Phys 396
Fall 2018

Quantum Information: Class Exam 1

9 October 2018

Name: _____ **Total:** **/70**

Instructions

- There are 5 questions on 8 pages.
- Show your reasoning and calculations and always explain your answers.

Question 1

Consider two binary digits, a and b. Let the NAND of these be denoted $\mathsf{NAND}(a, b)$. Show that for all a and b,

$$\mathsf{NAND}(a, b) = 1 \oplus (ab).$$

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Question 2

A spin-1/2 particle is subjected to a Stern-Gerlach measurement where the apparatus measures the spin component along the direction $\hat{\mathbf{n}} = \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}} + \frac{1}{2}\,\hat{\mathbf{z}}$. The outcome of the measurement is $S_n = +\hbar/2$

a) Express the state of the particle after this measurement in the basis $\{|+\hat{\boldsymbol{z}}\rangle\,, |-\hat{\boldsymbol{z}}\rangle\}$.

b) Immediately after this measurement the particle is subjected Stern-Gerlach measurement where the apparatus measures the spin component along the direction $\hat{\mathbf{y}}$. List the possible outcomes of this measurement and the probabilities with which they occur.

Question 2 continued ...

c) Immediately after the SG $\hat{\boldsymbol{y}}$ measurement the particle is subjected Stern-Gerlach measurement where the apparatus measures the spin component along the original direction $\hat{\mathbf{n}}$. Will the outcome $S_n = +\hbar/2$ occur again with certainty? Explain your answer.

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Question 3

Answer **either part a) or part b) for full credit.** If you answer both, each will be graded and you will be awarded the highest score that you obtained for one of the parts.

a) Consider the following polarization state of a single photon:

$$|\psi\rangle = \frac{1}{5}\Big(3i\,|{\rightarrow}\rangle + 4\,|{\uparrow}\rangle\Big).$$

Determine an expression, in terms of $\Big\{|{\rightarrow}\rangle\,, |{\uparrow}\rangle\Big\}$, for a normalized state that is orthogonal to $|\psi\rangle$.

Question 3 continued ...

b) Two possible single qubit measurements are proposed. For each measurement the two associated states are listed as follows.

$$\text{Possible measurement A:} \quad \left\{ \frac{1}{\sqrt{5}} \Big(|0\rangle - 2\,|1\rangle \Big), \frac{1}{\sqrt{5}} \left(2\,|0\rangle + |1\rangle \right) \right\}$$

$$\text{Possible measurement B:} \quad \left\{ \frac{1}{\sqrt{5}} \Big(|0\rangle - 2i\,|1\rangle \Big), \frac{1}{\sqrt{5}} \left(|0\rangle + 2i\,|1\rangle \right) \right\}$$

Determine whether each of these could represent a measurement. Explain your answer.

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Question 4

Two qubits are in the state

$$|\Psi\rangle = \frac{1}{2}\Big(|00\rangle - |01\rangle - |10\rangle + i\,|11\rangle\Big).$$

a) Determine whether this is an entangled state or a product state.

b) The "left" qubit is measured in the basis

$$\left\{\frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big), \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right\}$$

and the "right" qubit in the basis

$$\Big\{|0\rangle\,, |1\rangle\Big\}$$

Determine the probability with which all four measurement outcomes occur.

Question 4 continued ...

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Question 5

Consider a single qubit measurement in the basis

$$\left\{ \frac{1}{\sqrt{2}} \Big(|0\rangle + i\,|1\rangle \Big), \frac{1}{\sqrt{2}} \left(|0\rangle - i\,|1\rangle \right) \right\}$$

Construct the two associated projector operators and, for each, verify that $\hat{P}^2 = \hat{P}$.

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