Quantum Information: Class Exam II

13 November 2018

Name: _____ **Total:** **/60**

Instructions

- There are 5 questions on 8 pages.
- Show your reasoning and calculations and always explain your answers.

Physical constants and useful formulae

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\sigma}_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Question 1

Consider the evolution described by

$$\hat{U} = e^{-i\hat{\sigma}_y \varphi/2}.$$

a) Determine the matrix that represents $\hat{U}$ (in the standard $\{|0\rangle, |1\rangle\}$ basis).

Question 1 continued ...

b) Suppose that immediately prior to evolution the system is in the state $|0\rangle$. Determine φ so that immediately after the evolution, the system will be in the state $|1\rangle$.

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Question 2

Verify that

$$\hat{U} := |0\rangle \langle 0| \otimes \hat{I} + |1\rangle \langle 1| \otimes \hat{\sigma}_y$$

is unitary.

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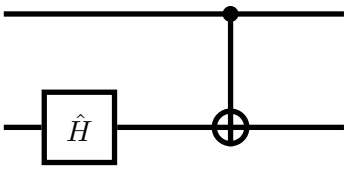
Question 3

Consider the illustrated circuit where $\hat{H}$ is a Hadamard gate.



a) Explain whether it is possible for this circuit to act on a system whose state is $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

b) Determine the effect of this circuit on each of the four computational basis states: $|00\rangle, |01\rangle, |10\rangle, |11\rangle$.

Question 3 continued ...

c) Suppose that the input state is

$$|\Psi\rangle := \frac{1}{2}\big(|00\rangle - |01\rangle + |10\rangle - |11\rangle\big) = \frac{1}{2}\big(|0\rangle + |1\rangle\big)\big(|0\rangle - |1\rangle\big).$$

Determine the output state produced by this circuit and verify whether it is entangled or not.

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Question 4

Consider a function that maps two bits to a single bit according to $f(x_1, x_0) = x_0 \oplus 1$. An oracle that corresponds to this function maps three qubits to three qubits and is defined via the computational basis map

$$\hat{U}_f \left|x_1 x_0\right\rangle \left|y\right\rangle := \left|x_1 x_0\right\rangle \left|y \oplus f(x_1, x_0)\right\rangle$$

where $x_0, x_1, y = 0, 1$. Provide a three qubit circuit for this oracle using CNOT and single qubit gates.

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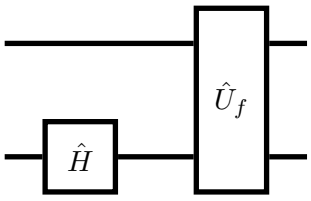
Question 5

Let f be a function that maps a single bit to a single bit and let $\hat{U}_f$ be the oracle defined on single qubit computational basis states as

$$\hat{U}_f \left|x\right\rangle \left|y\right\rangle := \left|x\right\rangle \left|y \oplus f(x)\right\rangle .$$

The lower (function register) qubit is initially in the state $\left|0\right\rangle$.



a) Suppose that f is constant. Determine the state after the oracle for any $x = 0$ or 1. Does this state depend on the particular constant function?

Question 5 continued ...

b) Suppose that $f(x) = x$. Is the state of the system after the oracle any different to that for constant f? Explain your answer.

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