Phys 396
Fall 2026

Quantum Information: Homework 2

Due: 28 August 2026

1 Ket operations

Consider the kets

$$\begin{aligned} |u\rangle &= 5\,|0\rangle + 12i\,|1\rangle \text{ and} \\ |v\rangle &= 3\,|0\rangle - 4i\,|1\rangle\,. \end{aligned}$$

a) Determine expressions of $\langle u|$ and $\langle v|$ in terms of $\langle 0|$ and $\langle 1|$.
b) Determine $\langle u|u\rangle$.
c) Determine $\langle u|v\rangle$.
d) Determine $\langle v|u\rangle$.
e) Determine $\langle v|v\rangle$.
f) Show how to normalize each ket.

2 Further ket operations

Consider the kets

$$\begin{aligned} |u_0\rangle &= \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}\,|1\rangle \text{ and} \\ |u_1\rangle &= \sin\left(\frac{\theta}{2}\right)|0\rangle - \cos\left(\frac{\theta}{2}\right)e^{i\phi}\,|1\rangle \end{aligned}$$

where θ and ϕ are real.

a) Show that these are orthonormal.
b) Express $|0\rangle$ as a combination of $|u_0\rangle$ and $|u_1\rangle$.

3 Basis kets

Consider the pair of kets

$$\begin{aligned} |u_0\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\,, \text{ and} \\ |u_1\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle \end{aligned}$$

and also the pair of kets

$$\begin{aligned} |v_0\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle + \frac{i}{\sqrt{2}}\,|1\rangle\,, \text{ and} \\ |v_1\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle - \frac{i}{\sqrt{2}}\,|1\rangle\,. \end{aligned}$$

Each set forms a commonly encountered basis for all kets.

a) Show that each of $|u_0\rangle$ and $|u_1\rangle$ is normalized and show that these are orthogonal.

b) Show that each of $|v_0\rangle$ and $|v_1\rangle$ is normalized and show that these are orthogonal.

c) Express $|0\rangle$ and $|1\rangle$ as linear combinations of $|u_1\rangle$ and $|u_1\rangle$. That is, write each in the form:

$$|0\rangle = \text{number}\,|u_0\rangle + \text{number}\,|u_1\rangle\,.$$

d) Express $|0\rangle$ and $|1\rangle$ as linear combinations of $|v_0\rangle$ and $|v_1\rangle$.

e) Let $|w\rangle = 4\,|0\rangle + 3i\,|1\rangle$. Express this in the basis $\{|u_0\rangle\,, |u_1\rangle\}$. That is write

$$|w\rangle = \text{number}\,|u_0\rangle + \text{number}\,|u_1\rangle\,.$$

f) Express $|w\rangle = 4\,|0\rangle + 3i\,|1\rangle$ in the basis $\{|v_0\rangle\,, |v_1\rangle\}$.

4 Matrix operations on kets

Let

$$\hat{R} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}.$$

a) For each of the following kets, $|v_j\rangle$ express $\hat{R}\,|v_j\rangle$ in terms of $|0\rangle$ and $|1\rangle$:

$$\begin{aligned}
|v_1\rangle &= |0\rangle\,, \\
|v_2\rangle &= |1\rangle\,, \\
|v_3\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\,, \\
|v_4\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}\,|1\rangle\,, \\
|v_5\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle + \frac{i}{\sqrt{2}}\,|1\rangle\,, \text{ and} \\
|v_6\rangle &= \frac{1}{\sqrt{2}}\,|0\rangle - \frac{i}{\sqrt{2}}\,|1\rangle\,.
\end{aligned}$$

b) Choose any pair of kets from the list of $|v_j\rangle$ such that the kets are orthogonal. Show that the pair of kets that results form the operation of $\hat{R}$ on your choice is also orthogonal.

c) Repeat the previous part for a different part of orthogonal kets.

