Electromagnetic Theory I: Homework 4

Due: 1 September 2026

This assignment will be graded immediately after the due date. If you get all problems correct, then you will receive 100%. If you have made any errors, then I will deduct 10%, point the errors out and you must submit a corrected assignment by 8 September 2026. If there are still errors, then I will deduct another 10% and you must submit the corrected assignment by 15 September 2026. This will continue until you **have solved every problem correctly.**

1 Charge density

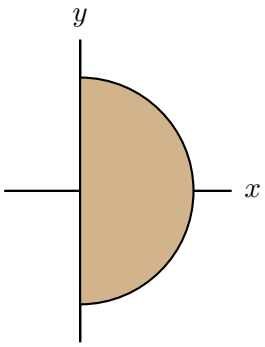
Charge is distributed with density

$$\rho = \frac{q}{a^4}\, x$$

where q is a constant with dimensions of charge.

a) Determine the total charge in the cube with sides of length a and corners at $(0,0,0)$, $(0,0,a)$, $(0,a,0)$, $(0,a,a)$, $(a,0,0)$, $(a,0,a)$, $(a,a,0)$, (a,a,a).

b) Determine $\int \rho \, \mathrm{d}\tau$ for the region which consists of half of a cylinder that is parallel to the z axis, ranges from $z = 0$ to $z = a$, has radius a and whose base in the xy plane is as illustrated.



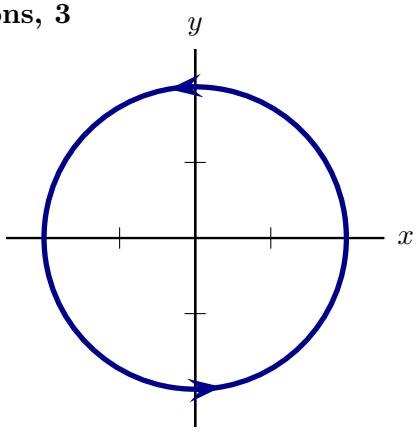
c) Explain, without integrating, what the total charge would be in a cylinder of the type described above but whose base is a full circle in the xy plane.

2 Line integrals along a closed loop in two dimensions, 3

Let

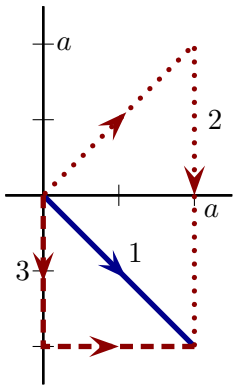
$$\mathbf{u} = y\hat{\mathbf{x}} - x\hat{\mathbf{y}} \quad \text{and}$$
$$\mathbf{v} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$$

Consider the line integral of each along the illustrated circular loop. Without actually computing the line integrals, describe whether the line integral of each vector field will be zero or not. Explain your answers. *Hint: Sketch the vector fields.*



3 Line integrals in two dimensions: straight segments, 4

Let $\mathbf{v} = y^2\hat{\mathbf{x}} - x^2\hat{\mathbf{y}}$. Three paths are indicated in the $z = 0$ plane.



a) Determine the line integral of $\mathbf{v}$ along line 1.

b) Determine the line integral of $\mathbf{v}$ along line 2: in the xy plane with straight line segments $(0, 0) \to (a, a) \to (a, -a)$.

c) Determine the line integral of $\mathbf{v}$ along line 3: in the xy plane with straight line segments $(0, 0) \to (0, -a) \to (a, -a)$.

4 Line integrals in three dimensions

Let $\mathbf{v} = y\hat{\mathbf{x}} - x\hat{\mathbf{y}} + z\hat{\mathbf{z}}$. In this exercise you will compute the line integral along two paths starting at the origin and that end at the point $(2, 2, 2)$.

a) Determine the line integral of $\mathbf{v}$ along the line with three straight segments perpendicular to each other. The first segment runs along the z axis for distance 2. The second starts at the end of the first and runs for distance 2 parallel to the y axis. The third starts at the end of the second and runs parallel to the x axis and suns for distance 2.

b) Determine the line integral of $\mathbf{v}$ along the line with a single straight segment from $(0, 0, 0)$ to $(2, 2, 2)$.

c) Do all line integrals from $(0, 0, 0)$ to $(2, 2, 2)$ give the same result regardless of the path taken? Explain your answer.

5 Fundamental theorem for gradients

Let

$$f(x, y, z) = x^2 - y^2 + z^2.$$

a) Determine $\boldsymbol{\nabla} f$.

b) Determine the line integral of $\boldsymbol{\nabla} f$ over the line consisting of the two straight line segments that run as follows: $(0, 0, 0) \to (1, 3, 0) \to (1, 3, 2)$. Do this by direct integration.

c) Check the fundamental theorem for gradients for this line integral.