of $|+\hat{\boldsymbol{z}}\rangle$ and $|-\hat{\boldsymbol{z}}\rangle$, the state which emerges from the measuring apparatus if it gave measurement outcome $S_n = +\hbar/2$. Repeat this for a particle emerging from the device if it yielded $S_n = -\hbar/2$. Demonstrate that the two states are orthogonal.

b) A Stern-Gerlach apparatus is oriented in the direction $\hat{\mathbf{n}} = (\hat{\mathbf{y}} + \hat{\boldsymbol{z}})/\sqrt{2}$. Express, as a superposition of $|+\hat{\boldsymbol{z}}\rangle$ and $|-\hat{\boldsymbol{z}}\rangle$, the state which emerges from the measuring apparatus if it gave measurement outcome $S_n = +\hbar/2$. Repeat this for a particle emerging from the device if it yielded $S_n = -\hbar/2$. Demonstrate that the two states are orthogonal.

2 States and subsequent Stern-Gerlach measurements

A particle in the state

$$|+\hat{\boldsymbol{n}}\rangle = \frac{1}{\sqrt{2}}|+\hat{\boldsymbol{z}}\rangle + \frac{3-4i}{5\sqrt{2}}|-\hat{\boldsymbol{z}}\rangle .$$

a) Use $\langle +\hat{\mathbf{y}}|$ to determine the probability $\mathrm{Prob}(S_y = +\hbar/2)$.

b) Use $\langle -\hat{\mathbf{y}}|$ to determine the probability $\mathrm{Prob}(S_y = -\hbar/2)$.

3 Successive Stern-Gerlach measurements

Suppose that $\hat{\mathbf{m}} = (\hat{\mathbf{y}} + \hat{\mathbf{z}})/\sqrt{2}$ and a particle is usbjected to a SG $\hat{\boldsymbol{m}}$ measurement. In one instance it emerges with $S_m = +\hbar/2$. After emerging, this particle is then subjected to an SG $\hat{\boldsymbol{n}}$ measuring device where $\hat{\mathbf{n}} = (\hat{\mathbf{x}} - \hat{\mathbf{y}})/\sqrt{2}$. Determine expressions for $\langle +\hat{\mathbf{n}}|$ and $\langle -\hat{\mathbf{n}}|$ and use these to compute the probabilities with which the outcomes $S_n = +\hbar/2$, and $S_n = -\hbar/2$ occur.

4 Global phase and measurement probabilities

A generic state of a spin-1/2 particle is

$$|\psi_1\rangle = a_+|+\hat{\boldsymbol{z}}\rangle + a_-|-\hat{\boldsymbol{z}}\rangle$$

where a_+ and a_- are complex numbers.

a) Suppose that the particle is subjected to a SG $\hat{\boldsymbol{z}}$ measurement. Explain using the probabilities of the two outcomes, why it is necessary that the state be normalized.

b) Suppose that a particle is in the state

$$|\psi_2\rangle = e^{i\alpha} |\psi_1\rangle$$

where α is real. Suppose that the particle is subjected to a SG $\hat{\boldsymbol{z}}$ measurement. Determine the probabilities of the measurement outcomes and show that they are independent of α.

The overall factor $e^{i\alpha}$ that multiples the entire state is called a *global phase*.

c) Suppose that a particle is in the state

$$|\psi_2\rangle = e^{i\alpha} |\psi_1\rangle$$

where α is real and this is subjected to SG $\hat{\boldsymbol{n}}$ for any direction $\hat{\mathbf{n}}$. Do the probabilities of the measurement outcomes depend on α? Explain your answer.

5 Measurement operators for SG $\hat{\boldsymbol{y}}$

a) Determine matrices for the measurement operators for the SG $\hat{\boldsymbol{y}}$ measurement.

b) Show that the two operators give the correct probabilities for a particle in state $|+\hat{.}\rangle$

6 Measurement operators and outcomes

Suppose that a single qubit is measured in the basis

$$\left\{ \frac{1}{\sqrt{5}} |+\hat{\boldsymbol{z}}\rangle + \frac{2i}{\sqrt{5}} |-\hat{\boldsymbol{z}}\rangle , \frac{2}{\sqrt{5}} |+\hat{\boldsymbol{z}}\rangle - \frac{i}{\sqrt{5}} |-\hat{\boldsymbol{z}}\rangle \right\}.$$

a) Construct matrices for the two measurement operators.

b) Verify that $\sum \hat{\Pi}_i = \hat{I}$.

c) Verify that $\hat{\Pi}_i^2 = \hat{\Pi}_i$ for each operator.

d) Verify that $\hat{\Pi}_i \hat{\Pi}_j = 0$ if $i \neq j$.

e) Suppose that a spin-1/2 particle is in the state $|-\hat{\boldsymbol{x}}\rangle$ prior to measurement. Determine the probabilities of the two measurement outcomes.

7 News item

Find a news item published within the last year that describes an advance in quantum computing or quantum information. Post a link to the item in the discussion thread for 4 September. Summarize (here) in a single paragraph what the article describes.