5 Unitary operations

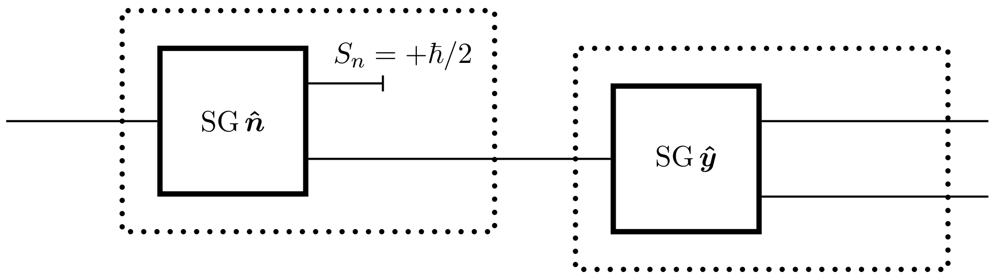
Consider

$$\hat{R} = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & i\sin\left(\frac{\theta}{2}\right) \\ i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

where θ is real. Check whether this is unitary.

6 Repeated Stern-Gerlach experiments

Two Stern-Gerlach devices are arranged as illustrated. The first is oriented in the direction $\hat{\mathbf{n}} = (\hat{\mathbf{x}} + 2\hat{\mathbf{y}})/\sqrt{5}$ and is such that the beam corresponding to $S_n = +\hbar/2$ is blocked. The second is oriented along the direction $\hat{\mathbf{y}}$. Neither of its outgoing beams are blocked.



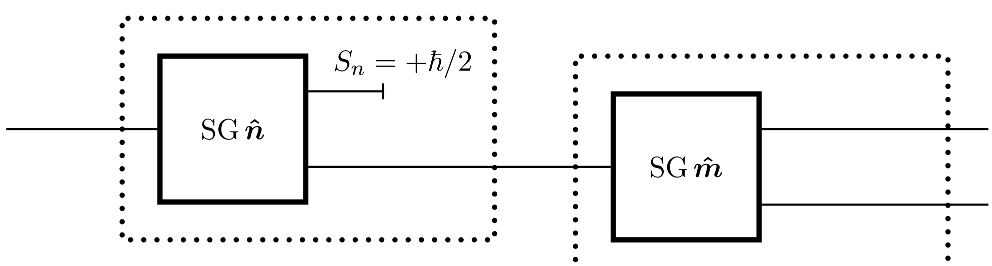
a) Suppose that a spin-1/2 particle initially in the state $|+\hat{\boldsymbol{z}}\rangle$ is subjected to the first device. Determine the probability with which it emerges from the leftmost dotted box.

b) Consider a particle that emerges from the first device and which is then subjected to the second Stern-Gerlach apparatus. List the possible outcomes of the second measurement, S_y, and the probabilities with which they will occur (for a particle which emerges from the first device with 100% certainty).

c) Suppose that a spin-1/2 particle is initially in the state $|+\hat{\boldsymbol{z}}\rangle$ prior to the first measurement. Determine the probability with which it yields $S_y = +\hbar/2$ after the second device. Determine the probability with which it yields $S_y = -\hbar/2$ after the second device. Do these probabilities add to 1? If not, why not?

7 Blocking repeated Stern-Gerlach experiments

Two Stern-Gerlach devices are arranged as illustrated. The first is oriented in the direction $\hat{\mathbf{n}} = (4\hat{\mathbf{x}} + 3\hat{\mathbf{y}})/5$ and is such that the beam corresponding to $S_n = +\hbar/2$ is blocked. The second is oriented along the direction $\hat{\mathbf{m}}$. Neither of its outgoing beams are blocked.



a) Explain the choice of direction $\hat{\mathbf{m}}$ such that no particles emerge from the lower beam of the second Stern-Gerlach apparatus.

b) Explain the choice of direction $\hat{\mathbf{m}}$ such that particle are equally likely to emerge from either output of the second Stern-Gerlach apparatus.

8 News item

Find a news item published within the last year that describes an advance in quantum computing or quantum information. Post a link to the item in the discussion thread for August 28, 2026. Summarize (here) in a single paragraph what the article describes.